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Mantle Convection and Plate Tectonics: A Primary Cause for Earthquakes

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Abstract:- The paper is given a philosophical touch. Starts with a *"Dedication"* followed by the historical aspects of *convection.* A brief review of Mantle Convection and Plate Tectonics facilitates further review of literature. Calculation of Slab velocity and a brief account of effective percentage trench for various plates have been worked out. Seduction boundary movements, Stress and Strain along a fault plane causing earthquakes have been dealt with. Mantle Convection leading to instability of convection has been mathematically analyzed with the help of Navier-Stoke's equation of Plasma Physics arriving at various Rayleigh numbers including the Rayleigh number for the Earth's mantle. The effect of Earth's rotation on Mantle Convection deciding the direction of platemotion, is also considered. The over-all treatment in the paper is such that the reader will feel the matter presented to be more of a specialized subject topic in Geophysics.

Keywords:- Convection, Convective instability, Convective vigor, Earthquakes, Mantle Convection, Navier-Stoke's Equation, Rayleigh number, Rotation of Earth, Slab velocity, Subductive boundary movements, Tectonic Plates.]

I. DEDICATION

Let me dedicate this Paper to RUAUMOKO [10] (Fig. 1), the Maori God of Earthquakes and Volcanoes. The Maoris of ancient New Zealand believed that praying and worshipping Ruaumoko would protect the mankind from the disasters caused by Earthquakes and Volcanoes so do I for the welfare of mankind.

II. PHILOSOPHY

■"The Earthquake" a verse written in about 1750 summarizes this view.[4]

"What powerful hand with force unknown,
Can these repeated tremblings make?
Or do the imprison'd vapors groan?
Or do the shores with fabled Tridents shake?
Ah no! the tread of impious feet,
The conscious Earth impatient bears;
And shuddering with the guilty weight,
One common grave for her bad race prepares."
-Anon

■*Thales of Miletus (6th century BC) believed that an agitation of the sea on which the Earth floats, produced earthquakes*
- Folklore Earthquake Myths

■God may save the King, but nothing can save Plate Tectonics

■*The Earth has a spirit of growth* - Leonardo Da Vinci

[Quoted by Don L Anderson at the start of Chapter 10 Isotopes, Theory of The Earthp.197]

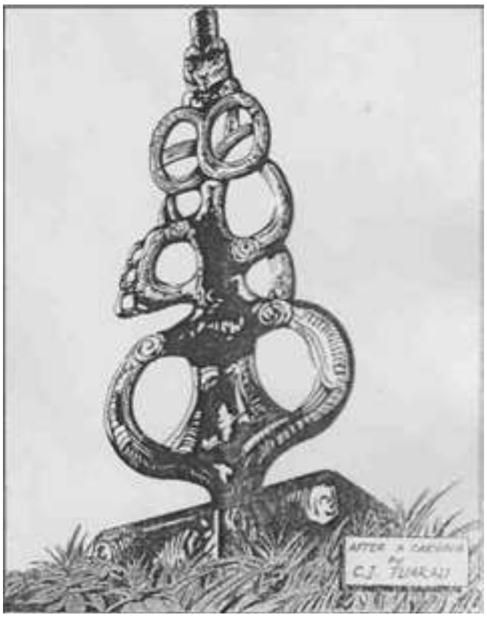


Fig. 1 Ruaumoko, the Maori God of Volcanoes and Earthquakes

Historical [1] The early natural philosophers had the idea of 'Flow' in the Earth's interior and only the outer surface was solid. Descartes imagined the Earth's crust to consist of sedimentary rocks lying over a shell of denser rocks called the mantle below which lies a metallic center called the core. Leibniz proposed that the Earth cooled from an initially molten state and that the deep interior remained molten as a relic of its formation. Edmond Halley has also contributed to the flow of liquids in a network of subsurface channels. Both Isaac Newton and Laplace interpreted the equatorial bulge to be consequence of a fluid like response to its rotation. According to G.H. Darwin (1898), Earth is not only solid to great depths but also "more rigid than steel".
The theory of 'Convection' dates back to the 19th century. The phenomenon of convection was first recognized by Count Rumford (1870) and James Thomson (1882) brother of Lord Kelvin. Exactly at the beginning of the 20th century Henry Benard (1900) noticed that as one surface of the liquid is open to the atmosphere, effect of surface tension has to be considered in addition to thermal convection. The results obtained were in agreement with what was shown later by Lord Rayleigh in 1916. Lord Rayleigh was the first to consider a linear problem of the onset of thermal convection in a horizontal layer. It was Arthur Holmes in 1931 who correctly identified convection as the ultimate driving force for continental motion. A more comprehensive analysis of the problem was given by Pellew and Southwell in the year 1940. Later in the year 1961 S. Chandrasekhar considered the effect of magnetic field and rotation of the Earth in a monograph. As the extensive pioneering work was done by both Lord Rayleigh and Henri Benard, the convection what we deal with is legitimately called "Rayleigh-Benard Convection"
Or Benard-Rayleigh convection or simply Rayleigh convection.

III. MANTLE CONVECTION AND PLATE TECTONICS

"*You all do know this mantle"*

- Shakespeare, JULIUS CAESAR

Mantle is a layer between crust and outer core of the Earth. It is a semi molten silicate rocky shell called magma with an average thickness of 2886 km and makes up about 84% of volume of Earth. Convection currents are created by radioactive decay in the core. Rheology and geometry are two important factors to be considered in the treatment of Earth's mantle. Convection phenomena i.e. the viscosity is strongly dependent on the temperature and the phenomena occurs in a spherical shell domain. Mantle convection is vital for our Earth system. Focusing the attention on these factors one can describe a total approach of numerical simulation of the mantle convection.Mathematical modeling, mathematical analysis, computational scheme, error analysis, etc.The Earth's crust is broken into pieces called plates which are moved by the convection currents. When the convection currents diverge near the Earth's crust, the plates move apart and when the convection currents converge, the plates come together. The movement of the plates and the activity taking place inside the Earth is called Plate Tectonics which is responsible for earthquakes and volcanoes. The point where two plates meet is called a plate boundary. Earthquakes and volcanoes occur either on the boundary or near the boundary.

