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Optimal Pricing Policy for a Manufacturing Inventory Model with Two Production Rates under the Effect of Quantity Incentive

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ABSTRACT: When a new product is launched, a manufacturer applies the strategy of offering a quantity incentive initially for some time to boost up the demand of the product. The present paper describes a manufacturing inventory model with price sensitive demand enhanced by a quantity incentive. Later on demand becomes time increasing also. Inventory cycle starts with low production rate which is followed by higher production rate when demand is boosted up. Shortages are not allowed in this model. Presentation of numerical examples, tables, graphs and sensitivity analysis describes the model very well. Lastly case without incentive illustrates that usually the quantity incentive offered initially is beneficial.

Keywords: Time dependent demand, price dependent demand, pricing policy, two production rates, quantity incentive

I. INTRODUCTION

Inventory models with constant demand rate have been developed by many researchers in the past. But the demand rate of many commodities may be variable in nature. It is commonly observed that due to the various reasons the regular demand rate is not always applicable to various household goods like garments, electronic gadgets, grocery, garments, medicines and edibles etc., therefore the demand rate changes from time to time. So it is essential to discuss the inventory model with time varying demand rate pattern. This work was initiated with Silver and Meal [1] who developed EOQ model with time-varying demand rate. Since then many researchers developed inventory models with time varying demand. The first analytical model for a linearly time-dependent demand was developed by Donaldson [2]. Later, inventory models with time-dependent demand were studied by McDonald [3], Mitra et al. [4], Ritchie [5], Deb and Chaudhuri [6] and Giri and Chaudhari [7]. Skouri et al [8] presented an inventory model with ramp type demand rate, partial backlogging & Weibull deterioration rate. Garg, Vaish and Gupta [9] developed an inventory model with variable production and linearly increasing demand. Karmarkar and Chaudhary [10] formulated inventory model with ramp type demand and partial backlogging. Kirtan and Gothi [11] established an EOQ model with constant deterioration rate and time dependent demand.

Further it is observed that the demand for physical goods may not always time dependent, it may depend on stock and price also. In the present situation of competitive market pricing policy has a significant importance .Adequate pricing policies may uplift the companies from bottom-line in the competitive market. Present time is the time where fashion changes very soon as new products are launched day by day. Therefore, it is essential to make such pricing policy which can ensure sale of the entire stock before the next cycle starts. Burwell et al [12] evaluated an economic lot size model for price-dependent demand under quantity discounts. Papachristos and Skouri [13] formulated an inventory model for deteriorating item where demand rate is a decreasing function of the selling price. Ray et al [14] developed an inventory model with deterministic price-sensitive demand. Roy [15] formulated an inventory model for deteriorating items with price dependent demand and time varying holding cost. Chang et al [16] proposed an inventory model with stock and price dependent demand. Hong and Kim [17] formulated a supply chain inventory model with pricing and ordering policies for price-dependent demand. He and Huang [18] developed an inventory model with optimizing inventory and pricing policy for seasonal deteriorating products using preservation technology investment. Zang et al [19] evaluated optimal pricing policy for deteriorating items with preservation technology investment. Hossen et al [20] formulated a fuzzy inventory model with price and time dependent demand under inflation. Giri and Roy [21] presented a supply chain inventory model with price dependent demand and controllable lead time. Mashud et al [22] developed an inventory model with two non-instantaneous deterioration rates, price and stock dependent demand and partial backlogging.

Many production inventory models have been developed by many researchers. Chao and Lin [23] established a production inventory model for variable demand and production. Samanta and Roy [24] formulated a production inventory model with deteriorating items and shortages. Rao et al [25] developed a production inventory model for deteriorating items with stock and time dependent demand. Su and Lin [26] formulated the optimal inventory policy of production management. Swaminathan and Muniappan [27] formulated a manufacturing inventory model for deteriorating items and determined optimum production inventory. Ukil and Uddin [28] developed a production inventory model with constant production rate, buffer stock and product with finite shelf life.

