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Interval Valued Intuitionistic Fuzzy Subrings of A Ring

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Abstract:- In this paper, we study some of the properties of interval valued intuitionistic fuzzy subring of a ring and prove some results on these.
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I. INTRODUCTION

Interval-valued fuzzy sets were introduced independently by Zadeh [13], Grattan-Guiness [5], Jahn [7], in the seventies, in the same year. An interval valued fuzzy set (IVF) is defined by an interval-valued membership function. Jun.Y.B and Kin.K.H[8] defined an interval valued fuzzy R-subgroups of nearrings. Solairaju.A and Nagarajan.R[10] defined the charactarization of interval valued Anti fuzzy Left h-ideals over Hemirings. M.G.Somasundara Moorthy and K. Arjunan[11] have defined an interval valued fuzzy subring of a ring under homomorphism. We introduce the concept of interval valued intuitionistic fuzzy subring of a ring and established some results.

1.PRELIMINARIES:

1.1 Definition: Let X be any nonempty set. A mapping $[M] : X \to D[0, 1]$ is called an interval valued fuzzy subset (briefly, IVFS) of X, where D[0,1] denotes the family of all closed subintervals of [0,1] and $[M](x) = [M^-(x), M^+(x)]$, for all x in X, where M^- and M^+ are fuzzy subsets of X such that $M^-(x) \le M^+(x)$, for all x in X. Thus $M^-(x)$ is an interval (a closed subset of [0,1]) and not a number from the interval [0,1] as in the case of fuzzy subset. Note that [0] = [0, 0] and [1] = [1, 1].

1.2 Definition: Let (R, +, ·) be a ring. An interval valued fuzzy subset [M] of R is said to be an **interval valued fuzzy subring(IVFSR)** of R if the following conditions are satisfied:

(i) $[M](x+y) \ge \text{rmin} \{ [M](x), [M](y) \}$,

(ii) $[M](-x) \ge [M](x)$,

(iii) $[M](xy) \ge \text{rmin} \{ [M](x), [M](y) \}$, for all x and y in R.

1.3 Definition: An **interval valued intuitionistic fuzzy subset** (IVIFS) [A] in X is defined as an object of the form $[A] = \{ < x, \mu_{[A]}(x), \nu_{[A]}(x) > / x \text{ in } X\}$, where $\mu_{[A]} : X \to D[0, 1]$ and $\nu_{[A]} : X \to D[0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $\mu_{[A]}^+(x) + \nu_{[A]}^+(x) \le 1$.

1.4 Definition: Let (R, +, .) be a ring. An interval valued intuitionistic fuzzy subset [A] of R is said to be an interval valued intuitionistic fuzzy subring of R if it satisfies the following axioms:

(i) $\mu_{[A]}(x - y) \ge \text{rmin} \{\mu_{[A]}(x), \mu_{[A]}(y)\} = [\min \{\mu_{[A]}^-(x), \mu_{[A]}^-(y)\}, \min\{\mu_{[A]}^+(x), \mu_{[A]}^+(y)\}]$

(ii) $\mu_{[A]}(xy) \ge \text{rmin} \{\mu_{[A]}(x), \mu_{[A]}(y)\} = [\min \{\mu_{[A]}^-(x), \mu_{[A]}^-(y)\}, \min\{\mu_{[A]}^+(x), \mu_{[A]}^+(y)\}]$

(iii) $\nu_{[A]}(x-y) \le \text{rmax}\{\nu_{[A]}(x), \nu_{[A]}(y)\} = [\max \{\nu_{[A]}^-(x), \nu_{[A]}^-(y)\}, \max\{\nu_{[A]}^+(x), \nu_{[A]}^+(y)\}]$

(iv) $\nu_{[A]}(xy) \le \text{rmax} \{\nu_{[A]}(x), \nu_{[A]}(y)\} = [\max\{\nu_{[A]}^-(x), \nu_{[A]}^-(y)\}, \max\{\nu_{[A]}^+(x), \nu_{[A]}^+(y)\}]$, for all x and y in R.

1.5 Definition: Let X and X′ be any two sets. Let $f : X \to X'$ be any function and [A] be an interval valued intuitionistic fuzzy subset in X, [V] be an interval valued intuitionistic fuzzy subset in f(X) = X′, defined by $\mu_{[V]}(y) = r\sup_{x \in f^{-1}(y)} \mu_{[A]}(x)$ and $\nu_{[V]}(y) = r\inf_{x \in f^{-1}(y)} \nu_{[A]}(x)$, for all x in X and y in X′. [A] is called a preimage of [V] under f and is denoted by $f^{-1}([V])$.

1.6 Definition: Let [A] be an interval valued intuitionistic fuzzy subring of a ring R and a in R. Then the **pseudo interval valued intuitionistic fuzzy coset** $(aA)^p$ is defined by $((a\mu_{[A]})^p)(x) = p(a)\mu_{[A]}(x)$ and $((a\nu_{[A]})^p)(x) = p(a)\nu_{[A]}(x)$, for every x in R and for some p in P.

1.7 Definition: Let [A] and [B] be interval valued intuitionistic fuzzy subsets of sets G and H, respectively. The **product** of [A] and [B], denoted by [A]×[B], is defined as $[A]\times[B] = \{\langle (x, y), \mu_{[A]\times[B]}(x,y), \nu_{[A]\times[B]}(x,y) \rangle$ / for all x in G and y in H }, where $\mu_{[A]\times[B]}(x, y)$ = rmin { $\mu_{[A]}(x)$, $\mu_{[B]}(y)$ } and $\nu_{[A]\times[B]}(x, y)$ = rmax { $\nu_{[A]}(x)$, $\nu_{[B]}(y)$ }.

1.8 Definition: Let [A] be an interval valued intuitionistic fuzzy subset in a set S. The **strongest interval valued intuitionistic fuzzy relation** on S, that is an interval valued intuitionistic fuzzy relation on [A] is [V] given by $\mu_{[V]}(x, y)$ = rmin { $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ } and $\nu_{[V]}(x, y)$ = rmax{ $\nu_{[A]}(x)$, $\nu_{[A]}(y)$ }, for all x, y in S.

II. SOME PROPERTIES

2.1 Theorem: Intersection of any two interval valued intuitionistic fuzzy subrings of a ring R is an interval valued intuitionistic fuzzy subring of R.