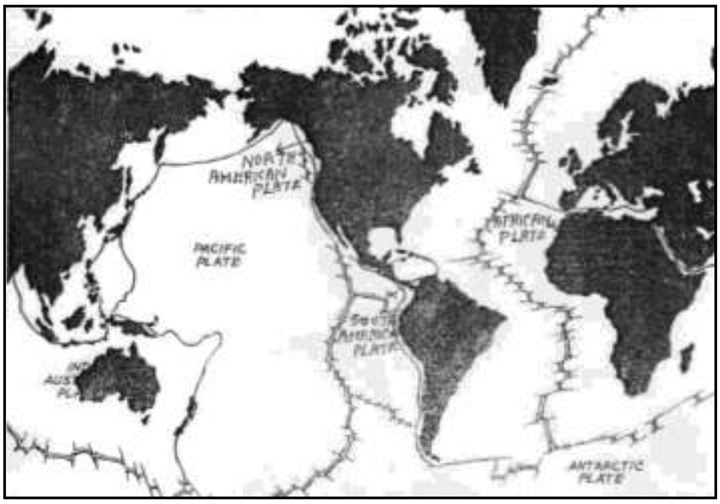


Fig. 2 Various Tectonic Plates

IV. REVIEW OF LITERATURE

When the earth got cooled, the lithosphere cracked and split into seven large and twelve small floating islands with ragged edges: the 'Tectonic Plate' which is a large relatively rigid segment of earth's lithosphere that moves in relation to other plates over the deep interior Plates meet in convergent zones and gets separated at divergent zones. The plates move continuously over the viscous mantle rubbing and pushing against each other and even trying to mount one over the other. (Fig. 2)[4] Most earthquakes occur near plate margins. According to geologists, these global geological or tectonic forces, though not understood in detail, are the consequences of temperature differences in the earth. The plates get heated at the mantle and rise above from where they get cooled and sink down back into the mantle as a subducting slab.That is, it is the sinking slab that pulls the plate. Quoting from David Bercovici'spaper [2], it is like the *'Convective wheels'* driving the tectonic '*Conveyor belt'*. About two thousand years ago Strabo the Greek geographer, historian and philosopher first observed that active seismic regions lay along coastal bandscalled "Rings of Fire" or "Circle of Fire", a name given as earthquakes were attributed to volcanic activity (Fig. 3)

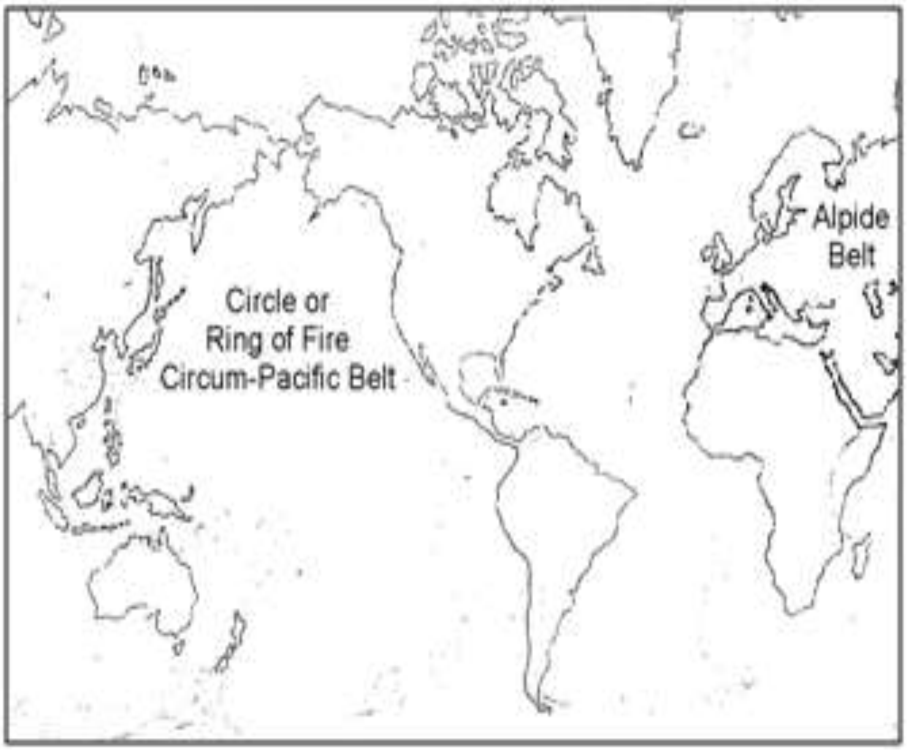


Fig. 3 Earthquake Zones: The Rings or Circle of Fire

The ‘circum pacific belt’ that rings the coasts of Pacific Ocean and the ‘Alpide belt along the southern boundary of the Eurasian plate, that cuts from the Atlantic Ocean across the Mediterranian into Asia. As the ancient western world was centered in the Mediterranean region along the Alpide belt, it was easy for the Greek philosophers to understand and explain the origin of earthquakes

David Bercovici[2] has extensively dealt with the “Generation of Plate Tectonics from Mantle Convection” and has hailed as the grand unifying principle of geology. It has been the focus of an active yet technically difficult field of research. Planetary interiors are gravitationally unstable. Being cold at the top than at the interior, they tend to convect. The convection currents drive the tectonic plates which on an average move about 10 cm per year over the mantle (Fig.4).

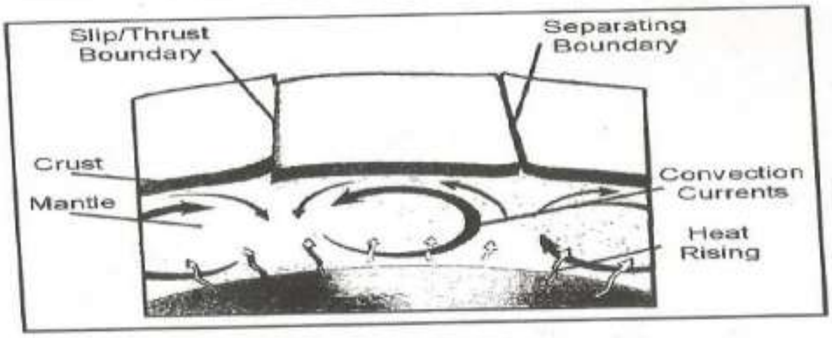


Fig. 4 Tectonic Plates driven by convection currents

The slab velocity can be worked out from the heat loss [2] due to injection of cold materials by subducting slabs. The equation for energy flux balance can be written as

