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A New Approach to Design a Reduced Order Observer

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Abstract:-In this paper, a new method for designing a reduced order observer for linear time-invariant system is proposed. The approach is based on matrix inversion with proper dimension. The arbitrariness associated with the method proposed by O'Reilly is presented here and has been reduced with the help of pole-placement technique. It also helps reducing the computations regarding the observer design parameters. Illustrative numerical examples with simulation results are also included.

Keywords:-Reduced order Observer, Full order Observer, Pole-placement technique, linear time invariant system, Matrix inversion technique

I. INTRODUCTION

When modern techniques are employed in the design of a control system for a dynamic process of plant, a feedback law often results that requires availability of the states of the plant. The state of the plant is not generally accessible in practice, however an estimator or observer of the plant state variable must be designed to estimate the non-available state.There are several authors who have discussed the problem of state estimation. In 1964, D.G. Luenberger proposed the full order observer that presuppose the observer structure[1] and assumes the output of the system to be in standard form[I:0]. He also presupposed reduced order observer [2,3] in which only immeasurable states(i.e. states not available for direct measurement) are estimated. The method proposed by Luenberger was simple and straightforward. O'Reilly also proposed a construction method of reduced order observer which also presuppose the observer structure[4]. The constraints imposed by O'Reilly for the observer to exist cannot be achieved easily in a proper way as it becomes hard to satisfy all the constraints at he same time. It also includes arbitrariness in the design of arbitrary parameters.

In this article, we obtain a simpler approach to design the observer parameters. Here the arbitrary matrix has been transformed into a analytical form by partitioning the matrix into proper dimensions and one element of the derived matrix is replaced by the observer gain matrix using the pole-placement technique. Thus the complexity in designing the observer has been reduced.

II. CONSTRUCTIONS

Let us consider a n-th order plant
$\dot{X} = AX + BU$(1)
With m independent outputs
Y=CX(2)
Where matrices A,B and C have dimensions compatible with X,Y and U.
Let the observer states are
Z=TX (3)
So we can write
$\begin{bmatrix} Y \\ Z \end{bmatrix} = \begin{bmatrix} C \\ T \end{bmatrix} X$(4)
From the above eqn we get X as$X = \begin{bmatrix} C \\ T \end{bmatrix}^{-1} \begin{bmatrix} Y \\ Z \end{bmatrix}$(5)
Now here we define [C T]' as

$\begin{bmatrix} C \\ T \end{bmatrix} = \begin{bmatrix} C_m & 0 \\ N & M \end{bmatrix}$(6)
Where C_m is a m x m matrix
N is a (n-m) x 1 matrix
and M is a (n-m) x (n-m) matrix
now we can find the non-singular matrix$\begin{bmatrix} C \\ T \end{bmatrix}^{-1}$ as

$\begin{bmatrix} C \\ T \end{bmatrix}^{-1} = \begin{bmatrix} C_m^{-1} & 0 \\ -M^{-1}NC_m^{-1} & M^{-1} \end{bmatrix}$(7)

We define the inverse of $\begin{bmatrix} C \\ T \end{bmatrix}$ *as*

$[V \quad P] = \begin{bmatrix} C_m^{-1} & 0 \\ -M^{-1}NC_m^{-1} & M^{-1} \end{bmatrix}$(8)

therefore we can writeV=$\begin{bmatrix} C_m^{-1} \\ M^{-1}NC_m^{-1} \end{bmatrix}$and P=$\begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}$

So from eqn(5) we get

X = [V P]$\begin{bmatrix} Y \\ Z \end{bmatrix}$

Or ,X=VY+PZ(9)

but from eqn 3 we get

$$\dot{Z} = T\dot{X}$$

=T(AX+BU)

=TAX +TBU

=TA(VY+PZ) +TVU

=TAPZ +TAVY + TBU

or, $[\dot{Z}]$ = TAPZ + TAVY +TBU(10)

as we knowT = [N M],A=$\begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix}$, $P = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}$ andV = $\begin{bmatrix} C_m^{-1} \\ M^{-1}NC_m^{-1} \end{bmatrix}$