It is the common mentality of most of the customers that they buy more than usual if they are benefited to get more quantity by expending less money. Therefore in the present competitive market in order to make his product popular and to gain optimum profit, an inventory manager applies many tactics like advertisement through media describing the qualities of the product, offering price discount (sale), quantity incentive (free bonus quantity) and quantity discount offered by the supplier on large purchase. When the demand of a product goes on decreasing continuously or when someone launches a new product in the competition of other established products of the same type in the market, he often advertises through media to offer a quantity incentive (free bonus quantity) on each unit of demand for sometimes initially. This enhances the demand and makes his product popular in the market. The examples can be seen in the products like oils, shampoos, soaps, biscuits and bread etc. On this feature of quantity incentive very rare inventory models may be found in literature.

In the present paper a manufacturing inventory model is developed for non- deteriorating items with quantity incentive (free bonus quantity),two production rates λ_1 and $\lambda_2 (\lambda_2 > \lambda_1)$,demand dependent on price, quantity incentive and time .Shortages are not allowed in the model. When a new product is launched then in order to make it popular and to boost up its demand, a quantity incentive is offered for some times initially and in the starting production rate is kept low but greater than demand. When the demand of the product is established in the market to large extent and goes on increasing with time, the production rate is raised high to fulfill the increased demand. Numerical examples are presented with tables, graphs and sensitivity analysis to describe the model very well. Lastly, a numerical example of case without incentive illustrates that usually the quantity incentive offered initially for some time is beneficial.

II. ASSUMPTIONS AND NOTATIONS

1: k ($0 < k < 1$) is the percentage of quantity incentive (free bonus quantity) offered on each demanded unit. $\alpha = (1-k)^{-n}$ ($n \in R$, the set of real numbers and $n \geq 1$) is the positive effect of quantity incentive on demand when $k \to 0, \alpha \to 1$ i.e. the demand returns to original form.

2: Demand is price sensitive and time dependent and it follows the pattern D (t) = $\alpha(a + bp^{-\eta}) + wtH(t - t_1)$ Where $H(t-t_1)$ is Heaviside Unit Step function defined by $H(t-t_1) = 1$ if $t \geq t_1$ and $H(t-t_1) = 0$ if $t < t_1$, p is selling price per unit, $a > 0$ is a scaling factor, $b > 0$, $w > 0$ and $\eta > 1$ is the index of price elasticity.

3: λ_1 is the initial production rate where quantity incentive is offered. λ_2 ($\lambda_2 > \lambda_1$) is the production rate when quantity incentive is stopped to offer.

4: c = production cost per unit

5: T = cycle length which is assumed to be infinite

6: t_1 = the time at which production rate changes from λ_1 to λ_2 and offering of quantity incentive is stopped

7: t_2= the time at which production is stopped

8: PQ = production quantity of each production cycle

9: h = holding cost per unit per unit time

10: A = set up cost of production

11: SR = sales revenue per production cycle

12: $I(t)$ = the inventory level at time t.

13: $F(t_1, T)$ = Profit per unit time

14: $p^*, t_1^*, T^*, F^*(t_1, T)$ *and* PQ^* represents the optimal values of p, t_1, T, $F(t_1, T)$ *and*, PQ respectively.

II. MODEL FORMULATION AND ANALYSIS

The behavior of the inventory level during cycle T is depicted in figure (1)

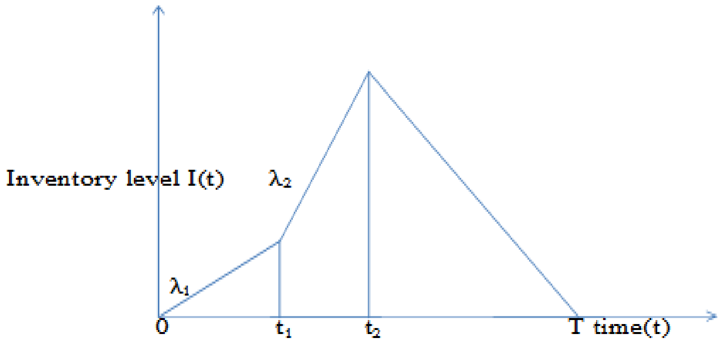

Figure (1)