$f Q = vA \rho c \Delta\theta$ ….(1)

where Q is the net heat output through the top of the mantle and can be approximated to 30 TW (=30 x 10^{12}W). As the remaining heat transport by mantle accounts for less than 10% of the net heat flux, the fraction f of Q accounting for by slab cooling can be taken as 0.9. The slab density $\rho = 3000\frac{\mathrm{kg}}{\mathrm{m}^3}$, the specific heat capacity c = $1000\frac{\mathrm{J}}{\mathrm{kg}\,{}^0\mathrm{K}}$. The average temperature difference can be taken as 700^0K. v the typical vertical velocity of the slab. The horizontal cross sectional area A of all slabs crossing at a particular depth δ = 100 km can be taken as $2\pi R\delta$, where R is the radius of theEarth approximated to 6000 km. Thus we get for the slab velocity from equation (1) as

$$v = \frac{fQ}{(\rho\, c\, \Delta\theta\, 2\, \pi\, R\, \delta)} \approx 10\frac{\mathrm{cm}}{yr} \ldots.\ (2)$$

is a typical value coinciding with the velocity of the Pacific Plate The India tectonic plate moves north at about 4.5 cm per year is pushing under the Eurasian plate beneath the Himalayas along a thrust fault line.

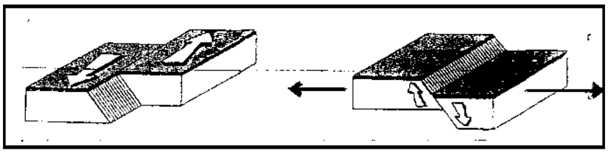
Even though the trench is just few cm per year, Masaki Ogawa [9]says that the kinetic energy of the convective flow is comparable to the kinetic energy of a fast moving car on a freeway.

The percentage of trench or subductive zone versus plate velocity v is given in Table No.1. (Credit: David Bercovici[2] Values approximated to from graph of Fig.2, p-110, “*Generation of Plate Tectonics from Mantle Convection”*)’

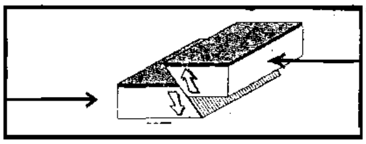
Table N0. 1

Name of Plate	**Effective Percentage Trench**	**Plate Velocity**
North American Plate	Less than 5%	Less than $2\frac{\mathrm{cm}}{\mathrm{Yr}}$
South American Plate	~2%	Less than $2\frac{\mathrm{cm}}{\mathrm{Yr}}$
African Plate	~3%	Less than $2\frac{\mathrm{cm}}{\mathrm{Yr}}$
Indo Australian Plate	~ 20%	Between $5\frac{\mathrm{cm}}{\mathrm{Yr}}$ and $10\frac{\mathrm{cm}}{\mathrm{Yr}}$
Philippine Plate	~28%	Between $5\frac{\mathrm{cm}}{\mathrm{Yr}}$ and $10\frac{\mathrm{cm}}{\mathrm{Yr}}$
Nazca Plate	~27%	Between $5\frac{\mathrm{cm}}{\mathrm{Yr}}$ and $10\frac{\mathrm{cm}}{\mathrm{Yr}}$
Pacific Plate	~24%	Between $5\frac{\mathrm{cm}}{\mathrm{Yr}}$ and $10\frac{\mathrm{cm}}{\mathrm{Yr}}$
Cocos Plate	~27.5%	Between $5\frac{\mathrm{cm}}{\mathrm{Yr}}$ and $10\frac{\mathrm{cm}}{\mathrm{Yr}}$

As one plate is driven against its neighbor, it will slip along the boundary (Fig.5a). It may even slide down. (Fig.5b). Sometimes, one plate, usually a continental plate will thrust up over the oceanic plate. (Fig.5c). These three sections occur atconvergent or subduction boundaries, regions where volcanic eruptions and earthquakes frequently occur.



a) The SLIP b) The Normal DIP



c) The THRUST

Fig. 5 Subduction Boundary Movements

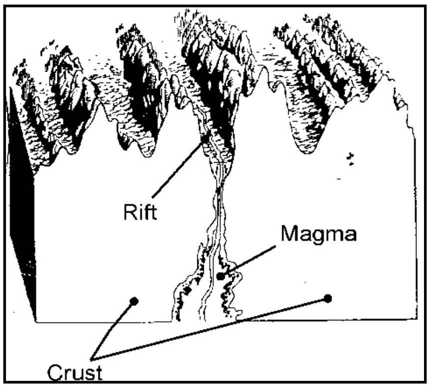


Fig. 6 The Mid Ocean Rift

At those tectonic boundaries where plates tend instead to separate mainly along the mid-ocean rifts (Fig. 6), red hot magma flows up from the mantle to fill the void left by the spreading plates and cools rapidly when meeting the frigid sea water, which also causes cracking in this new crustal material of basalt.

The process gets repeated, new material is constantly being added to the crust to make up for that which plunges back into the mantle and is re- melted along a subduction

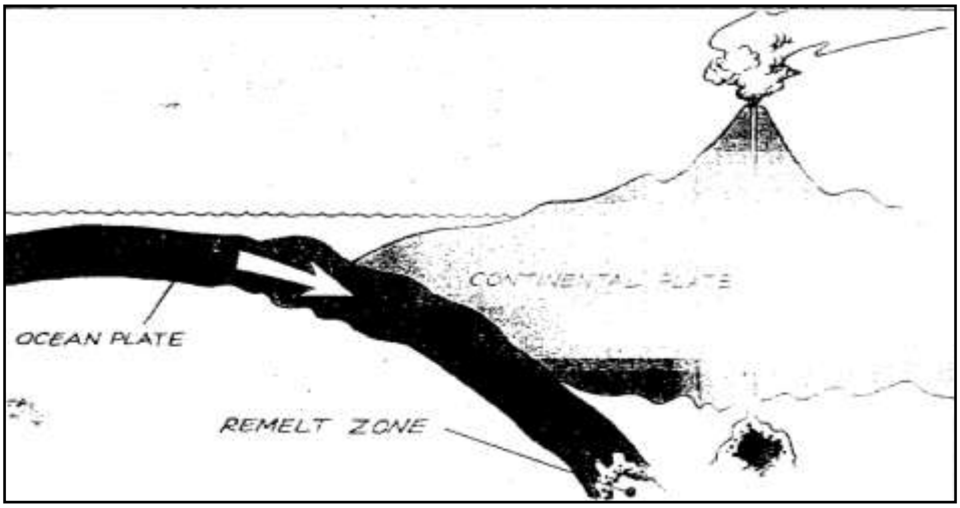
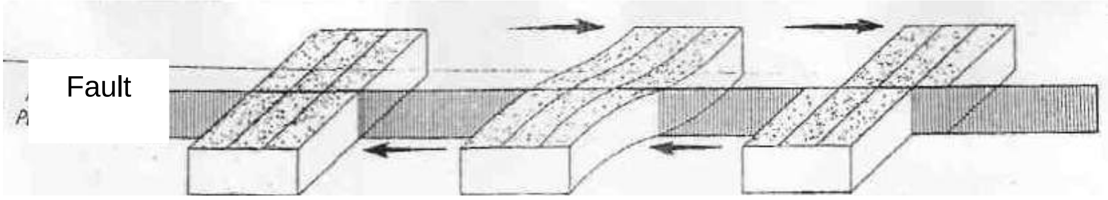