Proof: Let [A] and [B] be any two interval valued intuitionistic fuzzy subrings of a ring R and x, y in R. Let [A] = {(x, $\mu_{[A]}(x)$, $\nu_{[A]}(x)$)/ x∈R} and [B] = { (x, $\mu_{[B]}(x)$, $\nu_{[B]}(x)$) / x∈R} and also let [C] = [A]∩[B] = { (x, $\mu_{[C]}(x)$, $\nu_{[C]}(x)$) / x∈R}, where $\mu_{[C]}(x)$ = rmin {$\mu_{[A]}(x)$, $\mu_{[B]}(x)$ } and $\nu_{[C]}(x)$ = rmax {$\nu_{[A]}(x)$, $\nu_{[B]}(x)$ }. Now, $\mu_{[C]}(x-y)$ = rmin {$\mu_{[A]}(x-y)$, $\mu_{[B]}(x-y)$ }≥ rmin{ rmin { $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ }, rmin { $\mu_{[B]}(x)$, $\mu_{[B]}(y)$ }} = rmin {rmin{ $\mu_{[A]}(x)$, $\mu_{[B]}(x)$ }, rmin { $\mu_{[A]}(y)$, $\mu_{[B]}(y)$ }} = rmin { $\mu_{[C]}(x)$, $\mu_{[C]}(y)$ }. Therefore, $\mu_{[C]}(x-y)$ ≥ rmin { $\mu_{[C]}(x)$, $\mu_{[C]}(y)$ }, for all x, y in R. And, $\mu_{[C]}(xy)$ = rmin { $\mu_{[A]}(xy)$, $\mu_{[B]}(xy)$ } ≥ rmin { rmin{ $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ }, rmin { $\mu_{[B]}(x)$, $\mu_{[B]}(y)$ }} = rmin{ rmin {$\mu_{[A]}(x)$, $\mu_{[B]}(x)$ }, rmin { $\mu_{[A]}(y)$, $\mu_{[B]}(y)$ }} = rmin{ $\mu_{[C]}(x)$, $\mu_{[C]}(y)$ }. Therefore, $\mu_{[C]}(xy)$ ≥ rmin{ $\mu_{[C]}(x)$, $\mu_{[C]}(y)$ }, for all x, y in R. Now, $\nu_{[C]}(x-y)$ = rmax { $\nu_{[A]}(x-y)$, $\nu_{[B]}(x-y)$} ≤ rmax{rmax {$\nu_{[A]}(x)$, $\nu_{[A]}(y)$ }, rmax { $\nu_{[B]}(x)$, $\nu_{[B]}(y)$ }} = rmax{ rmax{$\nu_{[A]}(x)$, $\nu_{[B]}(x)$}, rmax { $\nu_{[A]}(y)$, $\nu_{[B]}(y)$}} = rmax { $\nu_{[C]}(x)$, $\nu_{[C]}(y)$ }. Therefore, $\nu_{[C]}(x-y)$ ≤ rmax { $\nu_{[C]}(x)$, $\nu_{[C]}(y)$ }, for all x, y in R. And, $\nu_{[C]}(xy)$ = rmax { $\nu_{[A]}(xy)$, $\nu_{[B]}(xy)$ } ≤ rmax { rmax {$\nu_{[A]}(x)$, $\nu_{[A]}(y)$ }, rmax { $\nu_{[B]}(x)$, $\nu_{[B]}(y)$ }}= rmax{ rmax { $\nu_{[A]}(x)$, $\nu_{[B]}(x)$ }, rmax {$\nu_{[A]}(y)$, $\nu_{[B]}(y)$ }} = rmax{ $\nu_{[C]}(x)$, $\nu_{[C]}(y)$}. Therefore, $\nu_{[C]}(xy)$ ≤ rmax{ $\nu_{[C]}(x)$, $\nu_{[C]}(y)$ }, for all x, y in R. Therefore [C] is an interval valued intuitionistic fuzzy subring of R. Hence the intersection of any two interval valued intuitionistic fuzzy subrings of a ring R is an interval valued intuitionistic fuzzy subring of R.

2.2 Theorem: The intersection of a family of interval valued intuitionistic fuzzy subrings of a ring R is an interval valued intuitionistic fuzzy subring of R.

Proof: Let { $[V]_i$: i∈I} be a family of interval valued intuitionistic fuzzy subrings of a ring R and let [A] = $\bigcap_{i\in I}[V]_i$. Let x and y in R. Then, $\mu_{[A]}(x-y)$ = $r\inf_{i\in I}$ $\mu_{[V]i}(x-y)$ ≥ $r\inf_{i\in I}$ rmin {$\mu_{[V]i}(x)$, $\mu_{[V]i}(y)$ } = rmin { $r\inf_{i\in I}$ $\mu_{[V]i}(x)$, $r\inf_{i\in I}$ $\mu_{[V]i}(y)$ }= rmin{$\mu_{[A]}(x)$, $\mu_{[A]}(y)$}.

Therefore, $\mu_{[A]}(x-y)$ ≥ rmin { $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ }, for all x, y in R. And, $\mu_{[A]}(xy)$ = $r\inf_{i\in I}$ $\mu_{[V]i}(xy)$ ≥ $r\inf_{i\in I}$ rmin{ $\mu_{[V]i}(x)$, $\mu_{[V]i}(y)$ } = rmin{ $r\inf_{i\in I}$ $\mu_{[V]i}(x)$, $r\inf_{i\in I}$ $\mu_{[V]i}(y)$ } = rmin {$\mu_{[A]}(x)$, $\mu_{[A]}(y)$ }. Therefore, $\mu_{[A]}(xy)$ ≥ rmin{ $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ }, for all x, y in R. Now, $\nu_{[A]}(x-y)$ = $r\sup_{i\in I}$ $\nu_{[V]i}(x-y)$ ≤ $r\sup_{i\in I}$ rmax { $\nu_{[V]i}(x)$, $\nu_{[V]i}(y)$ } = rmax { $r\sup_{i\in I}$ $\nu_{[V]i}(x)$, $r\sup_{i\in I}$ $\nu_{[V]i}(y)$ } = rmax{ $\nu_{[A]}(x)$, $\nu_{[A]}(y)$ }. Therefore, $\nu_{[A]}(x-y)$ ≤ rmax{ $\nu_{[A]}(x)$, $\nu_{[A]}(y)$ }, for all x, y in R. And, $\nu_{[A]}(xy)$ = $r\sup_{i\in I}$ $\nu_{[V]i}(xy)$ ≤ $r\sup_{i\in I}$ rmax{ $\nu_{[V]i}(x)$, $\nu_{[V]i}(y)$ } = rmax {$r\sup_{i\in I}$ $\nu_{[V]i}(x)$, $r\sup_{i\in I}$ $\nu_{[V]i}(y)$ } = rmax{ $\nu_{[A]}(x)$, $\nu_{[A]}(y)$ }. Therefore, $\nu_{[A]}(xy)$ ≤ rmax {$\nu_{[A]}(x)$, $\nu_{[A]}(y)$ }, for all x, y in R. That is, [A] is an interval valued intuitionistic fuzzy subring of R. Hence, the intersection of a family of interval valued intuitionistic fuzzy subrings of R is an interval valued intuitionistic fuzzy subring of R.

2.3 Theorem: If [A] is an interval valued intuitionistic fuzzy subring of a ring (R, +, ·), then $\mu_{[A]}(x)$ ≤ $\mu_{[A]}(e)$ and $\nu_{[A]}(x)$ ≥ $\nu_{[A]}(e)$, for x in R, the identity element e in R.

Proof: For x in R and e is the identity element of R. Now, $\mu_{[A]}(e)$ = $\mu_{[A]}(x-x)$ ≥ rmin {$\mu_{[A]}(x)$, $\mu_{[A]}(x)$ } = $\mu_{[A]}(x)$. Therefore, $\mu_{[A]}(e)$ ≥ $\mu_{[A]}(x)$, for x in R. And, $\nu_{[A]}(e)$ = $\nu_{[A]}(x-x)$ ≤ rmax { $\nu_{[A]}(x)$, $\nu_{[A]}(x)$ } = $\nu_{[A]}(x)$. Therefore, $\nu_{[A]}(e)$ ≤ $\nu_{[A]}(x)$, for x in R.

2.4 Theorem: If [A] is an interval valued intuitionistic fuzzy subring of a ring (R, +, ·), then (i) $\mu_{[A]}(x-y)$ = $\mu_{[A]}(e)$ gives $\mu_{[A]}(x)$ = $\mu_{[A]}(y)$, for x and y in R, e in R.

