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Some Comparative Growth Properties of Entire Functions with Zero Order as Defined by Datta and Biswas

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Abstract—*In this paper we compare the maximum term of composition of two entire functions with their corresponding left and right factors.*
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I. INTRODUCTION, DEFINITIONS AND NOTATIONS

Let f be an entire function defined in the open complex plane $\mathbb{C}$. The maximum term $\mu(r, f)$ of $f = \sum_{n=0}^{\infty} a_n z^n$ on $|z| = r$ is defined by $\mu(r, f) = \max_{n \geq 0}(|a_n| r^n)$. To start our paper we just recall the following definitions.

Definition 1 The order ρ_f and lower order λ_f of an entire function f is defined as follows:

$$\rho_f = \limsup_{r\to\infty} \frac{\log^{[2]} M(r, f)}{\log r} \quad \text{and} \quad \lambda_f = \liminf_{r\to\infty} \frac{\log^{[2]} M(r, f)}{\log r}$$

where

$$\log^{[k]} x = \log(\log^{[k-1]} x) \text{ for } k = 1,2,3,\ldots \text{ and } \log^{[0]} x = x.$$

If $\rho_f < \infty$ then f is of finite order. Also $\rho_f = 0$ means that f is of order zero. In this connection Liao and Yang [2] gave the following definition.

Definition 2 [2]Let f be an entire function of order zero. Then the quantities ρ_f^* and λ_f^* of an entire function f are defined as:

$$\rho_f^* = \limsup_{r\to\infty} \frac{\log^{[2]} M(r, f)}{\log^{[2]} r} \quad \text{and} \quad \lambda_f^* = \liminf_{r\to\infty} \frac{\log^{[2]} M(r, f)}{\log^{[2]} r}.$$

Datta and Biswas [1] gave an alternative definition of zero order and zero lower order of an entire function in the following way:

Definition 3 [1] Let f be an entire function of order zero. Then the quantities ρ_f^{**} and λ_f^{**} of f are defined by:

$$\rho_f^{**} = \limsup_{r\to\infty} \frac{\log M(r, f)}{\log r} \quad \text{and} \quad \lambda_f^{**} = \liminf_{r\to\infty} \frac{\log M(r, f)}{\log r}.$$

Since for $0 \leq r < R,$

$$\mu(r, f) \leq M(r, f) \leq \frac{R}{R-r}\mu(R, f) \qquad \{cf.[4]\} \tag{1}$$

it is easy to see that

$$\rho_f = \limsup_{r\to\infty} \frac{\log^{[2]} \mu(r,f)}{\log r}, \qquad \lambda_f = \liminf_{r\to\infty} \frac{\log^{[2]} \mu(r,f)}{\log r}$$

$$\rho_f^* = \limsup_{r\to\infty} \frac{\log^{[2]} \mu(r,f)}{\log^{[2]} r}, \qquad \lambda_f^* = \liminf_{r\to\infty} \frac{\log^{[2]} \mu(r,f)}{\log^{[2]} r}$$

and

$$\rho_f^{**} = \limsup_{r\to\infty} \frac{\log \mu(r,f)}{\log r}, \qquad \lambda_f^{**} = \liminf_{r\to\infty} \frac{\log \mu(r,f)}{\log r}.$$

In this paper we investigate some aspects of the comparative growths of maximum terms of two entire functions with their corresponding left and right factors. We do not explain the standard notations and definitions on the theory of entire function because those are available in [5].

II. LEMMAS

In this section we present some lemmas which will be needed in the sequel.

Lemma 1 [3] Let f and g be two entire functions with $g(0) = 0.$ Then for all sufficiently large values of r,

$$\mu(r, f\circ g) \geq \frac{1}{4}\mu(\frac{1}{8}\mu(\frac{r}{4}, g) - |g(0)|, f).$$

Lemma 2 [3] Let f and g be two entire functions. Then for every $\alpha > 0$ and $0 < r < R$,

$$\mu(r, f\circ g) \leq \frac{\alpha}{\alpha - 1}\mu(\frac{\alpha R}{R - r}\mu(R, g), f).$$

Lemma 3 [1] Let f and g be two entire functions such that $\rho_f < \infty$ and $\rho_g = 0.$ then $\rho_{fog} < \infty.$

III. THEOREMS

In this section we present the main results of the paper.