Fig.7 the Subduction Boundary (Fig.7)↓

Fig. 3.7 Subduction Boundary

The constant movement of the tectonic plates is resisted along their convergent boundaries by friction between their rough edges, akin to two bricks being rubbed against each other. Over a period of time this creates stress and strain along the boundary. This will continue till a stress level is reached sufficient to overcome the frictional resistance of the plates and ultimately causing a sudden slip (Fig.8). This is how an earthquake occurs.



Original Boundary **Strain Building** **Strain Release**

Fig. 8 Strain along a Fault Plane

Generally, many earthquakes occur along the convergent boundaries of the continental coastlines and their magnitudes are very high. Minor earthquakes occur along mid-ocean separating boundaries. Approximately 70% of the continental earthquakes take place along the perimeter of the Pacific plate and 20% along the alpide belt, with the remaining 10% scattered around the globe.

A country which is frequently struck by earthquakes and experiencing heavy damages almost every year is Japan. The earthquakes there are mostly caused by the rupture of active faults occurring in shallow part of the inland regions. An ‘active fault’ is a fault created by repeated earthquakes of the past in the same region and paving way for another major quake in that region. Toshikazu Yoshioka [13] has evaluated rupture probabilities of active faults using the Cascade Earthquake Model based on behavioral segmentation. In order to study critically the possibility of occurrence of earthquakes of large magnitudes that are dependent on the active faults, the author of the paper has divided active faults into behavioral segments and making use of the Earthquake Cascade Model. By cascade model is meant that the earthquake is caused by a single segment or multiple segments. The rupture probability of active faults can be very well determined by such method independent of the field data.

Irrespective of the value of the slippage of a fault, which if repeatedly happens to be in the same direction, a displacement of several meters might accumulate. If the displacement is in the vertical direction, the up thrust side will be converted into a mountain and the bottom side becomes a plain or basin. If there is side-way displacement, the valleys and peaks get bent. As great earthquakes are caused by rupture of active faults, an evaluation of rupture probability is very important.

After the earthquake of 1906 in California, USA, scientist Harry Fielding Reid, geologist at John Hopkins University, USA gave a successful explanation as to how energy get released during an earthquake based on Elastic Rebound Theory. As rocks on opposite sides of a fault are subjected to force and shift, they accumulate energy and slowly get deformed until their internal strength is exceeded. At that time, a sudden movement occurs along the fault releasing the accumulated energy and the rocks snap back to their original un-deformed shape. This energy causes an Earthquake which in turn releases elastic waves of tremendous energy called seismic waves throughout the Earth. The seismic waves generated by an earthquake are the best source for studying the interior of the Earth. The primary cause for tectonic plate movements is the mantle convection and the physical processes taking place is shown in the following cascade:

Mantle Convection → Plate Tectonics → Earthquake → Seismicity

Now, the rupture of active faults or the movement of the tectonic plates is an after-effect and as a physicist, following *“Cause and Effect Formalism”,* if the movement of the tectonic plates is the effect so as to cause an earthquake, then from the theoretical considerations described above, the cause for the movement of the tectonic plates is convection. More than convection, it is the *‘Instability of Convection’* creating a driving force that is a responsible for the formation of tectonic plates which is invariably a form of convectiveself-organization. The molten material such as the semi-fluid magma or lava from the core and mantle is akin to weakly ionized plasma, the motion of which falls in the realm of *Fluid Mechanics* the laws of which can be applied to both liquids and gases (fluids). Hans-Peter Bunge[8] says that the Earth’s mantle although stronger than steel capable of transmitting seismic shear waves can be treated as a fluid and can be dealt by hydrodynamic field equations expressing the fundamental principles of mass, momentum and energy conservation.

Convection currents develop in the earth’s mantle due to difference in temperatures between the crust and the core. The convection causes subduction boundary movements of the tectonic plates which ultimately is the cause for earthquakes. The question now is, “what is the cause for convection and for its instability?” For an answer, we have to deal with some mathematical treatment.

For a generalized mathematical treatment given below, it is legitimate to use the word, ‘Gas’ for the material of the molten fluid under consideration thereby falling in line with the usual treatment found elsewhere in which terms, ‘Gas’ and ‘Fluid’ are synonymously used.

We follow the theory given by B.M. Smirnov [12] and start with an equation first developed by French engineer, C.LM.H. Navier (1823) and Irish scientist, George. G. Stokes (1845) known as the Navier -Stoke’s equation:

$$\frac{\partial v}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{\text{grad } p}{\rho} + \frac{\eta}{\rho}\nabla^2\mathbf{v} + \frac{\eta}{3\rho}\text{ grad div }\mathbf{v} + \frac{\mathbf{F}}{M} \qquad \dots (3)$$

along with the equation of continuity:

$$\frac{\partial N}{\partial t} + \text{div } (N\mathbf{v}) = 0 \qquad \dots (4)$$

and the equation of heat transport;

$$\frac{\partial \theta}{\partial t} + \mathbf{v}\text{ grad }\theta = \frac{\chi}{Nc_v}\nabla^2\theta \qquad \dots (5)$$

where

N is the number density of gas
Particles; p, the gas pressure;
θ, the temperature; c_v the specific heat per molecule; η, the viscosity;
χ, the thermal conductivity; M, the mass of a gas molecule; **F,** the force acting on one molecule; **v,** the mean velocity referred to as the drift velocity and $\rho = MN$ is the mass density.