(ii) $\nu_{[A]}(x-y)$ = $\nu_{[A]}(e)$ gives $\nu_{[A]}(x)$ = $\nu_{[A]}(y)$, for x and y in R, e in R.

Proof: Let x and y in R, the identity e in R. (i) Now, $\mu_{[A]}(x)$ = $\mu_{[A]}(x-y+y)$ ≥ rmin {$\mu_{[A]}(x-y)$, $\mu_{[A]}(y)$ } = rmin { $\mu_{[A]}(e)$, $\mu_{[A]}(y)$ } = $\mu_{[A]}(y)$ = $\mu_{[A]}(x-(x-y))$ ≥ rmin {$\mu_{[A]}(x-y)$, $\mu_{[A]}(x)$ } = rmin { $\mu_{[A]}(e)$, $\mu_{[A]}(x)$ } = $\mu_{[A]}(x)$. Therefore, $\mu_{[A]}(x)$ = $\mu_{[A]}(y)$, for x, y in R. (ii) Now, $\nu_{[A]}(x)$ = $\nu_{[A]}(x-y+y)$ ≤ rmax { $\nu_{[A]}(x-y)$, $\nu_{[A]}(y)$ } = rmax {$\nu_{[A]}(e)$, $\nu_{[A]}(y)$ }= $\nu_{[A]}(y)$ = $\nu_{[A]}(x-(x-y))$ ≤ rmax { $\nu_{[A]}(x-y)$, $\nu_{[A]}(x)$ }= rmax { $\nu_{[A]}(e)$, $\nu_{[A]}(x)$ } = $\nu_{[A]}(x)$. Therefore, $\nu_{[A]}(x)$ = $\nu_{[A]}(y)$, for x and y in R.

2.5 Theorem: If [A] and [B] are any two interval valued intuitionistic fuzzy subrings of the rings R_1 and R_2 respectively, then [A]×[B] is an interval valued intuitionistic fuzzy subring of $R_1 \times R_2$.

Proof: Let [A] and [B] be two interval valued intuitionistic fuzzy subrings of the rings R_1 and R_2 respectively. Let x_1, x_2 in R_1 and y_1, y_2 in R_2. Then (x_1, y_1) and (x_2, y_2) are in $R_1 \times R_2$. Now, $\mu_{[A]\times[B]}[(x_1, y_1)-(x_2, y_2)] = \mu_{[A]\times[B]}(x_1- x_2, y_1- y_2) = \text{rmin}\{ \mu_{[A]}(x_1- x_2), \mu_{[B]}(y_1-y_2)\} \geq \text{rmin}\{ \text{rmin}\{ \mu_{[A]}(x_1), \mu_{[A]}(x_2) \}, \text{rmin}\{ \mu_{[B]}(y_1), \mu_{[B]}(y_2)\}\} = \text{rmin}\{ \text{rmin}\{\mu_{[A]}(x_1), \mu_{[B]}(y_1) \}, \text{rmin}\{\mu_{[A]}(x_2), \mu_{[B]}(y_2)\}\} = \text{rmin}\{ \mu_{[A]\times[B]}(x_1, y_1), \mu_{[A]\times[B]}(x_2, y_2)\}$. Therefore, $\mu_{[A]\times[B]}[(x_1, y_1)-(x_2, y_2)] \geq \text{rmin}\{ \mu_{[A]\times[B]}(x_1, y_1), \mu_{[A]\times[B]}(x_2, y_2)\}$. Also, $\mu_{[A]\times[B]}[(x_1, y_1)(x_2, y_2)] = \mu_{[A]\times[B]}(x_1x_2, y_1y_2) = \text{rmin}\{ \mu_{[A]}(x_1x_2), \mu_{[B]}(y_1y_2) \}\geq \text{rmin}\{\text{rmin}\{ \mu_{[A]}(x_1), \mu_{[A]}(x_2) \}, \text{rmin}\{ \mu_{[B]}(y_1), \mu_{[B]}(y_2)\}\} = \text{rmin}\{ \text{rmin}\{ \mu_{[A]}(x_1), \mu_{[B]}(y_1) \}, \text{rmin}\{ \mu_{[A]}(x_2), \mu_{[B]}(y_2)\}\} = \text{rmin}\{ \mu_{[A]\times[B]}(x_1, y_1), \mu_{[A]\times[B]}(x_2, y_2) \}$. Therefore, $\mu_{[A]\times[B]}[(x_1, y_1)(x_2, y_2)] \geq \text{rmin}\{ \mu_{[A]\times[B]}(x_1, y_1), \mu_{[A]\times[B]}(x_2, y_2)\}$. Now, $\nu_{[A]\times[B]}[(x_1, y_1)-(x_2, y_2)] = \nu_{[A]\times[B]}(x_1- x_2, y_1- y_2) = \text{rmax}\{ \nu_{[A]}(x_1- x_2), \nu_{[B]}(y_1- y_2) \} \leq \text{rmax}\{ \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[A]}(x_2)\}, \text{rmax}\{\nu_{[B]}(y_1), \nu_{[B]}(y_2)\}\} = \text{rmax}\{ \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[B]}(y_1) \}, \text{rmax}\{ \nu_{[A]}(x_2), \nu_{[B]}(y_2)\}\} = \text{rmax}\{ \nu_{[A]\times[B]}(x_1, y_1), \nu_{[A]\times[B]}(x_2, y_2) \}$. Therefore, $\nu_{A\times B}[(x_1, y_1)-(x_2, y_2)] \leq \text{rmax}\{\nu_{[A]\times[B]}(x_1, y_1), \nu_{[A]\times[B]}(x_2, y_2)\}$. Also, $\nu_{[A]\times[B]}[(x_1, y_1)(x_2, y_2)] = \nu_{[A]\times[B]}(x_1x_2, y_1y_2) = \text{rmax}\{ \nu_{[A]}(x_1x_2), \nu_{[B]}(y_1y_2) \} \leq \text{rmax}\{ \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[A]}(x_2) \}, \text{rmax}\{ \nu_{[B]}(y_1), \nu_{[B]}(y_2)\}\} = \text{rmax}\{ \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[B]}(y_1)\}, \text{rmax}\{ \nu_{[A]}(x_2), \nu_{[B]}(y_2)\}\} = \text{rmax}\{ \nu_{[A]\times[B]}(x_1, y_1), \nu_{[A]\times[B]}(x_2, y_2)\}$. Therefore, $\nu_{[A]\times[B]}[(x_1, y_1)(x_2, y_2)] \leq \text{rmax}\{ \nu_{[A]\times[B]}(x_1, y_1), \nu_{[A]\times[B]}(x_2, y_2)\}$. Hence [A]×[B] is an interval valued intuitionistic fuzzy subring of $R_1 \times R_2$.

2.6 Theorem: Let [A] be an interval valued intuitionistic fuzzy subset of a ring R and [V] be the strongest interval valued intuitionistic fuzzy relation of R. Then [A] is an interval valued intuitionistic fuzzy subring of R if and only if [V] is an interval valued intuitionistic fuzzy subring of R×R.