Theorem 1 Let f and g be two entire functions with $0 < \lambda_f^{**} \leq \rho_f^{**} < \infty$ and $\lambda_g > 0$. Then for any $A > 0,$

$$\lim_{r\to\infty} \frac{\log \mu(r, f\circ g)}{\log \mu(r^A, f)} = \infty.$$

Proof. In view of Lemma 1 we obtain for all sufficiently large values of r,

$$\log \mu(r, f\circ g) \geq \log\{\frac{1}{4}\mu(\frac{1}{8}\mu(\frac{r}{4}, g), f)\}$$

$$i.e.,\ \log \mu(r, f\circ g) \geq (\lambda_f^{**} - \varepsilon)\log\{\frac{1}{8}\mu(\frac{r}{4}, g)\} + O(1)$$

$$i.e.,\ \log \mu(r, f\circ g) \geq (\lambda_f^{**} - \varepsilon)\log \mu(\frac{r}{4}, g) + O(1)$$

$$i.e.,\ \log \mu(r, f\circ g) \geq (\lambda_f^{**} - \varepsilon)\left(\frac{r}{4}\right)^{\lambda_g - \varepsilon} + O(1). \qquad (2)$$

Again from the definition of ρ_f^{**} we have for arbitrary positive ε and for all sufficiently large values of r,

$$\log \mu(r^A, f) \leq A(\rho_f^{**} + \varepsilon)\ \log r. \qquad (3)$$

Therefore it follows from (2) and (3) for all sufficiently large values of r that

$$\frac{\log \mu(r, f \circ g)}{\log \mu(r^A, f)} \geq \frac{(\lambda_f^{**} - \varepsilon)\left(\frac{r}{4}\right)^{\lambda_g - \varepsilon} + O(1)}{A(\rho_f^{**} + \varepsilon) \log r}$$

$$i.e., \lim_{r \to \infty} \frac{\log \mu(r, f \circ g)}{\log \mu(r^A, f)} = \infty.$$

This completes the proof.

Remark 1 If we take $\rho_g > 0$ instead of $\lambda_g > 0$ in Theorem 1 and the other conditions remain the same, then in the line of Theorem 1 one can easily verify that

$$\limsup_{r \to \infty} \frac{\log \mu(r, f \circ g)}{\log \mu(r^A, f)} = \infty,$$

where $A > 0.$

Remark 2 Also if we consider $0 < \lambda_f^{**} < \infty$ or $0 < \rho_f^{**} < \infty$ instead of $0 < \lambda_f^{**} \leq \rho_f^{**} < \infty$ in Theorem 1 and the other conditions remain the same, then in the line of Theorem 1 one can easily verify that

$$\limsup_{r \to \infty} \frac{\log \mu(r, f \circ g)}{\log \mu(r^A, f)} = \infty,$$

where $A > 0.$

Theorem 2 Let f and g be two an entire functions such that $0 < \lambda_f^{**} < \infty$ and $0 < \lambda_g \leq \rho_g < \infty.$ Then

$$\lim_{r \to \infty} \frac{\log \mu(r, f \circ g)}{\log^{[2]} \mu(r^A, g)} = \infty,$$

where $A = 1,2,3... \ .$

Proof. Since ρ_g is the order of g, for given ε and for all sufficiently large values of r we get from the definition of order that

$$\log^{[2]} \mu(r^A, g) \leq A(\rho_g + \varepsilon) \log r. \tag{4}$$

Now we obtain from (2) and (4) for all sufficiently large values of r that

$$\frac{\log \mu(r, f \circ g)}{\log^{[2]} \mu(r^A, g)} \geq \frac{(\lambda_f^{**} - \varepsilon)\left(\frac{r}{4}\right)^{\lambda_g - \varepsilon} + O(1)}{A(\rho_g + \varepsilon) \log r}. \tag{5}$$

As $\lambda_g > 0$, the theorem follows from (5) .