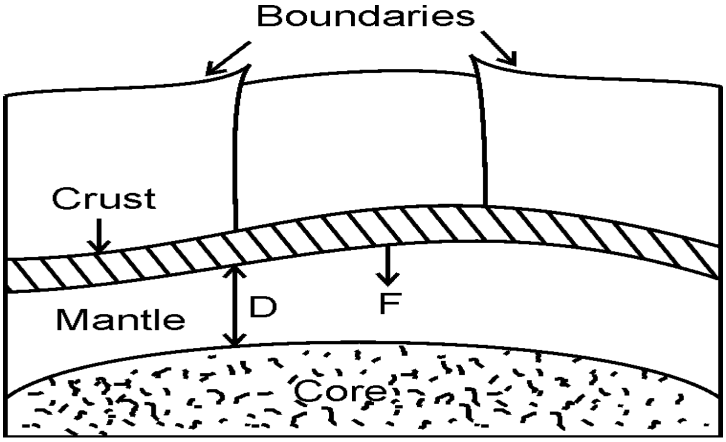


Fig. 9 Pertaining to Rayleigh Problem

When the temperature gradient is large in a gas which is in a field of external forces, there may appear a more effective mechanism of heat transport than thermal conduction and that is nothing but ‘Convection’. This process consists in the movement of the warmer gas into the cooler regions such as the crust of the earth and the cooler gas into the hot regions such as the mantle of earth.

We have to analyze the stability of gas at rest with the possibility of development of convection. Consider a gas at rest in which a temperature gradient is maintained in the field of external forces. The parameters of the gas are subjected to a small perturbation which is due to the slow motion of the gas and corresponds to convection. If this perturbation proves to be possible, convective instability can develop in such a gas. We have to find the conditions needed for the convective instability to occur so as to have convective heat transport.

The simplest problem can be studied by looking upon the so-called ‘Rayleigh problem’ of the weakly ionized plasma. The mantle lies between the crust and the core. The lower boundary of the crust and the upper boundary of the core can be taken as two infinite parallel walls and the hot fluid fills between them. The temperature of the lower wall is θ_1, the temperature of the upper wall is $\theta_2$with $\theta_1$higher than θ_2(Fig. 9). The force of the external field is directed downwards and perpendicular to the walls. Let D be the distance between the walls. We shall find out the conditions for convective instability.

Let us represent the parameters of the gas as sums of two terms: the first term is the parameter for the gas at rest and the second term is a small perturbation of the parameter due to the convective motion of the gas. Thus, the gas density is N + N’, the gas pressure is $p_0 + p'$, the gas temperature is $\theta + \theta'$ and the gas velocity is **v**, which is zero in the absence of convection. Now, insert these parameters into the stationary equations *of* continuity (4), of the Navier-Stoke’s equation (3) and of heat transport (5). The zero order approximation is,
$\mathrm{grad}\ p_0 = -\mathbf{F}N,\ \nabla^2\theta = 0$
In the first small-parameter approximation, these equations yield

$$\left.\begin{aligned} &\mathrm{div}\mathbf{v} = 0,\\ &-\frac{\mathrm{grad}\,(p_0 + p')}{(N + N')} + \frac{\eta\nabla^2 v}{(N + N')} + \mathbf{F} = 0\\ &v_z\left[\frac{(\theta_2 - \theta_1)}{D}\right] = \frac{\chi}{Nc_{\mathbf{v}}}\nabla^2\theta' \end{aligned}\right\} \quad \ldots (6)$$

The parameters of the above problem are used in the last equation. Here, the z-axis is perpendicular to the walls.

Transform the first term in the second equation of (6). Upto the first order of approximation, the term is,

$$\frac{\mathrm{grad}\,(p_0 + p')}{(N + N')} = \frac{\mathrm{grad}\,p_0}{N} + \frac{\mathrm{grad}\,p'}{N} - \frac{\mathrm{grad}\,p_0}{N}\left(\frac{N'}{N}\right) = \mathbf{F}\left(\frac{N - N'}{N}\right) + \frac{\mathrm{grad}\,p'}{N}$$

According to the equation of state, $p = N\theta$, $N = p/\theta$ and hence we find that

$$N' = \left(\frac{\partial N}{\partial \theta}\right)_p \theta' = -\left(\frac{N\theta'}{\theta}\right)$$

Inserting this relation into the second equation of (6), we can write the system of equations (6) as

$$\left.\begin{array}{l} \mathrm{div}\,\mathbf{v} = 0, \\ \dfrac{\mathrm{grad}\, p'}{N} - \mathbf{F}\left(\dfrac{\theta'}{\theta}\right) - \dfrac{\eta}{N}\nabla^2 v = 0 \\ v_z = \dfrac{\chi D}{N c_v (\theta_2 - \theta_1)} \nabla^2 \theta' \end{array}\right] \quad \ldots (7)$$

Let us reduce the system of equations (7) which connect the parameters of the gas, to an equation for one parameter. First, we apply to the second equation of (7) the operator ‘div’ and take into account the first equation of (7). We find that

$$\frac{\partial^2 p'}{N} - \left(\frac{\mathbf{F}}{\theta}\right)\frac{\partial \theta'}{\partial z} = 0 \quad \ldots (8)$$

As $(\theta_1 > \theta_2)$, $\left[\frac{(\theta_1 - \theta_2)}{\theta_1}\right] << 1$ and hence the undisturbed parameters of the gas do not vary much inside the volume being considered. We shall neglect their variation and assume that the unperturbed gas parameters are spatially constant.

Inserting w_z from the third equation of (7) into the zth component of the second equation and making use of the operator ∇^2,

$$\frac{1}{N}\frac{\partial}{\partial z}(\nabla^2 p') - \mathbf{F}\left(\frac{\nabla^2\theta'}{\theta}\right) + \left[\frac{\eta\chi D}{N^2 c_v (\theta_2 - \theta_1)}\right](\nabla^2)^3\theta' = 0$$

Combining this with equation (8) we get finally

$$(\nabla^2)^3\theta' = -\frac{R}{D^4}\left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)\theta' \quad \ldots (9)$$

Where the dimensionless combination of parameters

$$R = \left[\frac{(\theta_1 - \theta_2) c_v \mathbf{F} N^2 D^3)}{\eta\chi\theta}\right] \quad \ldots (10)$$