Proof: Suppose that [A] is an interval valued intuitionistic fuzzy subring of a ring R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ in R×R, we have, $\mu_{[V]}(x\text{-}y) = \mu_{[V]}[(x_1, x_2) -(y_1, y_2)] = \mu_{[V]}(x_1- y_1, x_2- y_2) = \text{rmin}\{ \mu_{[A]}(x_1-y_1), \mu_{[A]}(x_2-y_2)\} \geq \text{rmin}\{ \text{rmin}\{\mu_{[A]}(x_1), \mu_{[A]}(y_1) \}, \text{rmin}\{ \mu_{[A]}(x_2), \mu_{[A]}(y_2)\}\} = \text{rmin}\{ \text{rmin}\{ \mu_{[A]}(x_1), \mu_{[A]}(x_2) \}, \text{rmin}\{ \mu_{[A]}(y_1), \mu_{[A]}(y_2)\}\} = \text{rmin}\{ \mu_{[V]}(x_1, x_2), \mu_{[V]}(y_1, y_2) \} = \text{rmin}\{ \mu_{[V]}(x), \mu_{[V]}(y) \}$. Therefore, $\mu_{[V]}(x-y) \geq \text{rmin}\{ \mu_{[V]}(x), \mu_{[V]}(y)\}$, for all x, y in R×R. And, $\mu_{[V]}(xy) = \mu_{[V]}[(x_1, x_2)(y_1, y_2)] = \mu_{[V]}(x_1y_1, x_2y_2) = \text{rmin}\{ \mu_{[A]}(x_1y_1), \mu_{[A]}(x_2y_2) \} \geq \text{rmin}\{ \text{rmin}\{ \mu_{[A]}(x_1), \mu_{[A]}(y_1) \}, \text{rmin}\{ \mu_{[A]}(x_2), \mu_{[A]}(y_2)\}\} = \text{rmin}\{ \text{rmin}\{ \mu_{[A]}(x_1), \mu_{[A]}(x_2) \}, \text{rmin}\{ \mu_{[A]}(y_1), \mu_{[A]}(y_2)\}\} = \text{rmin}\{ \mu_{[V]}(x_1, x_2), \mu_{[V]}(y_1, y_2) \}= \text{rmin}\{ \mu_{[V]}(x), \mu_{[V]}(y) \}$. Therefore, $\mu_{[V]}(xy) \geq \text{rmin}\{\mu_{[V]}(x), \mu_{[V]}(y) \}$, for all x and y in R×R. We have, $\nu_{[V]}(x-y) = \nu_{[V]}[(x_1, x_2) -(y_1, y_2)] = \nu_{[V]}(x_1-y_1, x_2-y_2) = \text{rmax}\{ \nu_{[A]}(x_1- y_1), \nu_{[A]}(x_2- y_2) \} \leq \text{rmax}\{ \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[A]}(y_1) \}, \text{rmax}\{\nu_{[A]}(x_2), \nu_{[A]}(y_2)\}\} = \text{rmax}\{ \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[A]}(x_2) \}, \text{rmax}\{ \nu_{[A]}(y_1), \nu_{[A]}(y_2)\}\} = \text{rmax}\{ \nu_{[V]}(x_1, x_2), \nu_{[V]}(y_1, y_2)\} = \text{rmax}\{\nu_{[V]}(x), \nu_{[V]}(y) \}$. Therefore, $\nu_{[V]}(x-y) \leq \text{rmax}\{\nu_{[V]}(x), \nu_{[V]}(y) \}$, for all x, y in R×R. And, $\nu_{[V]}(xy) = \nu_{[V]}[(x_1, x_2)(y_1, y_2)] = \nu_{[V]}(x_1y_1, x_2y_2) = \text{rmax}\{ \nu_{[A]}(x_1y_1), \nu_{[A]}(x_2y_2) \} \leq \text{rmax}\{ \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[A]}(y_1) \}, \text{rmax}\{\nu_{[A]}(x_2), \nu_{[A]}(y_2)\}\} = \text{rmax}\{ \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[A]}(x_2) \}, \text{rmax}\{ \nu_{[A]}(y_1), \nu_{[A]}(y_2) \}\} = \text{rmax}\{ \nu_{[V]}(x_1, x_2), \nu_{[V]}(y_1, y_2) \}= \text{rmax}\{ \nu_{[V]}(x), \nu_{[V]}(y) \}$. Therefore, $\nu_{[V]}(xy) \leq \text{rmax}\{\nu_{[V]}(x), \nu_{[V]}(y) \}$, for all x, y in R×R. This proves that [V] is an interval valued intuitionistic fuzzy subring of R×R. Conversely assume that [V] is an interval valued intuitionistic fuzzy subring of R×R, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ in R×R, we have $\text{rmin}\{ \mu_{[A]}(x_1-y_1), \mu_{[A]}(x_2-y_2) \} = \mu_{[V]}(x_1- y_1, x_2- y_2) = \mu_{[V]}[(x_1, x_2) -(y_1, y_2)] = \mu_{[V]}(x-y) \geq \text{rmin}\{ \mu_{[V]}(x), \mu_{[V]}(y) \} = \text{rmin}\{ \mu_{[V]}(x_1, x_2), \mu_{[V]}(y_1, y_2) \} = \text{rmin}\{ \text{rmin}\{\mu_{[A]}(x_1), \mu_{[A]}(x_2) \}, \text{rmin}\{ \mu_{[A]}(y_1), \mu_{[A]}(y_2) \}\}$. If we put $x_2 = y_2 = 0$, we get, $\mu_{[A]}(x_1-y_1) \geq \text{rmin}\{\mu_{[A]}(x_1), \mu_{[A]}(y_1)\}$, for all x_1, y_1 in R. And, $\text{rmin}\{ \mu_{[A]}(x_1y_1), \mu_{[A]}(x_2y_2) \} = \mu_{[V]}(x_1y_1, x_2y_2) = \mu_{[V]}[(x_1, x_2)(y_1, y_2)] = \mu_{[V]}(xy) \geq \text{rmin}\{ \mu_{[V]}(x), \mu_{[V]}(y) \} = \text{rmin}\{\mu_{[V]}(x_1, x_2), \mu_{[V]}(y_1, y_2) \} = \text{rmin}\{ \text{rmin}\{ \mu_{[A]}(x_1), \mu_{[A]}(x_2) \}, \text{rmin}\{ \mu_{[A]}(y_1), \mu_{[A]}(y_2)\}\}$. If we put $x_2 = y_2 = 0$, we get $\mu_{[A]}(x_1y_1) \geq \text{rmin}\{ \mu_{[A]}(x_1), \mu_{[A]}(y_1)\}$, for all x_1, y_1 in R. We have $\text{rmax}\{ \nu_{[A]}(x_1-y_1), \nu_{[A]}(x_2- y_2) \} = \nu_{[V]}(x_1-y_1, x_2-y_2) = \nu_{[V]}[(x_1, x_2)-(y_1, y_2)] = \nu_{[V]}(x-y) \leq \text{rmax}\{ \nu_{[V]}(x), \nu_{[V]}(y) \}= \text{rmax}\{ \nu_{[V]}(x_1, x_2), \nu_{[V]}(y_1, y_2)\} = \text{rmax}\{ \text{rmax}\{\nu_{[A]}(x_1), \nu_{[A]}(x_2) \}, \text{rmax}\{\nu_{[A]}(y_1), \nu_{[A]}(y_2) \}\}$. If we put $x_2 = y_2 = 0$, we get $\nu_{[A]}(x_1-y_1) \leq \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[A]}(y_1)\}$, for all x_1, y_1 in R. And, $\text{rmax}\{ \nu_{[A]}(x_1y_1), \nu_{[A]}(x_2y_2) \} = \nu_{[V]}(x_1y_1, x_2y_2) = \nu_{[V]}[(x_1, x_2)(y_1, y_2)] = \nu_{[V]}(xy) \leq \text{rmax}\{\nu_{[V]}(x), \nu_{[V]}(y)\}= \text{rmax}\{\nu_{[V]}(x_1, x_2), \nu_{[V]}(y_1, y_2)\} = \text{rmax}\{ \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[A]}(x_2) \}, \text{rmax}\{ \nu_{[A]}(y_1), \nu_{[A]}(y_2) \}\}$. If we put $x_2 = y_2 = 0$, we get $\nu_{[A]}(x_1y_1) \leq \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[A]}(y_1) \}$, for all x_1, y_1 in R. Therefore [A] is an interval valued intuitionistic fuzzy subring of R.

