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# Manpower Model for a Single Grade System with Two Sources of Depletion and Two Components for Threshold

K.P.UMA[1], & P.SUDHARANI[2]
[1]*Associate Professor, Dept of Science and Humanities,*
*Hindustan College of Engineering and Technology, Coimbatore, Tamilnadu, India*
[2]*Professor and Head, Dept of Science and Humanities, Adithya Institute of Technology,*
*Coimbatore, Tamilnadu, India.*

**Abstract:-** In this paper, an organization with single grade subjected to exodus of personnel due to policy decisions taken by it, is considered. In order to avoid the crisis of the organization reaching a breakdown point, a suitable univariate policy recruitment based on shock model approach and cumulative damage process, is suggested. A mathematical model is constructed and a performance measure namely the mean and variance of time to recruitment are obtained.

**Keywords:-** Single grade system, Univariate policy of recruitment, shock model.

## I. INTRODUCTION

Frequent wastage or exit of personnel is common in many administrative and production oriented organization. Whenever the organizations announces revised policies regarding sales target, revision of wages, incentives and perquisites, the exodus is possible. Reduction in the total strength of marketing personnel adversely affects the sales turnover in the organization. As the recruitment involves several costs, it is usual that the organization has the natural reluctance to go in for frequent recruitments. Once the total amount of wastage crosses a certain threshold level, the organization reaches an uneconomic status which otherwise be called the breakdown point and recruitment is done at this point of time. The time to attain the breakdown point is an important characteristic for the management of the organization. Many models could be seen in, Barthlomew [**1**] and Barthlomew and Forbes [**2**]Researchers [**3**]and [**4**] have considered the problem of time to recruitment in a marketing organization under different conditions. In an organization, the depletion of manpower can occur due to two different cases:

1) whenever the policy decisions regarding pay, perquisites and work schedule are revised and
2) due to transfer of personnel to the other organization of the same management.

In the presence of these ***two different sources*** of depletion, Elangovan et.al [5] have studied the problem of time to recruitment for an organization consisting of one grade and obtained the variance of the time to recruitment using a univariate policy of recruitment when (i) the loss of man power in the organization due to the two sources of depletion and its threshold are independent and identically exponential random variables. (ii) Inter-policy decision times and inter-transfer decision times form the same renewal process. The Authors [6] have discussed a stochastic model for estimating the expected time to recruitment under the assumptions that i) the depletion of manpower is in terms of persons leaving at every epoch of decision making and at every epoch of transfer and hence they are in terms of discrete random variables. ii) the recruitment is carried out as and when the total depletion crosses a level called the threshold which is a discrete random variable .Usha et.al [7 ] proposed a model to determine the expected time to recruitment time under the assumption that the interarrival times between successive epochs of policy decisions are not independent but correlated, whereas the interarrival times between successive epochs of transfers are i.i.d random variables. Vijaysankar et.al [**8**] have constructed a stochastic model by assuming the threshold with two components namely the level of wastage which can be allowed and the manpower which is available from what is known as backup resource. Recently, Uma and Roja Mary [9] have considered a single grade system which is subjected to exodus of personnel due to policy decisions given by the organization and the explicit expression for the long run average cost is derived by considering survival time process, which is a geometric process and the threshold has two components.

In this paper, it is proposed to determine time to recruitment for a single grade manpower system with two sources of depletion and the renewal processes governing the inter-policy decisions times and that of the inter-transfer decisions times are same. Also it is assumed that threshold for the loss of manpower has two components namely the maximum allowable attrition and the maximum available manpower due to extra time work. The performance measures namely mean time to recruitment and the variances of time to recruitment are derived.

## II. MODEL DESCRIPTION

Consider an organization having single grade in which decisions are taken at random epoch. the depletion of manhours occurs at every decision making epoch and also due to transfer of personnel to the other organization of the same management. It is assumed that the interarrival times between successive epochs of policy decisions and that of transfers are i.i.d random variables . The renewal processes governing the inter-policy decisions times and that of the inter-transfer decisions times are same . It is assumed that the two sources of depletion are independent. There is a threshold level for the level of wastage and also available for the manhours due to extra time work. If the total loss of manhours crosses the sum of the threshold and the available manhours due to extra time work the break down occurs. The process that generates the loss of manhours and the threshold put together is linearly independent. Recruitment takes place only at decision points and time of recruitment is negligible. The recruitment is made whenever the cumulative loss of manhours exceeds the threshold of the organization.

**Notations:**

$L_{i1}$ ($L_{i2}$) : independent and identically distributed continuous random variable denoting the loss of manhours due to $i^{th}$ policy decision ( $j^{th}$ transfer ), i ,j ≥ 1. Its probability density function is f(.) (k(.)) and cumulative distribution is F(.) (K(.)).

$T_1$ ($T_2$) : continuous random variable representing the threshold level of policy decisions (transfer ) with probability density function $g_1(.)$ ($g_2(.)$) and cumulative distribution function $G_1(.)$ ($G_2(.)$).