The differential equations governing the inventory level *I(t)* at any time t of inventory cycle *T*are as follows :

$$\frac{dI(t)}{dt} = \lambda_1 - (1+k)\alpha(a+bp^{-\eta}) \qquad 0 \le t \le t_1 \qquad \dots (1)$$

$$\frac{dI(t)}{dt} = \lambda_2 - \{\alpha(a+bp^{-\eta}) + wt\} \qquad t_1 \le t \le t_2 \qquad \dots (2)$$

$$\frac{dI(t)}{dt} = -\{\alpha(a+bp^{-\eta}) + wt\} \quad t_2 \le t \le T \qquad \dots (3)$$

Boundary conditions are $I(0) = 0, I(T) = 0$

the solutions of the above differential equations are given by:

$$I(t) = (\lambda_1 - (1+k)X)t \quad 0 \le t \le t_1 \qquad \dots (4)$$

$$I(t) = (\lambda_2 - X)\text{t} + (\lambda_1 - \lambda_2 - kX)\text{t}_1 - \frac{w}{2}(t^2 - t_1^2) \quad t_1 \le t \le t_2 \qquad \dots (5)$$

$$I(t) = X(T-t) + \frac{w}{2}(T^2 - t^2) \quad t_2 \le t \le T \qquad \dots (6)$$

Where $X = \alpha(a+bp^{-\eta})$

From equations (5) and (6), equating value of $I(t_2)$, the value of t_2 can be obtained as

$$t_2 = \frac{w(T^2 - t_1^2) + 2XT + 2(\lambda_2 - \lambda_1 + kX)t_1}{2\lambda_2} \qquad \dots (7)$$

Various characters involved in determining the average profit are

Sales Revenue:

As k is % quantity incentive, sales revenue SR is obtained as:

$$SR = p[\int_0^{t_1} \alpha(a+bp^{-\eta})dt + \int_{t_1}^{T} (\alpha(a+bp^{-\eta}) + w\text{t})\,\text{dt}$$

$$SR = p\{\alpha(a+bp^{-\eta})T + \frac{w}{2}(T^2 - t_1^2)\} \qquad \dots (8)$$

Production cost:

$$PC = c[\lambda_1 t_1 + \lambda_2(t_2 - t_1)] \qquad \dots (9)$$

Holding Cost:
The holding cost *HC* is given by

$$HC = h[\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt + \int_{t_2}^{T} I(t)dt]$$

$$= h\{(-XT - \frac{w}{2}(T^2 - t_1^2) - \frac{(\lambda_2 - \lambda_1 + kX)t_1}{2})t_2 + \frac{(\lambda_2 - \lambda_1 + kX)t_1^2}{2} + \frac{XT^2}{2} + \frac{w}{3}(T^3 - t_1^3) + \frac{\lambda_2 t_2^2}{2}\} \qquad \dots (10)$$

Now, the profit per unit time for the system can be calculated as follows:

$$F(T, t_1) = [SR\text{-}PC - HC\text{-}A]$$

$$F(T, t_1) = \frac{1}{T}\{pXT + \frac{w}{2}(T^2 - t_1^2)\} - ht_2\{(-XT - \frac{w}{2}(T^2 - t_1^2) + (\lambda_1 - \lambda_2 - kX)t_1 + \frac{c\lambda_2}{h})$$

$$+ \frac{\lambda_2 t_2^2}{2} + \frac{(\lambda_1 - \lambda_2 - kX)t_1^2}{2} + \frac{XT^2}{2} + \frac{w}{3}(T^3 - t_1^3))\} - c(\lambda_1 - \lambda_2)t_1 - A\} \qquad \dots (11)$$

Where $t_2 = \dfrac{w(T^2 - t_1^2) + 2XT + 2(\lambda_2 - \lambda_1 + kX)t_1}{2\lambda_2}$

III. MATHEMATICAL SOLUTIONS

The following cases are considered in the model.