2.7 Theorem: If [A] is an interval valued intuitionistic fuzzy subring of a ring (R, +, ·), then $H = \{ x / x\in R: \mu_{[A]}(x) = [1], \nu_{[A]}(x) = [0]\}$ is either empty or is a subring of R.

Proof: If no element satisfies this condition, then H is empty. If x, y in H, then $\mu_{[A]}(x-y) \geq \text{rmin}\{ \mu_{[A]}(x), \mu_{[A]}(y) \} = \text{rmin}\{ [1], [1] \} = [1]$. Therefore, $\mu_{[A]}(x-y) = [1]$. And $\mu_{[A]}(xy) \geq \text{rmin}\{ \mu_{[A]}(x), \mu_{[A]}(y) \}= \text{rmin}\{ [1], [1] \} = [1]$. Therefore, $\mu_{[A]}(xy) = [1]$. Now, $\nu_{[A]}(x-y) \leq \text{rmax}\{ \nu_{[A]}(x), \nu_{[A]}(y) \} = \text{rmax}\{ [0], [0] \} = [0]$. Therefore, $\nu_{[A]}(x-y) = [0]$. And $\nu_{[A]}(xy) \leq \text{rmax}\{ \nu_{[A]}(x), \nu_{[A]}(y) \}= \text{rmax}\{ [0], [0] \} = [0]$. Therefore, $\nu_{[A]}(xy) = [0]$. We get x−y, xy in H. Therefore, H is a subring of R. Hence H is either empty or is a subring of R.

2.8 Theorem: Let [A] be an interval valued intuitionistic fuzzy subring of a ring

(R, +, ·). (i) If $\mu_{[A]}(x-y) = [0]$, then either $\mu_{[A]}(x) = [0]$ or $\mu_{[A]}(y) = [0]$, for all x, y in R. (ii) If $\nu_{[A]}(x-y) = [1]$, then either $\nu_{[A]}(x) = [1]$ or $\nu_{[A]}(y) = [1]$, for all x, y in R. (iii) If $\mu_{[A]}(xy) = [0]$, then either $\mu_{[A]}(x) = [0]$ or $\mu_{[A]}(y) = [0]$, for all x, y in R. (iv) If $\nu_{[A]}(xy) = [1]$, then either $\nu_{[A]}(x) = [1]$ or $\nu_{[A]}(y) = [1]$, for all x, y in R.

Proof: Let x and y in R. (i) By the definition $\mu_{[A]}(x-y) \geq \text{rmin}\{ \mu_{[A]}(x), \mu_{[A]}(y) \}$, which implies that $[0] \geq \text{rmin}\{ \mu_{[A]}(x), \mu_{[A]}(y) \}$. Therefore, either $\mu_{[A]}(x) = [0]$ or $\mu_{[A]}(y) = [0]$. (ii) By the definition $\nu_{[A]}(x-y) \leq \text{rmax}\{ \nu_{[A]}(x), \nu_{[A]}(y) \}$, which implies that $[1] \leq \text{rmax}\{\nu_{[A]}(x), \nu_{[A]}(y) \}$. Therefore, either $\nu_{[A]}(x) = [1]$ or $\nu_{[A]}(y) = [1]$. (iii) By the definition $\mu_{[A]}(xy) \geq \text{rmin}\{ \mu_{[A]}(x), \mu_{[A]}(y) \}$ which implies that $[0] \geq \text{rmin}\{ \mu_{[A]}(x), \mu_{[A]}(y) \}$. Therefore, either $\mu_{[A]}(x) = [0]$ or $\mu_{[A]}(y) = [0]$. (iv) By the definition $\nu_{[A]}(xy) \leq \text{rmax}\{\nu_{[A]}(x), \nu_{[A]}(y) \}$ which implies that $[1] \leq \text{rmax}\{ \nu_{[A]}(x), \nu_{[A]}(y) \}$. Therefore, either $\nu_{[A]}(x) = [1]$ or $\nu_{[A]}(y) = [1]$.

2.9 Theorem: If [A] is an interval valued intuitionistic fuzzy subring of a ring (R, +, ·), then □[A] is an interval valued intuitionistic fuzzy subring of R.

Proof: Let [A] be an interval valued intuitionistic fuzzy subring of a ring R. Consider $[A] = \{ \langle x, \mu_{[A]}(x), \nu_{[A]}(x) \rangle \}$, for all x in R, we take $□[A] = [B] = \{ \langle x, \mu_{[B]}(x), \nu_{[B]}(x) \rangle \}$, where $\mu_{[B]}(x) = \mu_{[A]}(x)$, $\nu_{[B]}(x) = [1]- \mu_{[A]}(x)$. Clearly, $\mu_{[B]}(x-y) \geq \text{rmin}\{\mu_{[B]}(x), \mu_{[B]}(y) \}$, for all x, y in R and $\mu_{[B]}(xy) \geq \text{rmin}\{ \mu_{[B]}(x), \mu_{[B]}(y) \}$, for all x, y in R. Since [A] is an interval valued intuitionistic fuzzy subring of R, we have $\mu_{[A]}(x-y) \geq \text{rmin}\{ \mu_{[A]}(x), \mu_{[A]}(y) \}$, for all x, y in R, which implies that $[1]- \nu_{[B]}(x-y) \geq \text{rmin}\{ [1]- \nu_{[B]}(x), [1]- \nu_{[B]}(y) \}$ which implies that $\nu_{[B]}(x-y) \leq [1]- \text{rmin}\{ [1]- \nu_{[B]}(x), [1]- \nu_{[B]}(y) \} = \text{rmax}\{\nu_{[B]}(x), \nu_{[B]}(y) \}$. Therefore, $\nu_{[B]}(x-y) \leq \text{rmax}\{ \nu_{[B]}(x), \nu_{[B]}(y) \}$, for all x, y in R. And $\mu_{[A]}(xy) \geq \text{rmin}\{ \mu_{[A]}(x), \mu_{[A]}(y) \}$, for all x, y in R, which implies that $[1]- \nu_{[B]}(xy) \geq \text{rmin}\{ [1]- \nu_{[B]}(x), [1]- \nu_{[B]}(y) \}$ which implies that $\nu_{[B]}(xy) \leq [1]- \text{rmin}\{ [1]- \nu_{[B]}(x), [1]- \nu_{[B]}(y) \} = \text{rmax}\{ \nu_{[B]}(x), \nu_{[B]}(y) \}$. Therefore, $\nu_{[B]}(xy) \leq \text{rmax}\{ \nu_{[B]}(x), \nu_{[B]}(y) \}$, for all x, y in R. Hence [B] = □[A] is an interval valued intuitionistic fuzzy subring of R.

