

## Closed-form formulas for the electromagnetic parameters of inverted microstrip line

Yamina Bekri<sup>1</sup>, Nasreddine Benahmed<sup>1</sup>, Nadia Benabdallah<sup>2</sup>, Kamila Aliane<sup>1</sup>  
and Fethi Tarik Bendimerad<sup>1</sup>

<sup>1</sup>Department of Telecommunications, University Abou Bekr Belkaid-Tlemcen,

<sup>2</sup>Department of Physics, Preparatory School of Sciences and Technology (EPST-Tlemcen), Tlemcen, Algeria.

**Abstract**—This article presents simple analytical expressions for the electromagnetic parameters (characteristic impedance ( $Z_c$ ), effective dielectric constant ( $\epsilon_{\text{eff}}$ ), inductance (L) and capacitance (C)) of inverted microstrip line (IML). Under quasi-TEM approximation, the analytical expressions can be deduced from rigorous analyses using finite element method (FEM) analysis and curve-fitting techniques. An analysis can be readily implemented in modern CAE software tools for the design of microwave and wireless components. For a dielectric material of  $\epsilon_r=2.22$ , this study presents rigorous and suitable general expressions for all inverted microstrip lines with a wide range of ( $w/h_1$ ) and ( $h_2/h_1$ ) ratios varying respectively between 0.01-9.5 and 0.01-1. An inverted microstrip branch line coupler operating at 3 GHz will be designed to demonstrate the usefulness of these design equations.

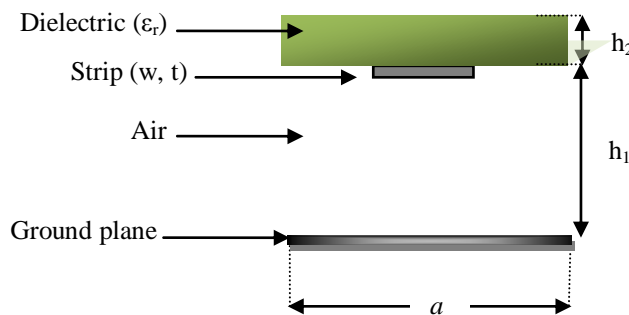
**Keywords**— Analytical expressions, EM parameters, FEM Results, frequency response, inverted microstrip line (IML), inverted microstrip branch line coupler, S-parameters.

### I. INTRODUCTION

Inverted substrate microstrip line (IML) is a very popular transmission media for millimeter and microwave applications. It has low attenuation, small effective dielectric constant, low propagation loss, and low insertion loss. This type of microstrip line is known to offer less stringent dimensional tolerances and provides less dispersion compared with the conventional microstrip lines [1-3]. The inhomogeneous structures may be used advantageously for the development of filters and couplers as compared to those using homogenous structures [4-5].

This article is a continuation of our previous paper that appeared in Computing Science and Technology International Journal [6]. In support of the analysis using FEM method, we developed rigorous and suitable general expressions for IML lines using duroid substrate ( $\epsilon_r=2.22$ ) with a wide range of ( $w/h_1$ ) and ( $h_2/h_1$ ) ratios varying respectively between 0.01-9.5 and 0.01-1.

Figure 1 shows the cross section of the shielded inverted microstrip line (IML). The electrical properties of lossless IML lines can be described in terms of the characteristic impedance ( $Z_c$ ), the effective dielectric constant ( $\epsilon_{\text{eff}}$ ) and the primary (L and C) parameters [7].



**Fig. 1** Cross section of the shielded inverted microstrip line.

Various numerical techniques can be used to determine the electromagnetic parameters (EM) of the IML line [4], [8]. However, they are too time-consuming for direct use in circuit design.

Closed-form analytical models are highly desirable in circuit design [9-11]. This article presents analytical expressions for the EM parameters ( $Z_c$ ,  $\epsilon_{\text{eff}}$ , L and C) of the IML line, deduced from analysis results of the structure by the finite element method (FEM) under freeFEM environment [12], and curve fitting techniques.