Case 1. In this case, unit time profit is assumed as a function of two variables *T* and *p*. To find out the optimal solution

$$\frac{\partial F(T,p)}{\partial p} = 0 \quad \frac{\partial F(T,p)}{\partial T} = 0 \qquad \dots (12)$$

The optimal values of T and p are obtained by solving these equations simultaneously provided

$$\frac{\partial^2 F(T,p)}{\partial T^2} \cdot \frac{\partial^2 F(T,p)}{\partial p^2} - \left(\frac{\partial^2 F(T,p)}{\partial T \partial p}\right)^2 > 0 \qquad \dots (13)$$

Numerical Illustration 1. This case of the model is discussed considering the following parametric values.
$b = 20$, $\lambda_1 = 530$ units/month, $\lambda_2 = 1550$units/month, $w = 50.3$, $k = 0.1$, $\alpha = (0.9)^{-2.2}$, $a = 200$,
$h = 0.4$rs/unit/time, $\eta = 1.2$, $A = 150$, $c = 300$rs, $t_1 = 4$ months.
Applying the solution procedure described above the optimal values obtained are as follows:
$T^* = 9.5864$ months, $p^* = 2470.525$rs, $F^*(T, p) = 975948$rs/time, $t_2^* = 5.4885$ months, $PQ^* = 1328150$ units.
Now effects of various parameters on total profit per unit time are observed.

Effects of parameter"λ_1" on Total Profit per Unit Time

% change in λ_1	λ_1	T	p	F(p,T)
-20%	424	9.58641	2470.21	975866
-15%	450.5	9.58641	2470.27	975880
-10%	477	9.58641	2470.35	975898
-5%	503.5	9.58641	2470.43	975921
0	530	9.58641	2470.53	975948
5%	556.5	9.58641	2470.63	975979
10%	583	9.58641	2470.74	976015
15%	609.5	9.58641	2470.86	976055
20%	636	9.58641	2470.99	976099

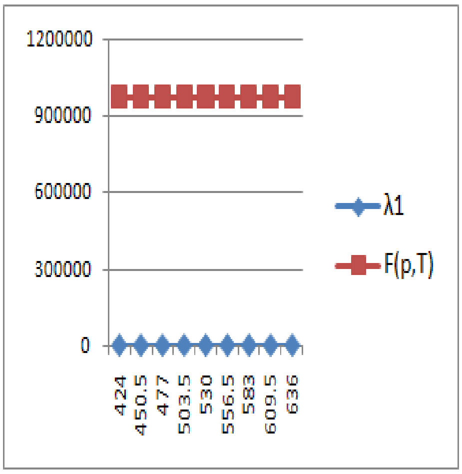

Table (1) Figure(2)

Effects of parameter"λ_2" on Total Profit per Unit Time

% change in λ_2	λ_2	T	p	$F(p,T)$
-20%	1240	9.58347	2496.27	987394
-15%	1317.5	9.59837	2483.95	982920
-10%	1395	9.58641	2481.67	980983
-5%	1472.5	9.58641	2475.8	978333
0	1550	9.58641	2470.53	975948
5%	1627.5	9.58641	2465.75	973789
10%	1705	9.57446	2462.76	972886
15%	1782.5	9.59735	2460.2	971980
20%	1860	9.59736	2456.57	970336

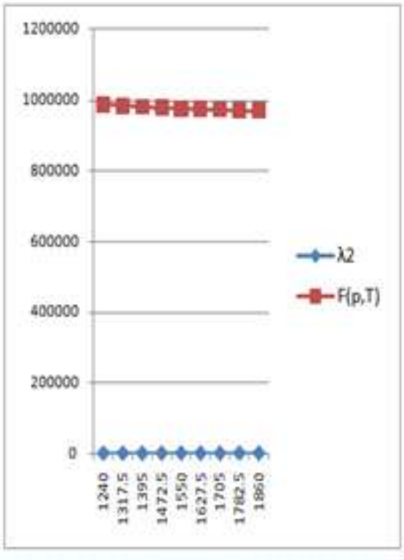

Table (2) **Figure (3)**

Effects of parameter" w" on Total Profit per Unit Time

%Change inw	w	T	p	$F(p, T)$
-20%	40.24	12.6346	2689.82	1146210
-15%	42.755	11.7856	2612.99	1095730
-10%	45.27	10.9976	2544.23	1047390
-5%	47.785	10.2741	2502.62	1010740
0	50.3	9.58641	2470.53	975948
5%	52.815	8.95359,	2446.62	943792
10%	55.33	8.31447	2465.34	924835
15%	57.845	7.75416	2427.56	882463
20%	60.36	7.16663	2436.4	852358