$T = T_1 + T_2$ : continuous random variable representing threshold level of the organization Its probability density function is g(.) and cumulative distribution function is G(.).

R : continuous random variable representing the time to the breakdown of the organization. Its probability density function is l(.) and cumulative distribution function is L(.).

v(.) : probability density function of inter-decision times ,$v_m(.)$ and $v^*(.)$ are its m-fold convolution and Laplace transform respectively.

w(.) : probability density function of inter-trans times with $w_n(.)$ and $w^*(.)$ are its n-fold convolution and Laplace transform respectively.

V(.) : cumulative distribution function of inter-decision times with $V_m(.)$ and $\overline{V}(.)$ are its k-fold convolution and Laplace-Stieltjes transform respectively.

W(.) : cumulative distribution function of inter-transfer times with $W_n(.)$ and $\overline{W}(.)$ are its n-fold convolution and Laplace-Stieltjes transform respectively.

$f_m(.)$ : probability density function of $\sum_{i=1}^{m} L_{i1}$, $f_m^*(.)$ is its Laplace transform

$k_n(.)$ : probability density function of $\sum_{i=1}^{n} L_{i2}$, $k_n^*(.)$ is its Laplace transform

$K_n(.)$ : cumulative distribution function of $\sum_{i=1}^{n} L_{i2}$, $\overline{K}_n(.)$ is its Laplace - Stieltjes transform

$\lambda_1, \lambda_2$ : parameters of the distribution function of thresholds $T_1$, $T_2$

$\sigma_1, \sigma_2$ : parameters of the distribution function of inter-decision time times and inter transfer times.

$\beta_1, \beta_2$ : parameters of the distribution function of the loss of manhours due to decision and due to transfer.

## III. EXPECTED TIME AND VARIANCE OF TIME TO RECRUITMENT

In this section, the analytical expressions for the expected time and the variance of time to recruitment are derived. The recruitment is done whenever the cumulative loss of manhours exceeds the sum $T_1 + T_2$.

Probability that the manpower system will fail only after time t is $P(R > t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}$ {probability that there are exactly m policy decisions in (0, t] in grade I} {Probability that there are exactly m policy decisions and n transfer of personnel in (0, t] } {Probability that the cumulative damage in the manpower system does not cross its threshold level}.

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} (V_m(t) - V_{m+1}(t))\,(W_n(t) - W_{n+1}(t))\; P\left[\left(\sum_{i=1}^{m} L_{i,1} + \sum_{i=1}^{n} L_{i,2}\right) < \left(T_1 + T_2\right)\right]$$

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} (V_m(t) - V_{m+1}(t))\,(W_n(t) - W_{n+1}(t))$$

$$\int_0^{\infty} P\left[(x_1 + x_2) < (T_1 + T_2) \,\middle|\, \sum_{i=1}^{m} L_{i,1} = x_1,\ \sum_{i=1}^{n} L_{i,2} = x_2\right] dG_m(x_1)\, dH_n(x_2) \qquad (1.1)$$

Since the threshold levels follow exponential distributions with parameters $\lambda_1$ and $\lambda_2$, the probability density function of $T_1 + T_2$ is

$$g(y) = \int_0^{y} g_1(x)\, g_2(y - x)\, dx$$

$$= \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}\left(e^{-\lambda_2 y} - e^{-\lambda_1 y}\right)$$

The equation (1.1) becomes

$$P(R > t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} (V_m(t) - V_{m+1}(t))\,(W_n(t) - W_{n+1}(t))$$

$$\int_0^{\infty}\int_0^{\infty}\int_{x_1+x_2}^{\infty} \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)}\left(e^{-\lambda_2 y} - e^{-\lambda_1 y}\right) dy\, dG_m(x_1)\, dH_n(x_2)$$

$$= \frac{1}{(\lambda_1 - \lambda_2)} \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} (V_m(t) - V_{m+1}(t))\,(W_n(t) - W_{n+1}(t))$$

$$\left\{\int_0^{\infty} \lambda_1\, e^{-\lambda_2 x_2}\, f_m^*(\lambda_2)\, d H_n(x_2) - \int_0^{\infty} \lambda_2\, e^{-\lambda_1 x_2}\, f_m^*(\lambda_1)\, d H_n(x_2)\right\}$$

$$= \frac{1}{(\lambda_1 - \lambda_2)} \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} (V_m(t) - V_{m+1}(t))\,(W_n(t) - W_{n+1}(t))$$

$$\left\{\lambda_1\, f_m^*(\lambda_2)\, k_n^*(\lambda_2) - \lambda_2\, f_m^*(\lambda_1)\, k_n^*(\lambda_1)\right\}$$

$$= \left(\frac{1}{\lambda_1 - \lambda_2}\right) \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} (V_m(t) - V_{m+1}(t))\,(W_n(t) - W_{n+1}(t))$$