2.10 Theorem: If [A] is an interval valued intuitionistic fuzzy subring of a ring (R, +, ·), then ◊[A] is an interval valued intuitionistic fuzzy subring of R.

Proof: Let [A] be an interval valued intuitionistic fuzzy subring of R. That is $[A] = \{ \langle x, \mu_{[A]}(x), \nu_{[A]}(x) \rangle \}$, for all x in R. Let $◊[A] = [B] = \{ \langle x, \mu_{[B]}(x), \nu_{[B]}(x) \rangle \}$, where $\mu_{[B]}(x) = [1]-\nu_{[A]}(x)$, $\nu_{[B]}(x) = \nu_{[A]}(x)$. Clearly, $\nu_{[B]}(x-y) \leq \text{rmax}\{ \nu_{[B]}(x), \nu_{[B]}(y) \}$, for all x, y in R and $\nu_{[B]}(xy) \leq \text{rmax}\{ \nu_{[B]}(x), \nu_{[B]}(y) \}$, for all x, y in R. Since [A] is an interval valued intuitionistic fuzzy subring of R, we have $\nu_{[A]}(x-y) \leq \text{rmax}\{ \nu_{[A]}(x), \nu_{[A]}(y) \}$, for all x, y in R, which implies that $[1]-\mu_{[B]}(x-y) \leq \text{rmax}\{ [1]- \mu_{[B]}(x), [1]- \mu_{[B]}(y) \}$ which implies that $\mu_{[B]}(x-y) \geq [1]- \text{rmax}\{ [1]- \mu_{[B]}(x), [1]- \mu_{[B]}(y)\} = \text{rmin}\{ \mu_{[B]}(x), \mu_{[B]}(y) \}$. Therefore, $\mu_{[B]}(x-y) \geq \text{rmin}\{ \mu_{[B]}(x), \mu_{[B]}(y) \}$, for all x, y in R. And $\nu_{[A]}(xy) \leq \text{rmax}\{\nu_{[A]}(x), \nu_{[A]}(y) \}$, for all x, y in R, which implies that $[1]-\mu_{[B]}(xy) \leq \text{rmax}\{ [1]-\mu_{[B]}(x), [1]-\mu_{[B]}(y) \}$ which implies that $\mu_{[B]}(xy) \geq [1]- \text{rmax}\{ [1]-\mu_{[B]}(x), [1]-\mu_{[B]}(y) \} = \text{rmin}\{\mu_{[B]}(x), \mu_{[B]}(y) \}$. Therefore, $\mu_{[B]}(xy) \geq \text{rmin}\{ \mu_{[B]}(x), \mu_{[B]}(y) \}$, for all x, y in R. Hence [B] = ◊[A] is an interval valued intuitionistic fuzzy subring of R.

2.11 Theorem: Let [A] be an interval valued intuitionistic fuzzy subring of a ring (R, +, .), then the pseudo interval valued intuitionistic fuzzy coset $(a[A])^p$ is an interval valued intuitionistic fuzzy subring of R, for every a in R.

Proof: Let [A] be an interval valued intuitionistic fuzzy subring of R. For every x, y in R, we have, $((a\mu_{[A]})^p)(x-y) = p(a)\mu_{[A]}(x-y) \geq p(a)\,\text{rmin}\{\mu_{[A]}(x), \mu_{[A]}(y) \} = \text{rmin}\{p(a)\mu_{[A]}(x), p(a)\mu_{[A]}(y) \}= \text{rmin}\{ ((a\mu_{[A]})^p)(x), ((a\mu_{[A]})^p)(y) \}$. Therefore, $((a\mu_{[A]})^p) (x-y) \geq \text{rmin}\{ ((a\mu_{[A]})^p)(x), ((a\mu_{[A]})^p)(y) \}$, for all x, y in R. Now, $((a\mu_{[A]})^p)(xy) = p(a)\mu_{[A]}(xy) \geq p(a)\,\text{rmin}\{\mu_{[A]}(x), \mu_{[A]}(y) \}= \text{rmin}\{ p(a)\mu_{[A]}(x), p(a)\mu_{[A]}(y) \} = \text{rmin}\{((a\mu_{[A]})^p)(x), ((a\mu_{[A]})^p)(y) \}$. Therefore, $((a\mu_{[A]})^p)(xy) \geq \text{rmin}\{ ((a\mu_{[A]})^p)(x), ((a\mu_{[A]})^p) (y) \}$, for all x, y in R. For every x, y in R, we have, $((a\nu_{[A]})^p)(x-y) = p(a)\, \nu_{[A]}(x-y) \leq p(a)\,\text{rmax}\{ \nu_{[A]}(x), \nu_{[A]}(y) \} = \text{rmax}\{ p(a)\nu_{[A]}(x), p(a)\nu_{[A]}(y) \} = \text{rmax}\{((a\nu_{[A]})^p)(x), ((a\nu_{[A]})^p)(y) \}$. Therefore, $((a\nu_{[A]})^p)(x-y) \leq \text{rmax}\{ ((a\nu_{[A]})^p)(x), ((a\nu_{[A]})^p)(y) \}$, for all x, y in R. Now, $((a\nu_{[A]})^p)(xy) = p(a)\nu_{[A]}(xy) \leq p(a)\,\text{rmax}\{ \nu_{[A]}(x), \nu_{[A]}(y) \} = \text{rmax}\{ p(a)\, \nu_{[A]}(x), p(a)\, \nu_{[A]}(y) \} = \text{rmax}\{ ((a\nu_{[A]})^p)(x), ((a\nu_{[A]})^p)(y) \}$. Therefore, $((a\nu_{[A]})^p)(xy) \leq \text{rmax}\{ ((a\nu_{[A]})^p)(x), ((a\nu_{[A]})^p)(y) \}$, for all x, y in R. Hence $(a[A])^p$ is an interval valued intuitionistic fuzzy subring of R.

2.12 Theorem: If *[A]* is an interval valued intuitionistic fuzzy subring of a ring R, then ?([*A]*) is an interval valued intuitionistic fuzzy subring of R.

Proof: For every x, y in R, we have $\mu_{?([A])}(x-y) = \text{rmin}\{ [½, ½], \mu_{[A]}(x-y) \} \geq \text{rmin}\{ [½, ½], \text{rmin}\{ \mu_{[A]}(x), \mu_{[A]}(y)\}\}= \text{rmin}\{ \text{rmin}\{[½, ½], \mu_{[A]}(x) \}, \text{rmin}\{[½, ½], \mu_{[A]}(y) \}\} = \text{rmin}\{\mu_{?([A])}(x), \mu_{?([A])}(y) \}$. Therefore $\mu_{?([A])}(x-y) \geq \text{rmin}\{\mu_{?([A])}(x), \mu_{?([A])}(y) \}$, for all x, y in R. And $\mu_{?([A])}(xy) = \text{rmin}\{ [½, ½], \mu_{[A]}(xy)\}\geq \text{rmin}\{ [½, ½], \text{rmin}\{\mu_{[A]}(x), \mu_{[A]}(y) \} \}= \text{rmin}\{ \text{rmin}\{ [½, ½], \mu_{[A]}(x) \}, \text{rmin}\{ [½, ½], \mu_{[A]}(y)\}\} = \text{rmin}\{\mu_{?([A])}(x), \mu_{?([A])}(y) \}$. Therefore $\mu_{?([A])}(xy) \geq \text{rmin}\{ \mu_{?([A])}(x), \mu_{?([A])}(y) \}$, for all x, y in R. Also $\nu_{?([A])}(x-y) = \text{rmax}\{ [½, ½], \nu_{[A]}(x-y) \} \leq \text{rmax}\{ [½, ½], \text{rmax}\{ \nu_{[A]}(x), \nu_{[A]}(y) \} \} = \text{rmax}\{ \text{rmax}\{[½, ½], \nu_{[A]}(x) \}, \text{rmax}\{[½, ½], \nu_{[A]}(y)\}\} = \text{rmax}\{ \nu_{?([A])}(x), \nu_{?([A])}(y) \}$. Therefore $\nu_{?([A])}(x-y) \leq \text{rmax}\{ \nu_{?([A])}(x), \nu_{?([A])}(y) \}$, for all x, y in R. And $\nu_{?([A])}(xy) = \text{rmax}\{ [½, ½], \nu_{[A]}(xy) \}\leq \text{rmax}\{ [½, ½], \text{rmax}\{ \nu_{[A]}(x), \nu_{[A]}(y) \}\}$