It is found that these expressions are suitable for calculating the EM parameters of IML line in a wide range of ( $w/h_1$ ) and ( $h_2/h_1$ ) ratios with a good accuracy.

## II. FEM RESULTS

In order to find the EM parameters of the IML line, we were interested in the analysis of the structure presented in figure 1 having a dielectric material of  $\epsilon_r=2.22$ . We applied the FEM-based numerical tool to the analysis of the IML line with FEM meshes of the structure shown in figure 2.

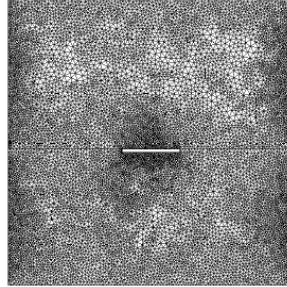


Fig. 2 FEM meshes of the IML line.

For different  $(w/h_1)$  and  $(h_2/h_1)$  ratios varying respectively between 0.01-9.5 and 0.01-1, the obtained results by the finite element method (FEM) are shown in figures 3 to 6.

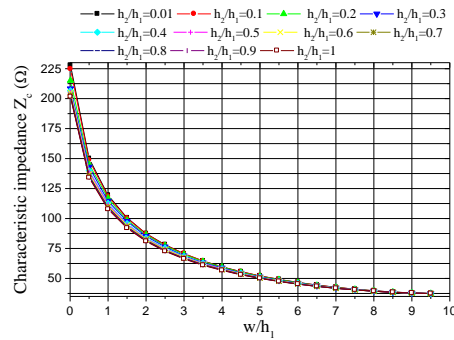


Fig. 3 Characteristic impedance of the IML as a function of  $w/h_1$  for various values of  $h_2/h_1$ .

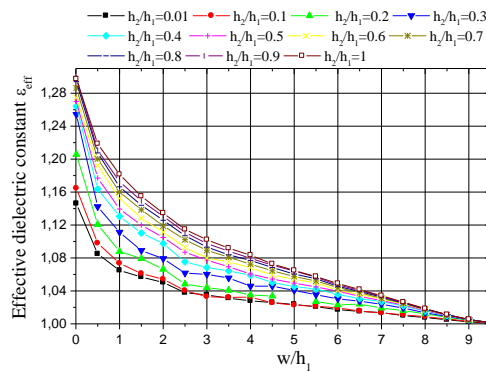


Fig. 4 Effective dielectric constant of the IML as a function of  $w/h_1$  for various values of  $h_2/h_1$ .

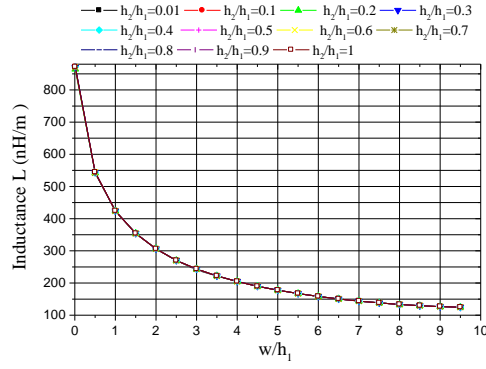


Fig. 5 Inductance of the IML as a function of  $w/h_1$  for various values of  $h_2/h_1$ .

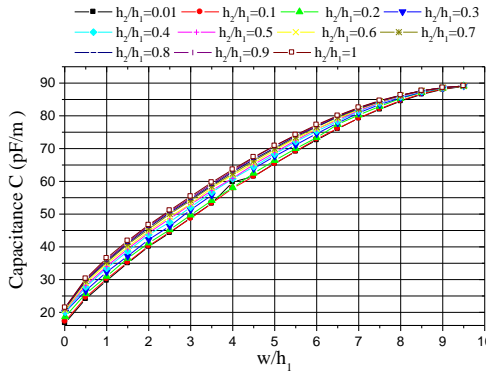


Fig. 6 Capacitance of the IML as a function of  $w/h_1$  for various values of  $h_2/h_1$ .

The obtained results show that the curves of figures 3 to 6 permit the design of IML lines at characteristic impedances varying between 35 and 225  $\Omega$ .