$$\left\{\lambda_1\, (f^*(\lambda_2))^m\, (k^*(\lambda_2))^n - \lambda_2\, (f^*(\lambda_1))^m\, (k^*(\lambda_1))^n\right\}$$

$$\left(\frac{1}{\lambda_1 - \lambda_2}\right) \sum_{m=0}^{\infty} (V_m(t) - V_{m+1}(t))\, \left(\lambda_1\, (f^*(\lambda_2))^m - \lambda_2\, (f^*(\lambda_1))^m\right)$$

$$-\lambda_1 (f^*(\lambda_2))^m \left( \sum_{n=1}^{\infty} W_n(t)(k^*(\lambda_2))^{n-1} - \sum_{n=1}^{\infty} W_n(t)(k^*(\lambda_2))^{n} \right)$$

$$+\lambda_2 (f^*(\lambda_1))^m \left( \sum_{n=1}^{\infty} W_n(t)(k^*(\lambda_1))^{n-1} - \sum_{n=1}^{\infty} W_n(t)(k^*(\lambda_1))^{n} \right)$$

$$= \left( \frac{1}{\lambda_1 - \lambda_2} \right) \sum_{m=0}^{\infty} (V_m(t) - V_{m+1}(t)) \{\lambda_1 (f^*(\lambda_2))^m - \lambda_2 (f^*(\lambda_1))^m$$

$$-\lambda_1 (f^*(\lambda_2))^m (1 - k^*(\lambda_2)) \sum_{n=1}^{\infty} W_n(t)(k^*(\lambda_2))^{n-1}$$

$$+\lambda_2 (f^*(\lambda_1))^m (1 - k^*(\lambda_1)) \sum_{n=1}^{\infty} W_n(t)(k^*(\lambda_1))^{n-1}\}$$

$$= \left( \frac{\lambda_1}{\lambda_1 - \lambda_2} \right) \left( 1 - (1 - k^*(\lambda_2)) \sum_{n=1}^{\infty} W_n(t)(k^*(\lambda_2))^{n-1} \right)$$

$$\left( 1 - (1 - f^*(\lambda_2)) \sum_{m=1}^{\infty} V_m(t)(f^*(\lambda_2))^{m-1} \right)$$

$$-\left( \frac{\lambda_2}{\lambda_1 - \lambda_2} \right) \left( 1 - (1 - k^*(\lambda_1)) \sum_{n=1}^{\infty} W_n(t)(k^*(\lambda_1))^{n-1} \right)$$

$$\left( 1 - (1 - f^*(\lambda_1)) \sum_{m=1}^{\infty} V_m(t)(f^*(\lambda_1))^{m-1} \right)$$

$$L(t) = 1 - P(R > t)$$

$$= \left( \frac{\lambda_1}{\lambda_1 - \lambda_2} \right) \left\{ (1 - f^*(\lambda_2)) \sum_{m=1}^{\infty} V_m(t)(f^*(\lambda_2))^{m-1} \right.$$

$$+ (1 - k^*(\lambda_2) \sum_{n=1}^{\infty} W_n(t)(f^*(\lambda_2))^{n-1}$$

$$\left. - (1 - f^*(\lambda_2))(1 - k^*(\lambda_2)) \left( \sum_{m=1}^{\infty} V_m(t)(f^*(\lambda_2))^{m-1} \right) \left( \sum_{n=1}^{\infty} W_n(t)(f^*(\lambda_2))^{n-1} \right) \right\}$$

$$-\left( \frac{\lambda_2}{\lambda_1 - \lambda_2} \right) \left\{ (1 - f^*(\lambda_1)) \sum_{m=1}^{\infty} V_m(t)(f^*(\lambda_1))^{m-1} \right.$$

$$+ (1 - k^*(\lambda_1) \sum_{n=1}^{\infty} W_n(t)(f^*(\lambda_1))^{n-1}$$

$$\left. - (1 - f^*(\lambda_1))(1 - k^*(\lambda_1)) \left( \sum_{m=1}^{\infty} V_m(t)(f^*(\lambda_1))^{m-1} \right) \left( \sum_{n=1}^{\infty} R_n(t)(f^*(\lambda_1))^{n-1} \right) \right\} \tag{1.2}$$

$$l(t) = \frac{d}{dt} L(t)$$

$$= \left( \frac{\lambda_1}{\lambda_1 - \lambda_2} \right) \left\{ (1 - f^*(\lambda_2)) \sum_{m=1}^{\infty} v_m(t)(f^*(\lambda_2))^{m-1} \right.$$

$$+ (1 - k^*(\lambda_2) \sum_{n=1}^{\infty} w_n(t)(k^*(\lambda_2))^{n-1}$$

$$-(1-f^*(\lambda_2))(1-k^*(\lambda_2))\left(\sum_{n=1}^{\infty} W_n(t)(k^*(\lambda_2))^{n-1}\right)\left(\sum_{m=1}^{\infty} v_m(t)(f^*(\lambda_2))^{m-1}\right)$$