Remark 3 If we take $0 < \rho_f^{**} < \infty$ instead of $0 < \lambda_f^{**} < \infty$ in Theorem 2 and the other conditions remain the same, then in the line of Theorem 2 one can easily verify that

$$\limsup_{r \to \infty} \frac{\log \mu(r, f \circ g)}{\log^{[2]} \mu(r^A, g)} = \infty,$$

where $A = 1,2,3... \ .$

Remark 4 Also if we consider $0<\lambda_g<\infty$. or $0<\rho_g<\infty$. instead of $0<\lambda_g\le\rho_g<\infty$. in Theorem 2 and the other conditions remain the same, then in the line of Theorem 2 one can easily verify that

$$\limsup_{r\to\infty}\frac{\log\mu(r,f\circ g)}{\log^{[2]}\mu(r^A,g)}=\infty,$$

where $A=1,2,3...$.

Theorem 3 Let f and g be two an entire functions such that $0<\lambda_f\le\rho_f<\infty$. and $\rho_g^{**}<\infty$. Then

$$\limsup_{r\to\infty}\frac{\log^{[2]}\mu(r,f\circ g)}{\log\mu(r^A,f)}=0,$$

where $A=1,2,3...$.

Proof. By Lemma 2 and Lemma 3 we get for all sufficiently large values of r that

$$\begin{aligned}
&\log^{[2]}\mu(r,f\circ g)\le\log^{[2]}\mu(\frac{\alpha R}{R-r}\mu(R,g),f)+O(1)\\
&i.e.,\ \log^{[2]}\mu(r,f\circ g)\le(\rho_f+\varepsilon)\log\left(\frac{\alpha R}{R-r}\mu(R,g)\right)+O(1)\\
&i.e.,\ \log^{[2]}\mu(r,f\circ g)\le(\rho_f+\varepsilon)\log\mu(R,g)+O(1)\\
&i.e.,\ \log^{[2]}\mu(r,f\circ g)\le(\rho_f+\varepsilon)(\rho_g^{**}+\varepsilon)\log R+O(1).
\end{aligned}\tag{6}$$

Again from the definition of λ_f we have for arbitrary positive $\lambda\varepsilon$ and for all sufficiently large values of r,

$$\begin{aligned}
&\log^{[2]}\mu(r^A,f)\ge A(\lambda_f-\varepsilon)\log r\\
&i.e.,\quad \log\mu(r^A,f)\ge r^{A(\lambda_f-\varepsilon)}.
\end{aligned}\tag{7}$$

Therefore it follows from (6) and (7) for all sufficiently large values of r that

$$\frac{\log^{[2]}\mu(r,f\circ g)}{\log\mu(r^A,f)}\le\frac{(\rho_f+\varepsilon)(\rho_g^{**}+\varepsilon)\log R+O(1)}{r^{A(\lambda_f-\varepsilon)}}.\tag{8}$$

As $\lambda_f>0$, the theorem follows from (8) .

Remark 5 If we take $0<\lambda_f<\infty$ or $0<\rho_f<\infty$ instead of $0<\lambda_f\le\rho_f<\infty$ in Theorem 3 and the other conditions remain the same, then in the line of Theorem 3 one can easily verify that

$$\liminf_{r\to\infty}\frac{\log^{[2]}\mu(r,f\circ g)}{\log\mu(r^A,f)}=0,$$

where $A=1,2,3...$.

Remark 6 Also if we take $\lambda_g^{**}<\infty$. instead of $\rho_g^{**}<\infty$. in Theorem 3 and the other conditions remain the same, then in the line of Theorem 3 one can easily verify that

$$\liminf_{r\to\infty}\frac{\log^{[2]}\mu(r,f\circ g)}{\log\mu(r^A,f)}=0,$$

where $A=1,2,3...$.

Theorem 4 Let f and g be two an entire functions such that $0<\lambda_f^{**}\le\rho_f^{**}<\infty$ and $\rho_g^{**}>0$. Then

$$\limsup_{r\to\infty}\frac{\log\mu(r,f\circ g)}{\log\mu(r^A,f)}\ge\frac{\lambda_f^{**}\rho_g^{**}}{A\rho_f^{**}},$$

where $A=1,2,3...$.