### III. DERIVATION OF ANALYTICAL EXPRESSIONS

#### A. Characteristic impedance

Using curve-fitting, it is found that the characteristic impedance of the IML can be expressed by:

$$Z_c = Z_{co} + A_1 e^{-u/t_1} + A_2 e^{-u/t_2} \quad (\Omega) \quad (1)$$

Where:

For  $0.01 \leq r < 0.5$

$$Z_{co} = 34.42 - 2.79 r + 11.74 r^2 - 12.05 r^3$$

$$A_1 = 83.32 + 13.47 r - 478.74 r^2 + 905.31 r^3$$

$$A_2 = -1769.1 + 1882.49 e^{-r/86.326}$$

$$t_1 = 0.4 - 0.0125e^{-r/0.165}$$

$$t_2 = 2.7 - 0.026 e^{-r/0.071}$$

$$u = w/h_1 ; r = h_2/h_1$$

For  $0.5 \leq r \leq 1$

$$Z_{co} = 34.73 - 0.33 e^{-(r-0.5)/0.15}$$

$$A_1 = 69.91 + 0.57 e^{-(r-0.5)/0.3}$$

$$A_2 = -64.02 + 166.71 e^{-(r-0.5)/19.7}$$

$$t_1 = 0.365 + 0.02 e^{-(r-0.5)/0.004}$$

$$t_2 = 2.65658 + 0.04347e^{-(r-0.5)/0.06}$$

$$u = w/h_1 ; r = h_2/h_1$$

### B. Effective dielectric constant

The effective dielectric constant can be given by the equation (2):

$$\varepsilon_{eff} = \varepsilon_{eff0} + A_1 e^{-u/t_1} + A_2 e^{-u/t_2} \quad (2)$$

Where:

For  $0.01 \leq r < 0.6$

$$\varepsilon_{eff0} = 0.99 - 0.974r + 17.1r^2 - 106.25r^3 + 257.11r^4 - 212.015r^5$$

$$A_1 = 0.071 + 0.717r - 9.8r^2 + 65.24r^3 - 168.35r^4 + 144.567r^5$$

$$A_2 = 0.0836 + 0.66r - 12.37r^2 + 83.2r^3 - 204.36r^4 + 168.06r^5$$

$$t_1 = 0.382 + 1.94r - 3.0r^2 - 47.81r^3 + 182.85r^4 - 176.95r^5$$

$$t_2 = 5.58 + 100.0r - 1727.58r^2 + 10728.63r^3 - 26237.13r^4 + 21858.7r^5$$

$$u = w/h_1 ; r = h_2/h_1$$

For  $0.6 \leq r \leq 1$

$$\varepsilon_{eff0} = 0.91 - 0.035e^{-(r-0.6)/0.007}$$

$$A_1 = 0.168 - 0.052r$$

$$A_2 = 8.56 - 41.4r + 76.6r^2 - 62.37r^3 + 18.92r^4$$

$$t_1 = 52 - 263.92r + 506.085r^2 - 429.12r^3 + 135.65r^4$$

$$t_2 = 1167.9 - 5725.83r + 10561.9r^2 - 8615.93r^3 + 2621.25r^4$$

$$u = w/h_1 ; r = h_2/h_1$$

### C. Inductance per unit length

The inductance of the IML line in (nH/m) is given by relation (3).

$$L = 115.563 + 352.98 e^{-u/0.364} + 412.57768 e^{-u/2.6} \quad (3)$$

Where:  $u = w/h_1$

### D. Capacitance per unit length

Finally the capacitance of the IML line can be expressed by relations (4) and (5).