$$-(1-f^*(\lambda_2))(1-k^*(\lambda_2))\left(\sum_{n=1}^{\infty} w_n(t)(k^*(\lambda_2))^{n-1}\right)\left(\sum_{m=1}^{\infty} V_m(t)(f^*(\lambda_2))^{m-1}\right)\Bigg\}$$

$$-\left(\frac{\lambda_1}{\lambda_1-\lambda_2}\right)\Bigg\{(1-f^*(\lambda_1))\sum_{m=1}^{\infty} v_m(t)(f^*(\lambda_1))^{m-1}$$

$$+(1-k^*(\lambda_1))\sum_{n=1}^{\infty} W_n(t)(k^*(\lambda_1))^{n-1}$$

$$-(1-f^*(\lambda_1))(1-k^*(\lambda_1))\left(\sum_{n=1}^{\infty} W_n(t)(k^*(\lambda_1))^{n-1}\right)\left(\sum_{m=1}^{\infty} v_m(t)(f^*(\lambda_1))^{m-1}\right)$$

$$-(1-f^*(\lambda_1))(1-k^*(\lambda_1))\left(\sum_{n=1}^{\infty} w_n(t)(k^*(\lambda_1))^{n-1}\right)\left(\sum_{m=1}^{\infty} R_m(t)(f^*(\lambda_1))^{m-1}\right)\Bigg\} \qquad (1.3)$$

As the inter-decision times and inter transfer times follow exponential distributions with parameters σ1 and $\sigma_2$respectively, we have

$$v_m(t) = \frac{\sigma_1^m e^{-\sigma_1 t} t^{m-1}}{(m-1)!} \text{ and } w_n(t) = \frac{\sigma_2^n e^{-\sigma_2 t} t^{n-1}}{(n-1)!}$$

Now, $\sum_{m=1}^{\infty} v_m(t)(f^*(\lambda_i))^{m-1} = \sum_{m=1}^{\infty} \frac{\sigma_1^m e^{-\sigma_1 t} t^{m-1}}{(m-1)!} (f^*(\lambda_i))^{m-1}$

$= \sigma_1 e^{-\sigma_1 t(1-f^*(\lambda_i))}$, i = 1, 2.

(1.4)

and $\sum_{n=1}^{\infty} r_n(t)(k^*(\lambda_i))^{n-1} = \sum_{m=1}^{\infty} \frac{\sigma_2^n e^{-\sigma_2 t} t^{n-1}}{(i-1)!} (k^*(\lambda_i))^{n-1}$

$= \sigma_2 e^{-\sigma_2 t(1-k^*(\lambda_i))}$, i = 1, 2

(1.5)

Using the equations (1.4) and (1.5), the equation (1.3) becomes

$$l(t) = \left(\frac{\lambda_1}{\lambda_1-\lambda_2}\right)\Bigg\{(1-f^*(\lambda_2))\sum_{m=1}^{\infty} v_m(t)(f^*(\lambda_2))^{m-1}$$

$$-(1-f^*(\lambda_2))(1-k^*(\lambda_2))\,\sigma_1 e^{-\sigma_1 t(1-f^*(\lambda_2))}\sum_{n=1}^{\infty} W_n(t)(k^*(\lambda_2))^{n-1}$$

$$+(1-k^*(\lambda_2))\sum_{n=1}^{\infty} w_n(t)(k^*(\lambda_2))^{n-1}$$

$$-(1-f^*(\lambda_2))(1-k^*(\lambda_2))\,\sigma_2 e^{-\sigma_2 t(1-k^*(\lambda_2))}\sum_{n=1}^{\infty} V_n(t)(f^*(\lambda_2))^{n-1}\Bigg\}$$

$$-\left(\frac{\lambda_2}{\lambda_1-\lambda_2}\right)\Bigg\{(1-f^*(\lambda_1))\sum_{m=1}^{\infty} v_m(t)(f^*(\lambda_1))^{m-1}$$

$$-(1-f^*(\lambda_1))(1-k^*(\lambda_1))\,\sigma_1 e^{-\sigma_1 t(1-f^{*(\lambda_1)})}\sum_{n=1}^{\infty} W_n(t)(k^*(\lambda_1))^{n-1}$$

$$+(1-k^*(\lambda_1))\sum_{n=1}^{\infty} w_n(t)(k^*(\lambda_1))^{n-1}$$

$$-(1-f^*(\lambda_1))(1-k^*(\lambda_1))\,\sigma_2\, e^{-\sigma_2 t(1-k^*(\lambda_1))}\left(\sum_{m=1}^{\infty} V_m(t)(f^*(\lambda_1))^{m-1}\right)\Bigg\}$$