Putting the value of value T, A, V and P in eqn (10) we get

$\dot{z} = [\mathrm{NM}] \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} z$+ [N M]$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} C_m^{-1} \\ -M^{-1}NC_m^{-1} \end{bmatrix}$ Y + [N M]$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$ U

$\dot{z}$=$[NA_{12}M^{-1} + MA_{22}M^{-1}]$ Z+ $[NA_{11}C_m^{-1} + MA_{21}C_m^{-1} - NA_{12}M^{-1}NC_m^{-1} - MA_{22}M^{-1}NC_m^{-1}]$ Y+ $[NB_1 + MB_2]\, U$ (11)

now putting N=-k in the above eqn we get

$\dot{Z} = [MA_{22}C_m^{-1} - KA_{12}M^{-1}]Z$ +$[MA_{21}C_m^{-1} - KA_{11}C_m^{-1} + KA_{12}M^{-1}NC_m^{-1} + MA_{22}M^{-1}KC_m^{-1}]$ Y + $[NB_1 + MB_2]$ U(12)

This is the dynamic equation of the observer

Now for the estimated state

$$X = \begin{bmatrix} C \\ T \end{bmatrix}^{-1} \begin{bmatrix} Y \\ Z \end{bmatrix}$$

$$X_1 = C_m^{-1}Y$$

$X_2 = -M^{-1}NC_m^{-1}Y + M^{-1}Z$(13)

this is the estimated state vector or observer state vector.

now the observer construction error is given by

∈= Z - TX

So the error dynamics will be

$\dot{\in} = \dot{Z} - T\dot{X}$(14)

fromeq(9) and eq(1) we get

$$\dot{\epsilon} = TAP\,\epsilon + (TAPT + TAVC - TA)X$$

now putting the value of T,A,P and C we get

TAPT + TAVC – TA=0(15)

then we obtain

$$\dot{\epsilon} = TAP\,\epsilon \qquad (16)$$

here the eigen value of TAP lies in the left half on the S-plane.

So as$t \to \infty, \epsilon \to 0$

and the main conditions for the observer to exist also satisfies.

III. NUMERICAL EXAMPLE

The system matrix have been taken as follows [6]

A=$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and C=[1 0 0]

The system has initial condition given by $X_0 = \begin{bmatrix} -.3 \\ -8 \\ 50 \end{bmatrix}$ at t=0 and subjects to unit step input.

The system poles are at [-1 -2 -3].

The solution for the state of the system is given by

$$X = \begin{bmatrix} \frac{18}{5}e^{-t} - \frac{83}{5}e^{-2t} + \frac{188}{15}e^{-3t} + \frac{1}{6} \\ \frac{166}{5}e^{-2t} - \frac{18}{5}e^{-t} - \frac{188}{5}e^{-3t} \\ \frac{18}{5}e^{-t} - \frac{332}{5}e^{-2t} + \frac{534}{5}e^{-3t} \end{bmatrix} \quad (17)$$

and by doing the mathematical calculations defines in the above procedure we obtain the observer eqn as

$$\tilde{X} = \begin{bmatrix} \frac{18}{5}e^{-t} - \frac{83}{5}e^{-2t} + \frac{188}{15}e^{-3t} + \frac{1}{6} \\ \frac{166}{5}e^{-2t} - \frac{18}{5}e^{-t} - \frac{188}{5}e^{-3t} + \frac{122}{5}e^{-15t} + \frac{162}{5}e^{-20t} \\ \frac{18}{5}e^{-t} - \frac{332}{5}e^{-2t} + \frac{534}{5}e^{-3t} + \frac{1708}{5}e^{-15t} + \frac{1458}{5}e^{-20t} \end{bmatrix} \quad (18)$$

Hence the observer is capable of following the state of the original system with the arbitrary initial condition.

IV. DISCUSSIONS

Simulation results of individual system states and corresponding observed states of the numerical examples are plotted using MATLAB and shown In figure 4.1-4.4 . In all of the plots the blue lines (X_i) indicate the system response while the green line (X_i hat) indicate the corresponding observed response.