For  $0.01 \leq u < 2$

$$C = C_o + A_1 e^{-u/t_1} \left( \frac{pF}{m} \right) \quad (4)$$

Where:

For  $0.01 \leq r < 0.6$

$$C_o = 64.36 - 519.95r + 7108.5r^2 - 32910.3r^3 + 62668.85r^4 - 42324.23r^5$$

$$\begin{aligned}
 A_1 &= -47.72 + 522.56r - 7129.36r^2 + 33595.44r^3 \\
 &\quad - 65159.186r^4 + 44812.66r^5 \\
 t_1 &= 3.0625 - 39r + 516.43r^2 - 2278.1r^3 \\
 &\quad + 4055.54r^4 - 2524.9r^5 \\
 u &= w/h_1 ; r = h_2/h_1 \\
 \text{For } 0.6 \leq r \leq 1 \\
 C_o &= -456.12 + 2491.91r - 4521.63r^2 + 3640.86r^3 \\
 &\quad - 1098r^4 \\
 A_1 &= 354.35 - 1875.8r + 3364.8r^2 - 2678.78r^3 \\
 &\quad + 799.67r^4 \\
 t_1 &= -2.58 + 15r - 15.63r^2 + 4.98r^3 \\
 u &= w/h_1 ; r = h_2/h_1
 \end{aligned}$$

For  $2 \leq u \leq 9.5$

$$C = A + B_1 u + B_2 u^2 \left( \frac{pF}{m} \right) \quad (5)$$

For  $0.01 \leq r < 0.5$

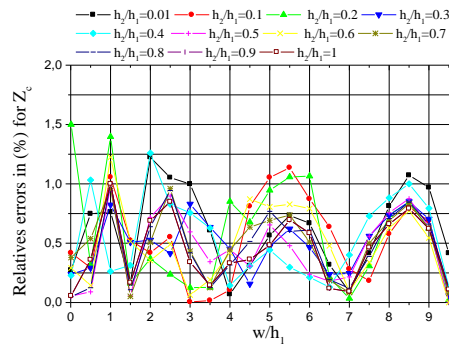
$$\begin{aligned}
 A &= 17.23 - 3.37r + 26.29r^2 \\
 B_1 &= 12.2 - 5.96r + 47.27r^2 - 78.4r^3 \\
 B_2 &= -0.48 + 0.7r - 5.37r^2 + 8.31r^3 \\
 u &= w/h_1 ; r = h_2/h_1
 \end{aligned}$$

For  $0.5 \leq r \leq 1$

$$\begin{aligned}
 A &= 21.345 - 10.0r + 24.62r^2 - 12.12r^3 \\
 B_1 &= 11.35 + 0.99e^{-(r-0.5)/10.524} \\
 B_2 &= -0.57 + 0.036e^{-(r-0.5)/0.22} \\
 u &= w/h_1 ; r = h_2/h_1
 \end{aligned}$$

### E. Comparison between analytical and numerical results

In figures 7 to 10 we show comparisons between our analytical and numerical results for the IML line.



**Fig. 7** Relatives errors between analytical and numerical results for the characteristic impedance of the IML line.

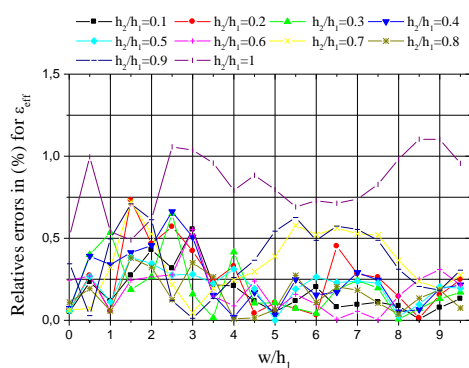


Fig. 8 Relatives errors between analytical and numerical results for the effective dielectric constant of the IML line.

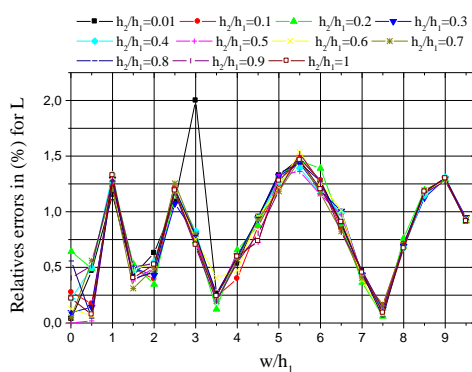


Fig. 9 Relatives errors between analytical and numerical results for the inductance of the IML line.