Taking Laplace transform on both sides and using the notations a = $(1-f^*(\lambda_1))\,\sigma_1$, b = $(1-f^*(\lambda_2))\,\sigma_1$, c = $(1-k^*(\lambda_1))\,\sigma_2$ and d = $(1-k^*(\lambda_2))\,\sigma_2$ we have,

$$l^*(s) = \left(\frac{\lambda_1}{\lambda_1-\lambda_2}\right)\Bigg\{\left((1-f^*(\lambda_2))\sum_{m=1}^{\infty}(f^*(\lambda_2))^{m-1}\, v_m^*(s)\right)$$

$$-(1-f^*(\theta_2))(1-h^*(\theta_2))\sum_{n=1}^{\infty}\sigma_1\,(k^*(\lambda_2))^{n-1}\,\overline{W}_n(b+s)$$

$$+(1-k^*(\lambda_2))\left(\sum_{n=1}^{\infty}(k^*(\lambda_2))^{n-1}\, w_n^*(s)\right)$$

$$-(1-f^*(\lambda_2))(1-k^*(\lambda_2))\left(\sum_{m=1}^{\infty}\sigma_2\,(f^*(\lambda_2))^{m-1}\,\overline{V}_m(d+s)\right)\Bigg\}$$

$$-\left(\frac{\lambda_2}{\lambda_1-\lambda_2}\right)\Bigg\{(1-f^*(\lambda_1))\left(\sum_{m=1}^{\infty}(f^*(\lambda_1))^{m-1}\, v_m^*(s)\right)$$

$$-(1-f^*(\lambda_1))(1-k^*(\lambda_1))\sum_{n=1}^{\infty}\sigma_1\,(k^*(\lambda_1))^{n-1}\,\overline{W}_n(a+s)$$

$$+(1-k^*(\lambda_1))\sum_{n=1}^{\infty}(k^*(\lambda_1))^{n-1}\, w_n^*(s)$$

$$-(1-f^*(\lambda_1))(1-k^*(\lambda_1))\left(\sum_{m=1}^{\infty}\sigma_2\,(f^*(\lambda_1))^{m-1}\,\overline{V}_m(c+s)\right)\Bigg\} \qquad (1.6)$$

Using the relation between the Laplace transform of the density function and Laplace transform of distribution function, we have

$$l^*(s) = \left(\frac{\lambda_1}{\lambda_1-\lambda_2}\right)\Bigg\{(1-f^*(\lambda_2))\sum_{m=1}^{\infty}(f^*(\lambda_2))^{m-1}\, v_m^*(s)$$

$$-(1-f^*(\lambda_2))(1-k^*(\lambda_2))\sum_{n=1}^{\infty}\sigma_1\,(k^*(\lambda_2))^{n-1}\,\frac{w_n^*(s+b)}{s+b}$$

$$+(1-k^*(\lambda_2))\sum_{n=1}^{\infty}(k^*(\lambda_2))^{n-1}\, w_n^*(s)$$

$$-(1-f^*(\lambda_2))(1-k^*(\lambda_2))\sum_{m=1}^{\infty}\sigma_2\,(f^*(\lambda_2))^{m-1}\,\frac{v_m^*(s+d)}{s+d}\Bigg\}$$

$$-\left(\frac{\lambda_2}{\lambda_1-\lambda_2}\right)\Bigg\{(1-f^*(\lambda_1))\sum_{m=1}^{\infty}(f^*(\lambda_1))^{m-1}\, v_m^*(s)$$

$$-(1-f^*(\lambda_1))(1-k^*(\lambda_1))\sum_{n=1}^{\infty}\sigma_1\,(k^*(\lambda_1))^{n-1}\,\frac{w_n^*(s+a)}{s+a}$$

$$+ (1-\mathrm{k}^*(\lambda_1)) \sum_{n=1}^{\infty} (\mathrm{k}^*(\lambda_1))^{n-1} \, \mathrm{w}_n^*(s)$$

$$- (1-\mathrm{f}^*(\lambda_1))\,(1-\mathrm{k}^*(\lambda_1)) \sum_{m=1}^{\infty} \sigma_2 \, (\mathrm{f}^*(\lambda_1))^{m-1} \, \frac{\mathrm{v}_m^*(s+c)}{s+c} \Bigg\}$$

$$= \left(\frac{\lambda_1}{\lambda_1-\lambda_2}\right) \left[ \frac{(1-\mathrm{f}^*(\lambda_2))\,\mathrm{v}^*(s)}{1-\mathrm{f}^*(\lambda_2)\,\mathrm{v}^*(s)} - \frac{\sigma_1\,(1-\mathrm{f}^*(\lambda_2))(1-\mathrm{k}^*(\lambda_2))\,\mathrm{w}^*(s+b)}{(s+b)\,(1-\mathrm{k}^*(\lambda_2)\,\mathrm{w}^*(s+b))} \right.$$