R is called as the Rayleigh Number and is a dimensionless quantity which can be brought to a standard form. Writing $\Delta\theta$ for $(\theta_1 - \theta_2)$ and for unit mass, $\rho = MN = N$ and the force $\mathbf{F} = g$, the acceleration due to gravity, we can write R as

$$R_a = \left[\frac{\Delta\theta c_v \mathbf{g}\rho^2 D^3)}{\eta\chi\theta}\right] \quad \ldots (10a)$$

The reciprocal of temperature θ appearing as a multiple in this equation is the volumetric expansion β of the fluid which brings in the effect of buoyancy in free convection and for an ideal gas we can write $\beta = \frac{1}{\theta}$, where θ is the absolute temperature of the gas* With this the Rayleigh number takes the standard form

$$R_a = \left[\frac{(\beta\Delta\theta c_v \mathbf{g}\rho^2 D^3)}{\eta\chi}\right] \quad \ldots (11)$$

R_a is thus the ratio, buoyant force divided by the product of the viscous drag and the rate of heat diffusion. In the treatment of mantle convection, $\Delta\theta$is the super adiabatic temperature across the shell of domain thickness D which is usually greater than 2500 km.

∗{It follows from elementary thermodynamics that by definition β is the rate of decrease of density with temperature per unit change of density leading to buoyancy. That is, $\beta = -[\frac{1}{\rho}(\frac{\partial\rho}{\partial T})_P]$. Minus sign indicating decrease. Now, for an ideal gas, pressure $P = \rho RT$ where T is the absolute temperature or $\rho = \frac{P}{RT}$. Differentiating by keeping the pressure constant, $(\frac{d\rho}{dT})_P = -\frac{P}{RT^2} = -(\frac{\rho RT}{RT^2}) = (-\frac{\rho}{T})$ and finally, $\beta = [-\left(\frac{1}{\rho}\right)(-\frac{\rho}{T})] = \frac{1}{T} = \frac{1}{\theta}$, where we have replaced T by θ}

Equation (9) shows that the Rayleigh number determines the possibility of development of convection. For example, in the Rayleigh problem, the boundary conditions at the walls are $\theta' = 0$, $v_z = 0$. Also the tangential forces, $\eta\left(\frac{\partial v_x}{\partial z}\right)$ and $\left(\frac{\partial v_y}{\partial z}\right)$ are zero at the walls. Differentiating the equation, div $\mathbf{v} = 0$ with respect to z and using the conditions for the tangential forces, we find that at the walls $\left(\frac{\partial^2 v_z}{\partial z^2}\right) = 0$. Hence, we have the following boundary conditions.

$$\theta' = 0, \quad v_z = 0 \quad \text{and} \left(\frac{\partial^2 v_z}{\partial z^2}\right) = 0$$

Denote by $z = 0$ the coordinate of the lower wall. The general solution of equation (7) with the boundary condition, $\theta' = 0$ at $z = 0$ can be expressed as

$$\theta' = C \exp [i(k_x x + k_y y)] \sin k_z z \qquad \dots (12)$$

The boundary condition, $\theta' = 0$ at $z = D$ yields $k_z D = \pi n$ where n is an integer. Inserting the solution given by equation (12) into equation (9), we get

$$R = [\frac{(k^2 D^2 + \pi^2 n^2)}{k^2 D^2}] \qquad \dots (13)$$

Where $k^2 = k_x{}^2 + k_y{}^2$. The solution given by equation (12) satisfies all boundary conditions.

Lord Rayleigh showed that the critical wave numberk_c for two stress-free boundaries is $\frac{\pi}{(D\sqrt{2})}$. With this and keeping n=1, we can get the value of critical Rayleigh number, R_{ac} from equation (10) as follows:

$$R_{ac} = \frac{(k_c{}^2 D^2 + \pi^2 n^2)^3}{k_c{}^2 D^2}$$

$= \frac{(\frac{\pi^2}{2} + \pi^2)^3}{\frac{\pi^2}{2}} = \frac{[\pi^2 (1+\frac{1}{2})]^3}{\frac{\pi^2}{2}} = \frac{2\pi^6 (\frac{3}{2})^3}{\pi^2} = 2\pi^4 (\frac{27}{8}) = (\frac{27}{4}) \pi^4 = 657.51 \approx 658$. Thus, 658 is the minimum Rayleigh number R_{min} required for convection. That is,

$$R_{a(min.)} \approx 658$$

This is the minimum value of Rayleigh number required for readily induced and free convection. The magnitude of R_{min}, however varies according to the geometry of the problem, but in all cases it is the Rayleigh number that characterizes the possibility of convection.

Instability of convective motion will occur only for Rayleigh numbers of very high values which can disturb the ordered convective flow and finally disrupts the stability of convective motion of gas giving rise to disordered or turbulent flow of gas even if it is contained in a resting closed system.. The types of motion for another two high Rayleigh numbers 2084 and 9215 are shown in Fig.11 and Fig.12 respectively

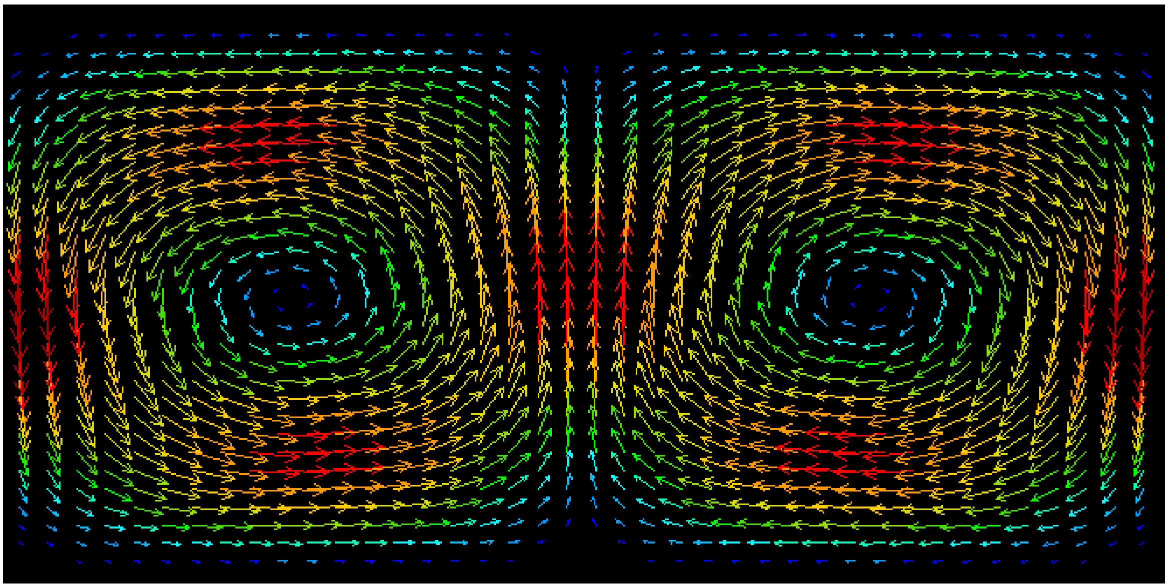


Fig. 11 The types of convective motion for Large Rayleigh number Ra=2084

The critical Rayleigh number for different types of boundaries is given Table No.2