Proof. Suppose $0<\varepsilon<\min\{\lambda_f^{**},\rho_g^{**}\}$.
Now from Lemma 1 we have for a sequence of values of r tending to infinity that

$$\log\mu(r,f\circ g)\ge\log\{\frac{1}{4}\mu(\frac{1}{8}\mu(\frac{r}{4},g),f)\}$$

$$i.e.,\ \log\mu(r,f\circ g)\ge(\lambda_f^{**}-\varepsilon)\log\{\frac{1}{8}\mu(\frac{r}{4},g)\}+O(1)$$

$$i.e.,\ \log\mu(r,f\circ g)\ge(\lambda_f^{**}-\varepsilon)\log\frac{1}{8}+(\lambda_f^{**}-\varepsilon)\log\mu(\frac{r}{4},g)+O(1)$$

$$i.e.,\ \log\mu(r,f\circ g)\ge(\lambda_f^{**}-\varepsilon)(\rho_g^{**}-\varepsilon)\log\frac{r}{4}+O(1)$$

$$i.e.,\ \log\mu(r,f\circ g)\ge(\lambda_f^{**}-\varepsilon)(\rho_g^{**}-\varepsilon)\log r+O(1). \tag{9}$$

So combining (3) and (9) we get for a sequence of values of r tending to infinity that

$$\frac{\log\mu(r,f\circ g)}{\log\mu(r^A,f)}\ge\frac{(\lambda_f^{**}-\varepsilon)(\rho_g^{**}-\varepsilon)\log r+O(1)}{A(\rho_f^{**}+\varepsilon)\ \log r}$$

$$i.e.,\ \limsup_{r\to\infty}\frac{\log\mu(r,f\circ g)}{\log\mu(r^A,f)}\ge\frac{\lambda_f^{**}\rho_g^{**}}{A\rho_f^{**}}.$$

This completes the proof.

Remark 7 Under the same conditions of Theorem 4 if $\lambda_f^{**}=\rho_f^{**}$,then

$$\limsup_{r\to\infty}\frac{\log\mu(r,f\circ g)}{\log\mu(r^A,f)}\ge\frac{\rho_g^{**}}{A}.$$

Remark 8 In Theorem 4 if we take $\lambda_g^{**}>0$ instead of $\rho_g^{**}>0$ and the other conditions remain the same then it can be shown that

$$\liminf_{r\to\infty}\frac{\log\mu(r,f\circ g)}{\log\mu(r^A,f)}\ge\frac{\lambda_f^{**}\rho_g^{**}}{A\rho_f^{**}}.$$

In addition if $\lambda_f^{**}=\rho_f^{**}$, then

$$\liminf_{r\to\infty}\frac{\log\mu(r,f\circ g)}{\log\mu(r^A,f)}\ge\frac{\rho_g^{**}}{A}.$$

Remark 9 Also if we consider $0<\lambda_f^{**}<\infty$ or $0<\rho_f^{**}<\infty$ instead of $0<\lambda_f^{**}\le\rho_f^{**}<\infty$ in Theorem 4 and the other conditions remain the same, then one can easily verify that

$$\limsup_{r\to\infty}\frac{\log\mu(r, f\circ g)}{\log\mu(r^A, f)}\geq\frac{\lambda_g^{**}}{A}.$$

Theorem 5 Let f and g be two an entire functions such that $0<\lambda_g^{**}\leq\rho_g^{**}<\infty$ and $\rho_f^{**}>0.$ Then

$$\limsup_{r\to\infty}\frac{\log\mu(r, f\circ g)}{\log\mu(r^A, g)}\geq\frac{\rho_f^{**}\lambda_g^{**}}{A\rho_g^{**}},$$

where $A>0$.

Proof. Let us choose ε in such a way that $0<\varepsilon<\min\{\rho_f^{**},\lambda_g^{**}\}.$
Now from Lemma 1 we have for a sequence of values of r tending to infinity that

$$\log\mu(r, f\circ g)\geq\log\{\frac{1}{4}\mu(\frac{1}{8}\mu(\frac{r}{4},g), f)\}$$
$$i.e.,\ \log\mu(r, f\circ g)\geq(\rho_f^{**}-\varepsilon)\log\{\frac{1}{8}\mu(\frac{r}{4},g)\}+O(1)$$
$$i.e.,\ \log\mu(r, f\circ g)\geq(\rho_f^{**}-\varepsilon)\log\frac{1}{8}+(\rho_f^{**}-\varepsilon)\log\mu(\frac{r}{4},g)+O(1)$$
$$i.e.,\ \log\mu(r, f\circ g)\geq(\rho_f^{**}-\varepsilon)(\lambda_g^{**}-\varepsilon)\log\frac{r}{4}+O(1)$$
$$i.e.,\ \log\mu(r, f\circ g)\geq(\rho_f^{**}-\varepsilon)(\lambda_g^{**}-\varepsilon)\log r+O(1). \tag{10}$$