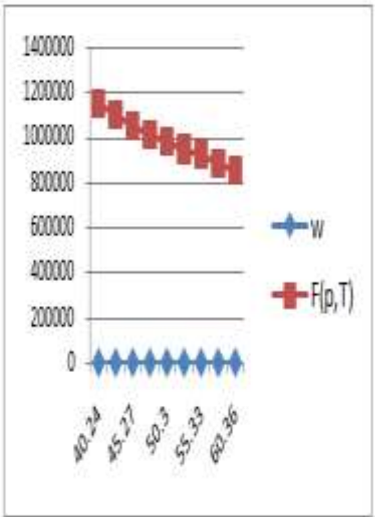

Table (3) **Figure (4)**

Effects of parameter "a" on Total Profit per Unit Time

%Change in a	a	T	p	F(p,T)
-20%	160	6.52846	2400.16	673624
-15%	170	7.46312	2426.81	727516
-10%	180	8.17972	2434.85	825430
-5%	190	8.9263	2462.06	890842
0	200	9.58641	2470.53	975948
5%	210	10.2368	2509.68	1063510
10%	220	10.8552	2555.14	1154020
15%	230	11.44	2621.31	1255780
20%	240	12.0498	2668.99	1352040

Table (4)

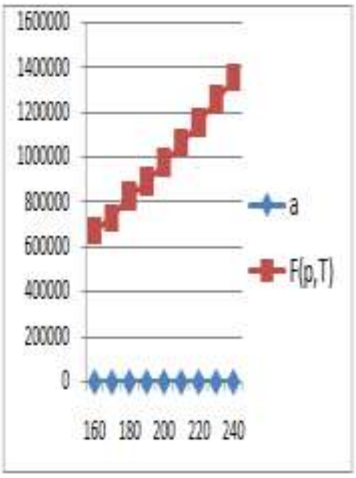

Figure (5)

Sensitivity Analysis Table:

parameter	% change	%change p	%change T	%change F(p,T)
λ_1	-20	-0.00012	0	-0.00008
	-10	-0.00007	0	-0.00005
	10	0.00008	0	0.00006
	20	0.00018	0	0.00015
w	-20	0.08876	0.31796	0.17445
	-10	0.02983	0.14720	0.07320
	10	-0.00210	-0.06601	-0.05237
	20	-0.01381	-0.25241	-0.12663
a	-20	0.02278	-0.31898	-0.30977
	-10	-0.00342	-0.14673	-0.15422
	10	0.03424	0.13235	0.18246
	20	0.08033	0.25696	0.38536
λ_2	-20	-0.00031	0.01041	0.011728
	-10	-0.00013	0.00450	0.005159
	10	0.00084	-0.00315	-0.00314
	20	0.001142	-0.00565	-0.00575

Table (5)

Observations:

1. From table (1) it is observed that as (λ_1) increases, *p* and unit time profit of the system increases

2. From table (2) it is observed that (λ_2) increases, the unit time profit of the system decreases.

3. From table (3) it is observed that (w) increases, the unit time profit of the system decreases

4. Table (4) reveals that (a) increases (p) and the unit time profit of the system also increases.

5. From sensitivity table (5) it has been noticed that T, p, and the unit time profit of the system are not sensitive to (λ_1) and are negligible sensitive to (λ_2).p shows small sensitivity sensitive to (*a*) & (*w*) but the unit time profit of the system is fairly sensitive to (*a*) & (*w*).