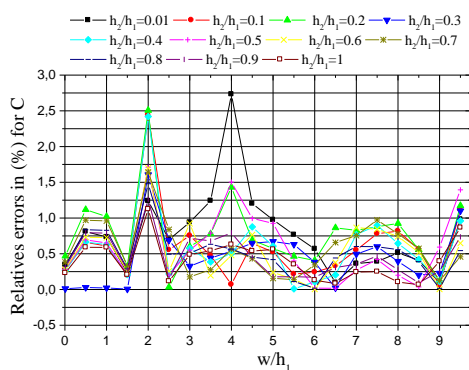


Fig. 10 Relatives errors between analytical and numerical results for the capacitance of the IML line.

From these figures it appears clearly that the relative errors between our analytical and numerical EM-results are less than 2% in a wide range, indicating the good accuracy of the closed-form expressions proposed for the IML line.

#### IV. DESIGN OF A 3 GHz IML BRANCH LINE COUPLER

The results of the proposed analytical expressions were used to design and build an IML branch line coupler operating at 3GHz. All the ports of the coupler are matched with  $Z_c=50\Omega$  (Fig. 11).

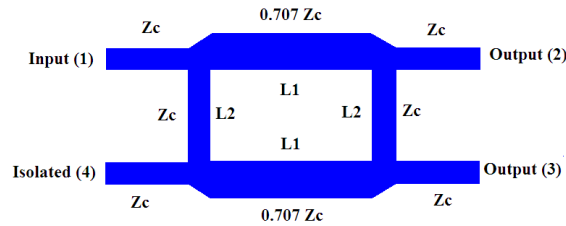


Fig. II Detailed illustration of the IML branch line coupler.

For the IML lines, the dielectric thickness was kept constant. The wide of the strip ( $w$ ) was varied as needed to change the characteristic impedance of the line. All of the dimensions and the EM parameters, obtained from the proposed analytical expressions, for the coupler lines are provided in table 1.

TABLE I. DESIGN PARAMETERS FOR A 3 GHZ BRANCH LINE COUPLER USING IML LINES.

Lines	$L_1$	$L_2$
$h_2/h_1$	0.01	0.01
$w/h_1$	9.5	5.0
$Z_c (\Omega)$	35.5	50.3
$\epsilon_{\text{eff}}$	1.	1.02
$L$ (nH/m)	124.9	177.7
$C$ (pF/m)	99.0	70.0
Length (mm)	24	24

The responses of the designed 3 GHz branch line coupler using IML lines are plotted in figure 12, using an adapted numerical model [13].

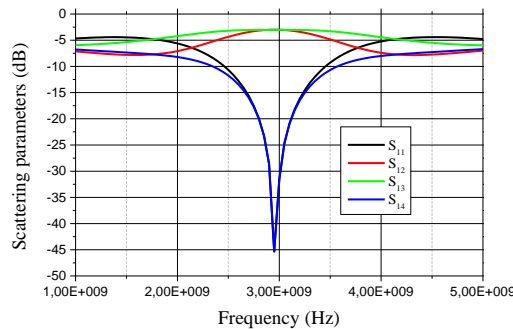


Fig. 12 Simulated S-parameters of the IML branch lines coupler.

Figure 12 shows the simulated S-parameters from 1GHz to 5GHz. The simulated insertion loss of the coupler ( $S_{13}$ ) and direct ( $S_{12}$ ) paths is better than (-5dB) over the 25% bandwidth from 2.5-3.5GHz. Return loss and isolation are better than (-10dB) over this bandwidth.

## V. CONCLUSION

In summary, the closed-form equations presented here provide simple calculations for the EM parameters of inverted microstrip line (IML) used for microwave and wireless components. These expressions deduced from the finite element method are valid in a wide range of ( $w/h_1$ ) and ( $h_2/h_1$ ) ratios varying respectively between 0.01-9.5 and 0.01-1. The formulas were used as the basis for designing an IML branch line coupler operating at 3GHz.

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