Table No. 2 [7]

Type of Boundary	**Critical Wave number k_c**	**Critical Rayleigh number R_{ac}**
2 Stress-free boundaries	$\frac{\pi}{\sqrt{2}} = 2.221$	$\frac{27}{4}\pi^4 = 657.51 \approx 658$
2 Rigid boundaries	3.117	1707.8
1 Rigid and 1 stress-free boundaries	2.682	1100.66

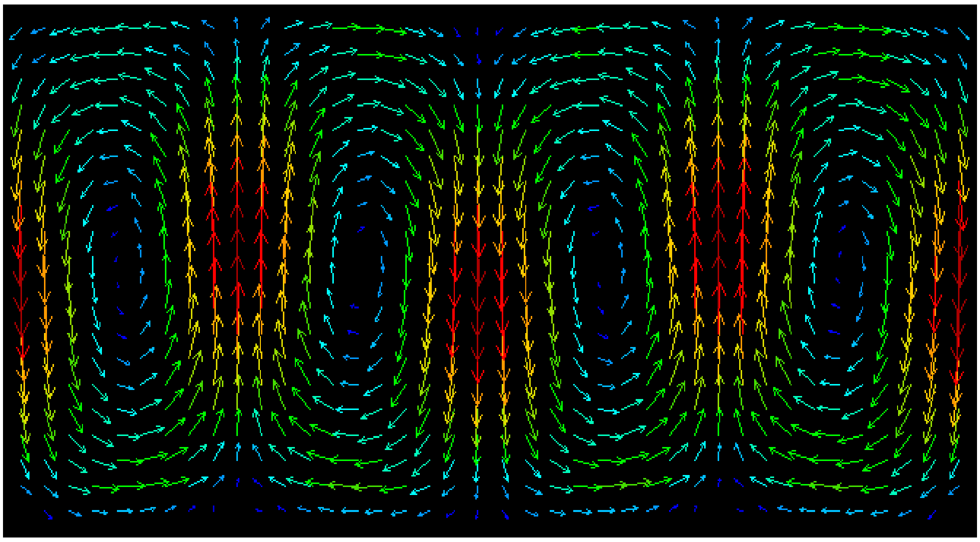


Fig.12 The type of convective motion for a very large Rayleighnumber, $R_a = 9215$

As convection leading to turbulence is ideal for tectonic plate movements, let us analyze the development of turbulent gas flow.

Going for a still higher value of R_a, say $R_a = 108\pi^4 = 10520$ which is some 16 times R_{min}. In this case there can simultaneously develop two different types of convective motion. Fig.13[12] shows two types of convective motion for the Rayleigh number $108\pi^4$ corresponding to the wave number $k_1 = \frac{9.4}{D}$ for n = l and $k_2 = \frac{4.7}{D}$ for n = 2. The mixing of the gas flows travelling in opposite directions finally results in a random gas motion or turbulence. The higher the R_a the convection becomes vigorous. In other words, R_a is a measure of convective vigor

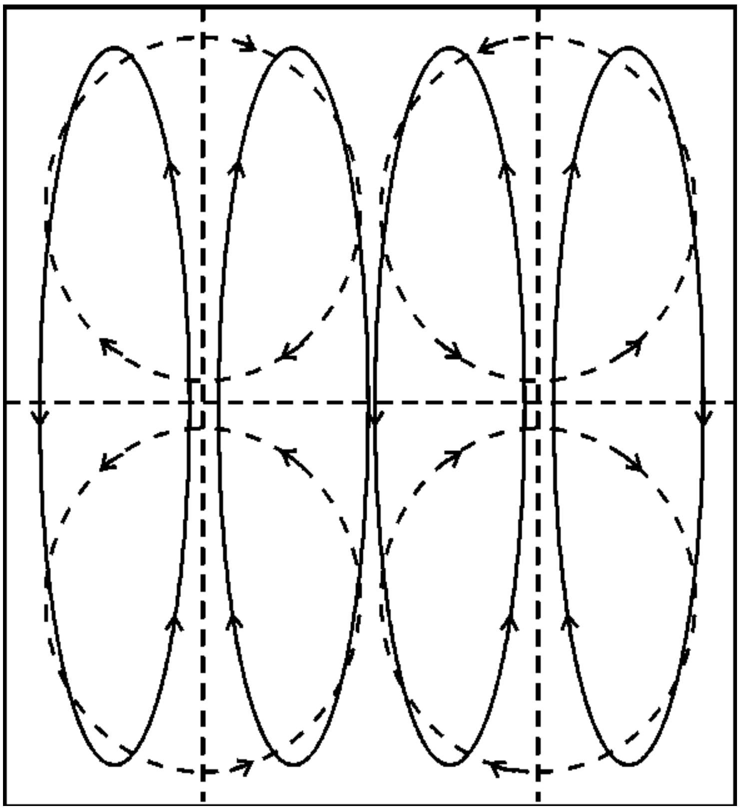


Fig. (13) Paths of the gas elements for Rayleigh number, $R_a = 108\pi^4 = 10520$

It is of interest now to work out the Rayleigh number for the Earth's mantle.