= rmax { rmax { [½, ½], $\nu_{[A]}$(x) }, rmax { [½, ½], $\nu_{[A]}$(y) }} = rmax { $\nu_{?([A])}$ (x), $\nu_{?([A])}$ (y) }. Therefore $\nu_{?([A])}$(xy) ≤ rmax { $\nu_{?([A])}$(x), $\nu_{?([A])}$(y) }, for all x, y in R. Hence ?([*A*]) is an interval valued intuitionistic fuzzy subring of R.

2.13 Theorem: If [*A*] is an interval valued intuitionistic fuzzy subring of a ring R, then !([*A*]) is an interval valued intuitionistic fuzzy subring of R.

Proof: For every x, y in R, we have $\mu_{!([A])}$ (x–y) = rmax { [½, ½], $\mu_{[A]}$(x–y) }≥ rmax {[½, ½], rmin { $\mu_{[A]}$(x), $\mu_{[A]}$(y) } }= rmin { rmax { [½, ½], $\mu_{[A]}$(x) }, rmax { [½, ½], $\mu_{[A]}$(y) } } = rmin { $\mu_{!([A])}$ (x), $\mu_{!([A])}$(y) }. Therefore $\mu_{!([A])}$(x–y) ≥ rmin { $\mu_{!([A])}$(x), $\mu_{!([A])}$(y) }, for all x, y in R. And $\mu_{!([A])}$(xy) = rmax { [½, ½], $\mu_{[A]}$(xy) } ≥ rmax { [½, ½], rmin { $\mu_{[A]}$(x), $\mu_{[A]}$(y) } } = rmin { rmax { [½, ½], $\mu_{[A]}$(x) }, rmax { [½, ½], $\mu_{[A]}$(y) }} = rmin { $\mu_{!([A])}$(x), $\mu_{!([A])}$(y) }. Therefore $\mu_{!([A])}$(xy) ≥ rmin { $\mu_{!([A])}$(x), $\mu_{!([A])}$(y) }, for all x, y in R. Also $\nu_{!([A])}$ (x–y) = rmin { [½, ½], $\nu_{[A]}$(x–y) } ≤ rmin { [½, ½], rmax { $\nu_{[A]}$(x), $\nu_{[A]}$(y) } } = rmax { rmin { [½, ½], $\nu_{[A]}$(x) }, rmin { [½, ½], $\nu_{[A]}$(y) } } = rmax {$\nu_{!([A])}$(x), $\nu_{!([A])}$(y) }. Therefore $\nu_{!([A])}$(x–y) ≤ rmax { $\nu_{!([A])}$(x), $\nu_{!([A])}$(y) }, for all x, y in R. And $\nu_{!([A])}$ (xy) = rmin { [½, ½], $\nu_{[A]}$(xy) } ≤ rmin { [½, ½], rmax { $\nu_{[A]}$(x), $\nu_{[A]}$(y) }} = rmax { rmin{[½, ½], $\nu_{[A]}$(x) }, rmin { [½, ½], $\nu_{[A]}$(y) }} = rmax { $\nu_{!([A])}$ (x), $\nu_{!([A])}$ (y) }.

Therefore $\nu_{!([A])}$(xy) ≤ rmax { $\nu_{!([A])}$(x), $\nu_{!([A])}$(y) }, for all x, y in R. Hence !([*A*]) is an interval valued intuitionistic fuzzy subring of R.

2.14 Theorem: If [*A*] is an interval valued intuitionistic fuzzy subring of a ring R, then $Q_{\alpha,\beta}$([*A*]) is an interval valued intuitionistic fuzzy subring of R.

Proof: For every x, y in R, for α, β ∈ D[0,1] and α +β ≤ [1], we have $\mu_{Q_{\alpha,\beta}([A])}$(x–y) = rmin { α, $\mu_{[A]}$(x–y) }≥ rmin { α, rmin { $\mu_{[A]}$(x), $\mu_{[A]}$(y) }} = rmin { rmin { α, $\mu_{[A]}$(x) }, rmin { α, $\mu_{[A]}$(y) }} = rmin { $\mu_{Q_{\alpha,\beta}([A])}$(x), $\mu_{Q_{\alpha,\beta}([A])}$(y) }. Therefore $\mu_{Q_{\alpha,\beta}([A])}$(x–y) ≥ rmin { $\mu_{Q_{\alpha,\beta}([A])}$(x), $\mu_{Q_{\alpha,\beta}([A])}$(y) }, for all x, y in R. And $\mu_{Q_{\alpha,\beta}([A])}$(xy) = rmin { α, $\mu_{[A]}$(xy) } ≥ rmin { α, rmin { $\mu_{[A]}$(x), $\mu_{[A]}$(y) } } = rmin { rmin { α, $\mu_{[A]}$(x) }, rmin { α, $\mu_{[A]}$(y) }} = rmin { $\mu_{Q_{\alpha,\beta}([A])}$(x), $\mu_{Q_{\alpha,\beta}([A])}$(y) }. Therefore $\mu_{Q_{\alpha,\beta}([A])}$(xy) ≥ rmin { $\mu_{Q_{\alpha,\beta}([A])}$(x), $\mu_{Q_{\alpha,\beta}([A])}$(y) }, for all x, y in R. Also $\nu_{Q_{\alpha,\beta}([A])}$(x–y) = rmax { β, $\nu_{[A]}$(x–y) } ≤ rmax { β, rmax { $\nu_{[A]}$(x), $\nu_{[A]}$(y) }} = rmax { rmax { β, $\nu_{[A]}$(x) }, rmax { β, $\nu_{[A]}$(y) }} = rmax { $\nu_{Q_{\alpha,\beta}([A])}$(x), $\nu_{Q_{\alpha,\beta}([A])}$(y) }. Therefore $\nu_{Q_{\alpha,\beta}([A])}$(x–y) ≤ rmax { $\nu_{Q_{\alpha,\beta}([A])}$(x), $\nu_{Q_{\alpha,\beta}([A])}$(y) }, for all x, y in R. And $\nu_{Q_{\alpha,\beta}([A])}$(xy) = rmax { β, $\nu_{[A]}$(xy) } ≤ rmax { β, rmax { $\nu_{[A]}$(x), $\nu_{[A]}$(y) } } = rmax { rmax { β, $\nu_{[A]}$(x) }, rmax { β, $\nu_{[A]}$(y) }} = rmax { $\nu_{Q_{\alpha,\beta}([A])}$(x), $\nu_{Q_{\alpha,\beta}([A])}$(y) }. Therefore $\nu_{Q_{\alpha,\beta}([A])}$(xy) ≤ rmax { $\nu_{Q_{\alpha,\beta}([A])}$(x), $\nu_{Q_{\alpha,\beta}([A])}$(y) }, for all x, y in R. Hence $Q_{\alpha,\beta}$([*A*]) is an interval valued intuitionistic fuzzy subring of R.