Also we have for all sufficiently large values of r,

$$\log\mu(r^A, g)\leq A(\rho_g^{**}+\varepsilon)\ \log r. \tag{11}$$

Therefore from (10) and (11) we get for a sequence of values of r tending to infinity that

$$\frac{\log\mu(r, f\circ g)}{\log\mu(r^A, g)}\geq\frac{(\rho_f^{**}-\varepsilon)(\lambda_g^{**}-\varepsilon)\log r+O(1)}{A(\rho_g^{**}+\varepsilon)\ \log r}$$

$$i.e.,\ \limsup_{r\to\infty}\frac{\log\mu(r, f\circ g)}{\log\mu(r^A, g)}\geq\frac{\rho_f^{**}\lambda_g^{**}}{A\rho_g^{**}}.$$

This proves the theorem.

Remark 10 Under the same conditions of Theorem 5 if $\lambda_g^{**}=\rho_g^{**}$, then

$$\limsup_{r\to\infty}\frac{\log\mu(r, f\circ g)}{\log\mu(r^A, g)}\geq\frac{\rho_f^{**}}{A}.$$

Remark 11 In Theorem 5 if we take $\lambda_f^{**}>0.$ instead of $\rho_f^{**}>0.$ and the other conditions remain the same then it can be shown that

$$\liminf_{r\to\infty}\frac{\log\mu(r, f\circ g)}{\log\mu(r^A, g)}\geq\frac{\lambda_f^{**}\lambda_g^{**}}{A\rho_g^{**}},$$

In addition if $\lambda_g^{**}=\rho_g^{**}$, then

$$\liminf_{r\to\infty} \frac{\log \mu(r, f \circ g)}{\log \mu(r^A, g)} \geq \frac{\lambda_f^{**}}{A}.$$

Remark 12 Also if we consider $0 < \lambda_g^{**} < \infty$ or $0 < \rho_g^{**} < \infty$ instead of $0 < \lambda_g^{**} \leq \rho_g^{**} < \infty$ in Theorem 5 and the other conditions remain the same, then one can easily verify that

$$\limsup_{r\to\infty} \frac{\log \mu(r, f \circ g)}{\log \mu(r^A, g)} \geq \frac{\lambda_f^{**}}{A}.$$

Theorem 6 Let f and g be two an entire functions such that $0 < \lambda_f^{**} \leq \rho_f^{**} < \infty$ and $\rho_g^{**} > 0$. Then

$$\limsup_{r\to\infty} \frac{\log \mu(r, f \circ g)}{\log \mu(r^A, f)} \leq \frac{\rho_f^{**} \rho_g^{**}}{A\lambda_f^{**}},$$

where $A = 1,2,3...$.

Proof. From the definition of λ_f^{**} we have for arbitrary positive ε and for all sufficiently large values of r,

$$\log \mu(r^A, f) \geq A(\rho_f^{**} - \varepsilon) \log r. \tag{12}$$

Again by Lemma 2 it follows for all sufficiently large values of r that

$$\log \mu(r, f \circ g) \leq \log \mu\left(\frac{\alpha R}{R-r} \mu(R,g), f\right) + O(1)$$

$$i.e.,\ \log \mu(r, f \circ g) \leq (\rho_f^{**} + \varepsilon) \log\left(\frac{\alpha R}{R-r} \mu(R,g)\right) + O(1)$$

$$i.e.,\ \log \mu(r, f \circ g) \leq (\rho_f^{**} + \varepsilon) \log \mu(R,g) + O(1)$$

$$i.e.,\ \log \mu(r, f \circ g) \leq (\rho_f^{**} + \varepsilon)(\rho_g^{**} + \varepsilon) \log R + O(1). \tag{13}$$

Now combining (12) and (13) we get for all sufficiently large values of r,

$$\frac{\log \mu(r, f \circ g)}{\log \mu(r^A, f)} \leq \frac{(\rho_f^{**} + \varepsilon)(\rho_g^{**} + \varepsilon) \log R + O(1)}{A(\rho_f^{**} - \varepsilon) \log r}$$

$$i.e.,\ \limsup_{r\to\infty} \frac{\log \mu(r, f \circ g)}{\log \mu(r^A, f)} \leq \frac{\rho_f^{**} \rho_g^{**}}{A\lambda_f^{**}}.$$

This completes the proof.