$$\left. + \frac{(1-\mathrm{k}^*(\lambda_2))\,\mathrm{w}^*(s)}{1-\mathrm{k}^*(\lambda_2)\,\mathrm{w}^*(s)} - \frac{\sigma_2\,(1-\mathrm{f}^*(\lambda_2))(1-\mathrm{k}^*(\lambda_2))\,\mathrm{v}^*(s+d)}{(s+d)\,(1-\mathrm{f}^*(\lambda_2)\,\mathrm{v}^*(s+d))} \right]$$

$$- \left(\frac{\lambda_2}{\lambda_1-\lambda_2}\right) \left[ \frac{(1-\mathrm{f}^*(\lambda_1))\,\mathrm{v}^*(s)}{1-\mathrm{f}^*(\lambda_1)\,\mathrm{v}^*(s)} - \frac{\sigma_1\,(1-\mathrm{f}^*(\lambda_1))(1-\mathrm{k}^*(\lambda_1))\,\mathrm{v}^*(s+a)}{(s+a)\,(1-\mathrm{k}^*(\lambda_1)\,\mathrm{w}^*(s+a))} \right.$$

$$\left. + \frac{(1-\mathrm{k}^*(\lambda_1))\,\mathrm{w}^*(s)}{1-\mathrm{k}^*(\lambda_1)\,\mathrm{w}^*(s)} - \frac{\sigma_2\,(1-\mathrm{f}^*(\lambda_1))(1-\mathrm{k}^*(\lambda_1))\,\mathrm{v}^*(s+c)}{(s+c)\,(1-\mathrm{f}^*(\lambda_1)\,\mathrm{v}^*(s+c))} \right] \quad (1.7)$$

Since f(.) and k(.) follow exponential distributions with parameters $\beta_1$ and $\beta_2$ respectively, we have

$$\mathrm{f}^*(\lambda_i) = \frac{\beta_1}{\beta_1+\lambda_i} \quad \text{and} \quad \mathrm{k}^*(\lambda_i) = \frac{\beta_2}{\beta_2+\lambda_i}, \quad i = 1, 2 \quad (1.8)$$

Taking

$A_1(s) = (\sigma_1 + s)(\alpha_1 + \lambda_2) - \sigma_1 \beta_1$ ; $A_2(s)=(\sigma_2 + s)(\beta_2 + \lambda_2) - \lambda_2 \beta_2$

$A_3(s) = (\sigma_1 + s)(\beta_1 + \lambda_1) - \sigma_1 \beta_1$ ; $A_4(s)=(\sigma_2 + s)(\beta_2 + \lambda_1) - \sigma_2 \beta_2$

$B_1(s) = \lambda_2 \sigma_1 + s(\beta_1 + \lambda_2)$; $B_2(s)=\lambda_2 \sigma_2 + s(\beta_2 + \lambda_2)$

$B_3(s) = \lambda_1 \sigma_1 + s(\beta_1 + \lambda_1)$ ; $B_4(s)=\lambda_1 \sigma_2 + s(\beta_2 + \lambda_1)$

$C_1(s) = (\sigma_2 + s)(\beta_1 + \lambda_2)(\beta_2 + \lambda_2) + \sigma_1 \lambda_2 (\beta_2 + \lambda_2) - \sigma_2 \beta_2 (\beta_1 + \lambda_2)$

$C_2(s) = (\sigma_1 + s)(\beta_1 + \lambda_2)(\beta_2 + \lambda_2) + \sigma_2 \lambda_2 (\beta_1 + \lambda_2) - \sigma_1 \beta_1 (\beta_2 + \lambda_2)$

$C_3(s) = (\sigma_2 + s)(\beta_1 + \lambda_1)(\beta_2 + \lambda_1) + \sigma_1 \lambda_1 (\beta_2 + \lambda_1) - \sigma_2 \beta_2 (\beta_1 + \lambda_1)$

$C_4(s) = (\sigma_1 + s)(\beta_1 + \lambda_1)(\beta_2 + \lambda_1) + \sigma_2 \lambda_1 (\beta_1 + \lambda_1) - \sigma_1 \beta_1 (\beta_2 + \lambda_1)$

and using (1.8) the equation (1.7) becomes

$$\mathrm{l}^*(\mathrm{s}) = \left(\frac{\lambda_1}{\lambda_1-\lambda_2}\right) \left\{ \frac{\lambda_2\,\sigma_1}{A_1(\mathrm{s})} + \frac{\lambda_2\,\sigma_2}{\mathrm{A}_2(\mathrm{s})} - \lambda_2^2\,\sigma_1\,\sigma_2 \left[ \frac{\beta_1+\lambda_2}{B_1(s)\,\mathrm{C}_1(s)} + \frac{\beta_2+\lambda_2}{B_2(s)\,\mathrm{C}_2(s)} \right] \right\}$$