Case 2: In this case the unit time profit is assumed as a function of two variables T and t_1. To find out the optimal solution

$$\frac{\partial F(T,t_1)}{\partial t_1}=0 \quad \frac{\partial F(T,t_1)}{\partial T}=0 \qquad \ldots (14)$$

The optimal values of T and $t_1$are obtained by solving the above equations simultaneously provided

$$\frac{\partial^2 F(T,t_1)}{\partial T^2}.\frac{\partial^2 F(T,t_1)}{\partial t_1^2}-\left(\frac{\partial^2 F(T,t_1)}{\partial T\partial t_1}\right)^2>0 \qquad \ldots (15)$$

Numerical Illustration2: This case of the model is discussed considering the following parametric values. b = 50, λ_1 = 250 units/month, λ_2 = 950 units/month, w = 3.8, k = 0.2rs/unit, α = $(0.8)^{-2.2}$, a = 100, h = 3.2 rs/unit/time, η =2.8, A=100, c = 200rs, p = 800. Applying the solution procedure described above the optimal values obtained is as follows: T^* = 143.165 months, t_1^*= 40.339, $F^*(T,t_1)$ = 207228 rs/time, t_2^*=93.47, PQ^*= 1211195 units.

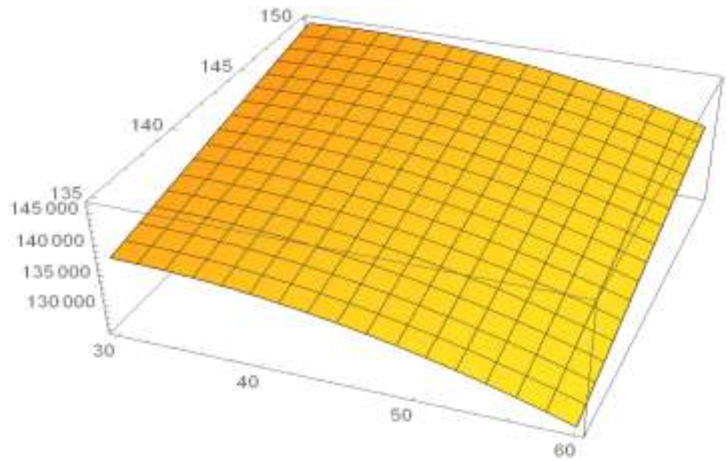

Figure (6)

Above 3D Graph shows the concavity of the unit time profit function F (t_1,T)

Effects of parameter" c" on Total Profit per Unit Time

% change in c	c	t_1	T	$F(t_1,T)$
-20%	160	35.2142	132.576	215768
-15%	170	36.4242	135.133	213683
-10%	180	37.6814	137.75	211567
-5%	190	38.9862	140.428	209415
0	200	40.339	143.165	207228
5%	210	41.7401	145.96	205001
10%	220	43.1894	148.811	202734
15%	230	44.6869	151.716	200423
0%	240	46.2323	154.673	198066

Table (6)

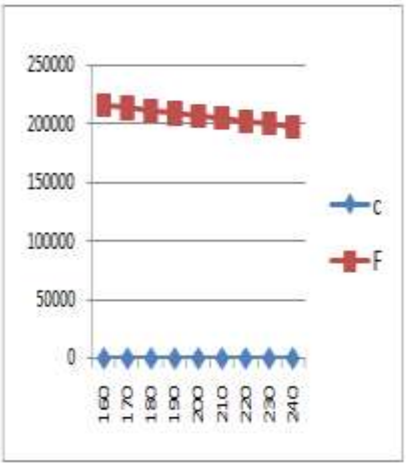

Figure (7)

Effects of parameter" λ_1" on Total Profit per Unit Time

% change in λ_1	λ_1	t_1	T	$F(t_1,T)$
-20%	200	42.0789	138.761	207327
-15%	212.5	41.6402	139.853	207293
-10%	225	41.2045	140.952	207265
-5%	237.5	40.7709	142.056	207243
0	250	40.339	143.165	207228
5%	262.5	39.9082	144.28	207211
10%	275	39.4779	145.4	207213
15%	287.5	39.0474	146.525	207214
20%	300	38.6162	147.655	207219

Table (7)