Similarly we may state the following theorem without proof for the right factor g of the composite function $f \circ g$:

Theorem 7 Let f and g be two an entire functions such that $\rho_f^{**} < \infty$ and $0 < \lambda_g^{**} \leq \rho_g^{**} < \infty$. Then

$$\limsup_{r\to\infty} \frac{\log \mu(r, f \circ g)}{\log \mu(r^A, g)} \leq \frac{\rho_f^{**} \rho_g^{**}}{A\lambda_g^{**}},$$

where $A = 1,2,3...$.

Remark 13 Under the same conditions of Theorem 6 if $\lambda_f^{**} = \rho_f^{**}$, then

$$\limsup_{r\to\infty}\frac{\log\mu(r,f\circ g)}{\log\mu(r^A,f)}\le\frac{\rho_g^{**}}{A}.$$

Remark 14 In Theorem 6 if we take $\lambda_g^{**}>0$ instead of $\rho_g^{**}>0$ and the other conditions remain the same then it can be shown that

$$\liminf_{r\to\infty}\frac{\log\mu(r,f\circ g)}{\log\mu(r^A,f)}\le\frac{\rho_f^{**}\lambda_g^{**}}{A\lambda_f^{**}},$$

In addition if $\lambda_f^{**}=\rho_f^{**}$, then

$$\liminf_{r\to\infty}\frac{\log\mu(r,f\circ g)}{\log\mu(r^A,f)}\le\frac{\lambda_g^{**}}{A}.$$

Remark 15 If we take $0<\lambda_f^{**}<\infty$ or $0<\rho_f^{**}<\infty$ instead of $0<\lambda_f^{**}\le\rho_f^{**}<\infty$ in Theorem 6 and the other conditions remain the same, then one can easily verify that

$$\liminf_{r\to\infty}\frac{\log\mu(r,f\circ g)}{\log\mu(r^A,f)}\le\frac{\rho_g^{**}}{A}.$$

Remark 16 Under the same conditions of Theorem 7 if $\lambda_g^{**}=\rho_g^{**}$, then

$$\limsup_{r\to\infty}\frac{\log\mu(r,f\circ g)}{\log\mu(r^A,g)}\le\frac{\rho_f^{**}}{A}.$$

Remark 17 In Theorem 7 if we take $\lambda_f^{**}<\infty$ instead of $\rho_f^{**}<\infty$ and the other conditions remain the same then it can be shown that

$$\liminf_{r\to\infty}\frac{\log\mu(r,f\circ g)}{\log\mu(r^A,g)}\le\frac{\lambda_f^{**}\rho_g^{**}}{A\lambda_g^{**}},$$

In addition if $\lambda_g^{**}=\rho_g^{**}$, then

$$\liminf_{r\to\infty}\frac{\log\mu(r,f\circ g)}{\log\mu(r^A,g)}\le\frac{\lambda_f^{**}}{A}.$$

Remark 18 If we take $0<\lambda_g^{**}<\infty$ or $0<\rho_g^{**}<\infty$ instead of $0<\lambda_g^{**}\le\rho_g^{**}<\infty$ in Theorem 7 and the other conditions remain the same, then one can easily verify that

$$\liminf_{r\to\infty}\frac{\log\mu(r,f\circ g)}{\log\mu(r^A,g)}\le\frac{\rho_f^{**}}{A}.$$

REFERENCES

[1]. S.K. Datta and T. Biswas, *On the definition of a meromorphic function of order zero*, Int. Math. Forum, Vol.4, No. 37(2009) pp.1851-1861.

[2]. L. Liao and C. C. Yang, *On the growth of composite entire functions*, Yokohama Math. J., Vol. 46(1999), pp. 97-107.

[3]. A. P. Singh, *On maximum term of composition of entire functions*, Proc. Nat. Acad. Sci. India, Vol. 59(A), Part I (1989), pp. 103-115.

[4]. A. P. Singh and M. S. Baloria, *On maximum modulus and maximum term of composition of entire functions*, Indian J. Pure Appl. Math., Vol. 22(1991), No 12, pp. 1019-1026.

[5]. G. Valiron, *Lectures on the general theory of integral functions*, Chelsea Publishing Company (1949).