$$- \left(\frac{\lambda_2}{\lambda_1-\lambda_2}\right) \left\{ \frac{\lambda_1\,\sigma_1}{A_3(\mathrm{s})} + \frac{\lambda_1\,\sigma_2}{\mathrm{A}_4(\mathrm{s})} - \lambda_1^2\,\sigma_1\,\sigma_2 \left[ \frac{\beta_1+\lambda_1}{B_3(s)\,\mathrm{C}_3(s)} + \frac{\beta_2+\lambda_1}{B_4(s)\,\mathrm{C}_4(s)} \right] \right\}$$

$$\frac{d\mathrm{l}^*(\mathrm{s})}{ds} = \left(\frac{\lambda_1}{\lambda_1-\lambda_2}\right) \left\{ \frac{-\lambda_2\,\sigma_1\,(\beta_1+\lambda_2)}{[\mathrm{A}_1(\mathrm{s})]^2} - \frac{\lambda_2\,\sigma_2\,(\beta_2+\lambda_2)}{[\mathrm{A}_2(\mathrm{s})]^2} \right.$$

$$+ \lambda_2^2\,\sigma_1\,\sigma_2 \left[ \frac{(\beta_1+\lambda_2)[\mathrm{B}_1(\mathrm{s})(\beta_1+\lambda_2)(\beta_2+\lambda_2)+\mathrm{C}_1(\mathrm{s})(\beta_1+\lambda_2)]}{[\mathrm{B}_1(\mathrm{s})C_1(\mathrm{s})]^2} \right.$$

$$\left.\left. + \frac{(\beta_2+\lambda_2)[\mathrm{B}_2(\mathrm{s})(\beta_1+\lambda_2)(\beta_2+\lambda_2)+\mathrm{C}_2(\mathrm{s})(\beta_2+\lambda_2)]}{[\mathrm{B}_2(\mathrm{s})\;\mathrm{C}_2(\mathrm{s})]^2} \right] \right\}$$

$$- \left(\frac{\lambda_2}{\lambda_1-\lambda_2}\right) \left\{\frac{-\lambda_1\,\sigma_1\,(\beta_1+\lambda_1)}{[A_3(s)]^2} - \frac{\lambda_1\,\sigma_2\,(\beta_2+\lambda_1)}{[A_4(s)]^2}\right.$$

$$+ \lambda_1^2\,\sigma_1\,\sigma_2 \left[\frac{(\beta_1+\lambda_1)[B_3(s)(\beta_1+\lambda_1)(\beta_2+\lambda_1)+C_3(s)(\beta_1+\lambda_1)]}{[B_3(s)C_3(s)]^2}\right.$$

$$\left.\left.+ \frac{(\beta_2+\lambda_1)[B_4(s)(\beta_1+\lambda_1)(\beta_2+\lambda_1)+C_4(s)(\beta_2+\lambda_1)]}{[B_4(s)\,C_4(s)]^2}\right]\right\}$$

The mean time to recruitment is

$$E(R) = \left(\frac{\lambda_1}{\lambda_1-\lambda_2}\right)\left\{\frac{\beta_1+\lambda_2}{\sigma_1\,\lambda_2} + \frac{\beta_2+\lambda_2}{\sigma_2\,\lambda_2}\right.$$

$$- \frac{\sigma_2\,(\beta_1+\lambda_2)^2}{\sigma_1\,\lambda_2}\left[\frac{2\,\sigma_1\,(\beta_2+\lambda_2)+\sigma_2\,(\beta_1+\lambda_2)}{[\sigma_2\,(\beta_1+\lambda_2)+\sigma_1\,(\beta_2+\lambda_2)]^2}\right]$$

$$\left.- \frac{\sigma_1\,(\beta_2+\lambda_2)^2}{\sigma_2\,\lambda_2}\left[\frac{2\,\sigma_2\,(\beta_1+\lambda_2)+\sigma_1\,(\beta_2+\lambda_2)}{[\sigma_1\,(\beta_2+\lambda_2)+\sigma_2\,(\beta_1+\lambda_2)]^2}\right]\right\}$$

$$- \left(\frac{\lambda_2}{\lambda_1-\lambda_2}\right)\left\{\frac{\beta_1+\lambda_1}{\sigma_1\,\lambda_1} + \frac{\beta_2+\lambda_1}{\sigma_2\,\lambda_1}\right.$$

$$- \frac{\sigma_2\,(\beta_1+\lambda_1)^2}{\sigma_1\,\lambda_1}\left[\frac{2\,\sigma_1\,(\beta_2+\lambda_1)+\sigma_2\,(\beta_1+\lambda_1)}{[\sigma_2\,(\beta_1+\lambda_1)+\sigma_1\,(\beta_2+\lambda_1)]^2}\right]$$