2.15 Theorem: If [*A*] is an interval valued intuitionistic fuzzy subring of a ring R, then $P_{\alpha,\beta}$([*A*]) is an interval valued intuitionistic fuzzy subring of R.

Proof: For every x, y in R, for α, β ∈D[0,1] and α +β ≤ [1], we have $\mu_{P_{\alpha,\beta}([A])}$(x–y) = rmax { α, $\mu_{[A]}$(x–y) } ≥ rmax { α, rmin { $\mu_{[A]}$(x), $\mu_{[A]}$(y) }} = rmin { rmax { α, $\mu_{[A]}$(x) }, rmax {α, $\mu_{[A]}$(y) } } = rmin { $\mu_{P_{\alpha,\beta}([A])}$(x), $\mu_{P_{\alpha,\beta}([A])}$(y) }. Therefore $\mu_{P_{\alpha,\beta}([A])}$(x–y) ≥ rmin { $\mu_{P_{\alpha,\beta}([A])}$(x), $\mu_{P_{\alpha,\beta}([A])}$(y) }, for all x, y in R. And $\mu_{P_{\alpha,\beta}([A])}$(xy) = rmax { α, $\mu_{[A]}$(xy)} ≥ rmax { α, rmin { $\mu_{[A]}$(x), $\mu_{[A]}$(y) } } = rmin { rmax { α, $\mu_{[A]}$(x) }, rmax { α, $\mu_{[A]}$(y) }} = rmin { $\mu_{P_{\alpha,\beta}([A])}$(x), $\mu_{P_{\alpha,\beta}([A])}$(y) }. Therefore $\mu_{P_{\alpha,\beta}([A])}$(xy) ≥ rmin { $\mu_{P_{\alpha,\beta}([A])}$(x), $\mu_{P_{\alpha,\beta}([A])}$(y) }, for all x, y in R. Also $\nu_{P_{\alpha,\beta}([A])}$(x–y) = rmin { β, $\nu_{[A]}$(x–y) } ≤ rmin { β, rmax { $\nu_{[A]}$(x), $\nu_{[A]}$(y) } } = rmax { rmin { β, $\nu_{[A]}$(x) }, rmin { β, $\nu_{[A]}$(y) }} = rmax { $\nu_{P_{\alpha,\beta}([A])}$(x), $\nu_{P_{\alpha,\beta}([A])}$(y) }. Therefore $\nu_{P_{\alpha,\beta}([A])}$(x–y) ≤ rmax { $\nu_{P_{\alpha,\beta}([A])}$(x), $\nu_{P_{\alpha,\beta}([A])}$(y) }, for all x, y in R. And $\nu_{P_{\alpha,\beta}([A])}$(xy)= rmin {β, $\nu_{[A]}$(xy)} ≤ rmin { β, rmax {$\nu_{[A]}$(x), $\nu_{[A]}$(y)}} = rmax { rmin { β, $\nu_{[A]}$(x) }, rmin { β, $\nu_{[A]}$(y) } } = rmax { $\nu_{P_{\alpha,\beta}([A])}$(x), $\nu_{P_{\alpha,\beta}([A])}$(y) }. Therefore $\nu_{P_{\alpha,\beta}([A])}$(xy)≤ rmax{ $\nu_{P_{\alpha,\beta}([A])}$(x), $\nu_{P_{\alpha,\beta}([A])}$(y)}, for all x, y in R. Hence $P_{\alpha,\beta}$([*A*]) is an interval valued intuitionistic fuzzy subring of R.

2.16 Theorem: If [*A*] is an interval valued intuitionistic fuzzy subring of a ring R, then $G_{\alpha,\beta}$([*A*]) is an interval valued intuitionistic fuzzy subring of R.

Proof: For every x, y in R, for α, β ∈D[0,1] and α +β ≤ [1], we have $\mu_{G_{\alpha,\beta}([A])}$ (x–y) = α $\mu_{[A]}$ (x–y) ≥ α (rmin { $\mu_{[A]}$(x), $\mu_{[A]}$(y) }) = rmin { α $\mu_{[A]}$(x), α $\mu_{[A]}$(y)}= rmin { $\mu_{G_{\alpha,\beta}([A])}$ (x), $\mu_{G_{\alpha,\beta}([A])}$ (y) }. Therefore $\mu_{G_{\alpha,\beta}([A])}$ (x–y) ≥ rmin { $\mu_{G_{\alpha,\beta}([A])}$ (x), $\mu_{G_{\alpha,\beta}([A])}$ (y) }, for all x, y in R. And $\mu_{G_{\alpha,\beta}([A])}$ (xy) = α $\mu_{[A]}$(xy) ≥ α (rmin { $\mu_{[A]}$(x), $\mu_{[A]}$(y) }) = rmin { α $\mu_{[A]}$(x), α $\mu_{[A]}$(y) } = rmin { $\mu_{G_{\alpha,\beta}([A])}$ (x), $\mu_{G_{\alpha,\beta}([A])}$ (y) }. Therefore $\mu_{G_{\alpha,\beta}([A])}$ (xy) ≥ rmin { $\mu_{G_{\alpha,\beta}([A])}$ (x), $\mu_{G_{\alpha,\beta}([A])}$ (y) }, for all x, y in R. Also $\nu_{G_{\alpha,\beta}([A])}$ (x-y) = β $\nu_{[A]}$(x–y) ≤ β (rmax { $\nu_{[A]}$(x), $\nu_{[A]}$(y) }) = rmax { β $\nu_{[A]}$(x), β $\nu_{[A]}$(y) } = rmax { $\nu_{G_{\alpha,\beta}([A])}$ (x), $\nu_{G_{\alpha,\beta}([A])}$ (y) }. Therefore $\nu_{G_{\alpha,\beta}([A])}$ (x-y) ≤ rmax { $\nu_{G_{\alpha,\beta}([A])}$ (x), $\nu_{G_{\alpha,\beta}([A])}$ (y) }, for all x, y in R. And $\nu_{G_{\alpha,\beta}([A])}$ (xy) = β $\nu_{[A]}$(xy) ≤ β rmax { $\nu_{[A]}$(x), $\nu_{[A]}$(y) }) = rmax { β $\nu_{[A]}$(x), β $\nu_{[A]}$(y) }= rmax { $\nu_{G_{\alpha,\beta}([A])}$ (x), $\nu_{G_{\alpha,\beta}([A])}$ (y) }. Therefore $\nu_{G_{\alpha,\beta}([A])}$ (xy) ≤ rmax { $\nu_{G_{\alpha,\beta}([A])}$ (x), $\nu_{G_{\alpha,\beta}([A])}$ (y) }, for all x, y in R. Hence $G_{\alpha,\beta}([A])$ is an interval valued intuitionistic fuzzy subring of R.

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