Typical values taken from David Bercovici [3] is as follows:

Density $\rho \approx 4000 \frac{\mathrm{kg}}{\mathrm{m}^{3,}}$; acceleration due to gravity g $\approx 10 \frac{m}{s^2}$; cubical expansion $\beta \approx 3 \mathrm{x} 10^{-5}/{}^0\mathrm{K}$; $\Delta\theta = 3000^0\mathrm{K}$; D $\approx$2900 km; viscosity $\eta \approx 10^{22}$ Pas (lower mantle); Thermal diffusivity $\mathrm{k} \approx 10^{-6} \frac{m^2}{s}$.. With these values, the standard form for $\mathrm{R_a}$ given by equation (10 a) requires modification in order to introduce thermal diffusivity in the equation. As thermal diffusivity, $\mathrm{k} = \frac{\text{Thermal conductivi ty}}{(\text{Specific heat})(\text{Density})} = \frac{\varkappa}{c_{\mathrm{v}}\rho}$, the equation (10 a) takes the form $\mathrm{R_a} = \frac{\beta\,\Delta\theta\,g\,\rho\,D^3}{\eta(\frac{\chi}{c_{\mathrm{v}}\rho})} = \frac{\beta\,\Delta\theta\,g\,\rho\,D^3}{\eta\mathrm{k}}$ where k is the thermal diffusivity. Substituting for the values we get for $\mathrm{R_a} = \frac{3 \mathrm{~x~} 10^{-5} \mathrm{x~} 3000 \mathrm{~x~} 10 \mathrm{~x~} 4000 \mathrm{~x~} [2900 \mathrm{~x~} 10^3]^3}{10^{22} \mathrm{x} 10^{-6}} = \frac{8.78 \mathrm{~x~} 10^{22}}{10^{16}} = 8.78 \mathrm{~x~} 10^6 \approx 10^7$ which is beyond supercritical.

The fact that increasing the Rayleigh number facilitates the convective flow to become turbulent as other wise the system has an ordered convective flow except that a small perturbation somewhere in one of the regions of the gas volume gives rise to another type of flow. At the boundary wall of the region two opposite gas flows meet such that the kinetic energy of motion of the gas flows gets transformed into the thermal energy of the gas. This results in a disordered motion of the gas. The development of turbulence not only changes the character of heat transport but facilitates tectonic plate movements.

III. Effect of Rotation of the Earth

The generation of Plate Tectonics from Mantle Convection is also influenced by the rotation of the Earth as mathematically explained by Shuping Chen [11] The mantle convection and rotation of the Earth affect each other, the former the power source in breaking the lithosphere plate and the latter the direction of convection and plate motions. The type of plate motion depends on the mantle up wells which are controlled by the rotation of the Earth. The geometric shapes of the plate boundaries can adjust the direction of plate movement. The author in his research paper has represented the relationship between convection and Earth's rotation by a cartoon model, called the "Earth's Dynamic Car" in which the convection is shown as Power Source similar to engine of the car responsible for plate breaking and the Steering Wheel corresponds to rotation of the Earth determining the direction of plate movement is shown Fig.14.

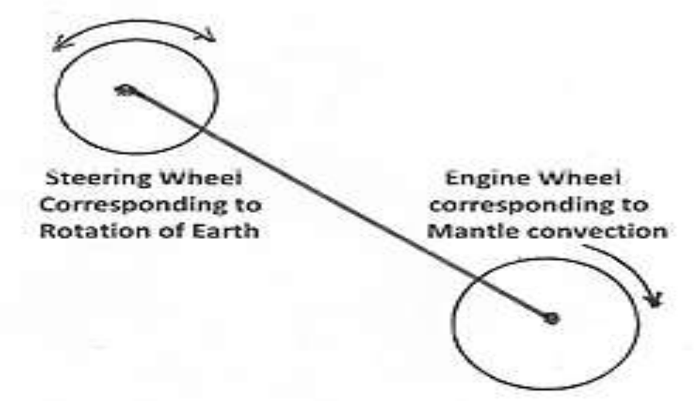


Fig.14 Shuping Chen's [11] depiction of Mantle Convection and Rotation of Earth

The author further says in his paper that mantle convection is relevant to heat-content of the Earth and where the convection actually takes place, there is a redistribution of interior substance of the Earth resulting the rotational angular speed of the Earth getting altered. This develops an increment in Earth's rotation inertia force. This along with the force of mantle convection decides the orientation or direction of plate movement. The geometric shapes of the plate boundaries adjust the direction of plate movement. The Shuping Chen's argument is further supported by geophysicists Carlo Doglioni and Roberto Sabadini [5] These authors are of the view that mantle convection alone seems not able to generate plate tectonics. A more robust contribution in combining mantle convection with Earth's rotation could be envisaged. The Earth's rotation is also able to generate a possible polarity in the kinematics of the core, mantle and lithosphere, a sort of railway path, they say.

V. CONCLUSION

The cascade process of increase of Rayleigh number, development of convection creating turbulence resulting in the movement of tectonic plates which ultimately is the cause for earthquakes, remains a mystery of nature. The earthquakes and volcanoes on various parts of the earth stand as a proof for the physical process that takes place inside the earth. The molten lava that comes out of the volcanoes which can be visually seen is the material of the molten fluid in the earth's mantle the convection of which we were dealing with in the above treatment. The formation of earthquakes and volcanoes stands as the only experimental proof for the mathematical treatment we have been dealing with.

Once again I would like to quote David Bercovici [3] that one fails to understand the cooling of Earth because of incomplete knowledge of radiogenic heat sources as well as complex Physics of mantle flow. At the end of his paper, he concludes: "*Thus mantle convection remaining one of the grand unifying physical theories of how the Earth works, many of the major questions and mysteries about the mantle remain unsolved and are thus ripe for discovery by future generations of Earth Scientists"*

There are two aspects, '*Character'* and *'Characteristic'*. Among all planets Earth has a well-behaved and well-set character of maintaining its physical processes. Plate Tectonics, Earthquakes and Volcanoesare various characteristics which are in-born with the formation of Earth and nobody can change it.

Let us pray *'Ruaumoko'* the Maori God of earthquakes and volcanoes in particular and the *nature almighty* in general to control such phenomena for the peace and welfare of mankind.

ACKNOWLEDGEMENT

The basic material in this paper is from Introduction to Plasma Physics by B.M. Smirnov (Ref. No.[12]). The classical treatment given by Smirnov is really excellent.. All the three papers of David Bercovici (Ref. Nos.[1], [2] and [3]) are highly exhaustive pertaining to their respective titles and are really master piece in the topic on Mantle Convection. Very few authors in modern research in Mantle Convection have considered the effect of Rotation of Earth on Mantle Convection, but Shuping Chen's (Ref. No. [11]) mathematical treatment explaining direction of motion of Tectonic Plates is to be appreciated. Last, but not the least, I hereby acknowledge the work of rest of the authors in the reference ([4], [6], [7], [8], [9], [10], [12] and [13]) for their excellent presentation so as to enable me to complete this paper.

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