$$\left.- \frac{\sigma_1\,(\beta_2+\lambda_1)^2}{\sigma_2\,\lambda_1}\left[\frac{2\,\sigma_2\,(\beta_1+\lambda_1)+\sigma_1\,(\beta_2+\lambda_1)}{[\sigma_1\,(\beta_2+\lambda_1)+\sigma_2\,(\beta_1+\lambda_1)]^2}\right]\right\} \qquad (1.9)$$

$$\frac{d^2 l^*(s)}{ds^2} = \left(\frac{\lambda_1}{\lambda_1-\lambda_2}\right)\left\{\frac{2\,\lambda_2\,\sigma_1\,(\beta_1+\lambda_2)}{(A_1(s))^3} + \frac{2\,\lambda_2\,\sigma_2\,(\beta_2+\lambda_2)}{(A_2(s))^3}\right.$$

$$- \lambda_2^2\,\sigma_1\,\sigma_2\left[\frac{2\,(\beta_1+\lambda_2)^3}{(B_1(s)\,C_1(s))^3}\left[B_1(s)C_1(s)(\beta_2+\lambda_2) - (B_1(s)(\beta_2+\lambda_2)+C_1(s))^2\right]\right.$$

$$\left.\left.+ \frac{2\,(\beta_2+\lambda_2)^3}{(B_2(s)\,C_2(s))^3}\left[B_2(s)C_2(s)(\beta_1+\lambda_2) - (B_2(s)(\beta_1+\lambda_2)+C_2(s))^2\right]\right]\right\}$$

$$- \left(\frac{\lambda_2}{\lambda_1-\lambda_2}\right)\left\{\frac{2\,\lambda_1\,\sigma_1\,(\beta_1+\lambda_1)}{(A_3(s))^3} + \frac{2\,\lambda_1\,\sigma_2\,(\beta_2+\lambda_1)}{(A_4(s))^3}\right.$$

$$+ \lambda_1^2\,\sigma_1\,\sigma_2\left[\frac{2\,(\beta_1+\lambda_1)^3}{(B_3(s)\,C_3(s))^3}\left[B_3(s)C_3(s)(\beta_2+\lambda_1) - (B_3(s)(\beta_2+\lambda_1)+C_3(s))^2\right]\right.$$

$$\left.\left.+ \frac{2\,(\beta_2+\lambda_1)^3}{(B_4(s)\,C_4(s))^3}\left[B_4(s)C_4(s)(\beta_1+\lambda_1) - (B_4(s)(\beta_1+\lambda_1)+C_4(s))^2\right]\right]\right\}$$

Let $C_{1i} = \left(C_i(s)\right)_{s=0}$, i = 1, 2

Then,

$$E(R^2) = \left(\frac{\lambda_1}{\lambda_1-\lambda_2}\right)\left\{\frac{2(\beta_1+\lambda_2)^2}{(\sigma_1\,\lambda_2)^2}+\frac{2(\beta_2+\lambda_2)^2}{(\sigma_2\,\lambda_2)^2}\right.$$

$$-\lambda_2^2\,\sigma_1\,\sigma_2\left[\frac{2(\beta_1+\lambda_2)^3}{(C_{11}\,\lambda_2\,\sigma_1)^3}(C_{11}\lambda_2\sigma_1\,(\beta_2+\lambda_2)\right.$$

$$+(\lambda_2\sigma_1)^2\,(\beta_2+\lambda_2)^2+(C_{11})^2)+\frac{2(\beta_2+\lambda_2)^3}{(C_{12}\,\lambda_2\,\sigma_2)^3}(C_{12}\,\lambda_2\,\sigma_2\,(\beta_1+\lambda_2)$$

$$\left.\left.+(\lambda_2\,\sigma_2)^2\,(\beta_1+\lambda_2)^2+(C_{12})^2)\right]\right\}$$

$$-\left(\frac{\lambda_2}{\lambda_1-\lambda_2}\right)\left\{\frac{2(\alpha_1+\theta_1)^2}{(\mu_1\,\theta_1)^2}+\frac{2(\alpha_2+\theta_1)^2}{(\mu_2\,\theta_1)^2}\right.$$

$$-\lambda_1^2\,\sigma_1\,\sigma_2\left[\frac{2(\beta_1+\lambda_1)^3}{(C_{13}\,\lambda_1\,\sigma_1)^3}(C_{13}\lambda_1\sigma_1\,(\beta_2+\lambda_1)\right.$$

$$+(\lambda_1\sigma_1)^2\,(\beta_2+\lambda_1)^2+(C_{13})^2)+\frac{2(\beta_2+\lambda_1)^3}{(C_{14}\,\lambda_1\,\sigma_2)^3}(C_{14}\,\lambda_1\,\sigma_2\,(\beta_1+\lambda_1)$$

$$\left.\left.+(\lambda_1\,\sigma_2)^2\,(\beta_1+\lambda_1)^2+(C_{14})^2)\right]\right\} \qquad (1.10)$$

Using the equations (1.9) and (1.10) the variance of time to recruitment can be calculated.

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