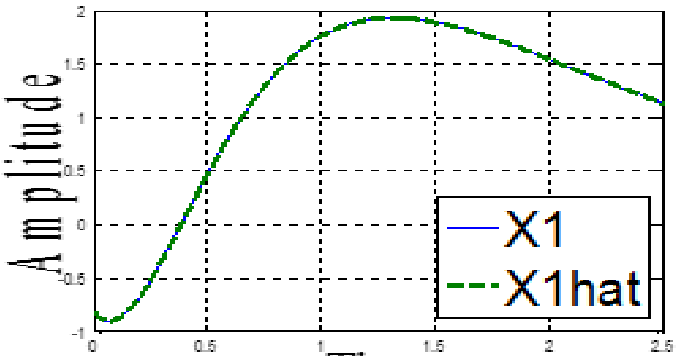

Fig 4.1: Estimated State X1

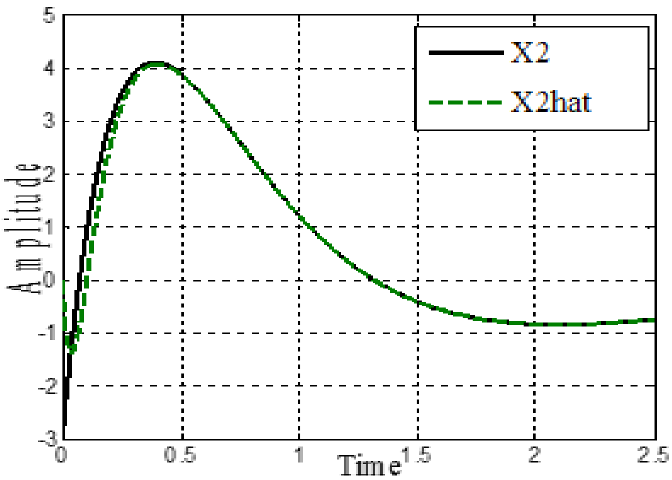

Fig 4.2: Estimated state X2

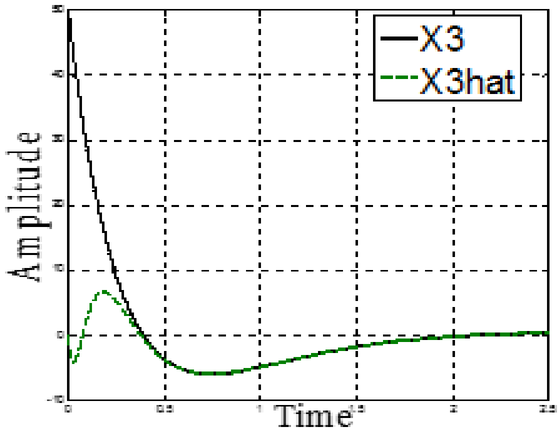

Fig 4.3: Estimated State X3

The responses indicate the observer dynamics is starting from Zero initial condition and converges with system dynamics.

V. CONCLUSIONS

This article presents simple and straightforward design of reduced order observer. The proposed construction method does not presuppose the observer structure. It also imposes no constraint on the rank of the output matrix except the pair {A,C} to be observable.

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REFERENCES

[1]. D.G.Luenberger,"Observing the state of a linear system,IEEE Trk. Mil. Electronics," Vol. MIL-8,pp. 74-80, April. 1964.
[2]. D.G. Luenberger, "Observing the state of a linear system," IEEE Transaction on Automatic control, vol. AC-11, pp. 190-197,April. 1966.
[3]. D.G.Luenberger,"An introduction to observers," IEEE Transaction on Automatic Control, Vol. AC-16, no. 6, pp. 596-602, December ,1971.
[4]. Jhon O'Reilly, "Observer for linear system," New York: Academic, 1983.
[5]. F.A. Graybill, "Introduction to Matrices with application inStatistics, Belmont, CA: Wadsworth,1969
[6]. K.Ogata, "Modern Control Engineering," 5th edition, New Delhi, Prentice Hall of India Pvt Ltd. 2010