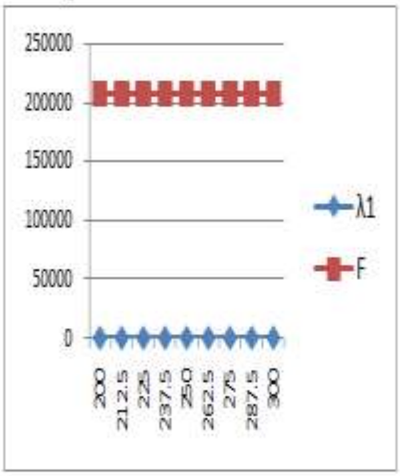

Figure (8)

Effects of parameter" w" on Total Profit per Unit Time

% change in w	w	t_1	T	$F(t_1,T)$
-20%	3.04	60.6622	184.975	201213
-15%	3.23	54.3809	172.51	202979
-10%	3.42	49.004	161.544	204551
-5%	3.61	44.3663	151.828	205960
0	3.8	40.339	143.165	207228
5%	3.99	36.8203	135.396	208374
10%	4.18	33.7288	128.393	209414
15%	4.37	30.9985	122.049	210361
20%	4.56	28.5759	116.279	211227

Table (8)

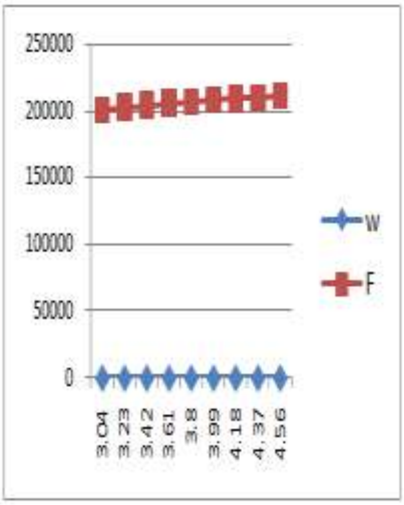

Figure (9)

Effects of parameter"*p*" on Total Profit per Unit Time

% change in p	p	t_1	T	$F(t_1,T)$
-20%	640	56.6818	162.298	151031
-15%	680	51.7564	156.685	165323
-10%	720	47.4447	151.668	179437
-5%	760	43.6633	147.181	193398
0	800	40.339	143.165	207228
5%	840	37.408	139.564,	220948
10%	880	34.8151	136.327	234577
15%	920	32.5132	133.412	248128
20%	960	30.4619	130.779	261614

Table (9)

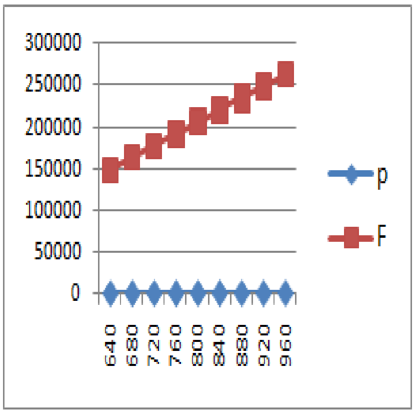

Figure (10)

Observations:

1. From table (6) it is observed that as production cost (c) increases, the unit time profit of the system decreases.
2. From table (7) it is observed that (λ_1) increases, the unit time profit of the system decreases.
3. From table (8) it has been noticed that as (w) increases (T) decreases and the unit time profit of the system increases.
4. Table (9) reveals that as the selling price (p) increases, (T) decreases and the unit time profit of the system increases.

Case 3: Quantity Incentive is profitable:

In this case the profit function per unit time is considered for single variable t_1.
The optimal value of t_1 is obtained by solving the equation

$$\frac{dF(t_1)}{dt_1} = 0 \quad \text{… (16)}$$

Provided $\frac{d^2F(t_1)}{dt_1^2} < 0$ … (17)

Numerical Illustration-3: To illustrate case (3) the following parametric values are considered.
T = 150 days, $\lambda_1 = 250$, $\lambda_2 = 950$, p = 800rs, $b = 50$, $a = 100$, $A = 100$, $\alpha = (0.80)^{-2.}$

h =3.2 rs/unit/time, c = 200 rs, k = 0.20, η = 2.8, w = 3.8
Applying the solution procedure described above the optimal values obtained are as follows:
t_1*= 42.9294 days, t_2*= 100.22 days, $F^*(t_1)$ =212254rs, PQ^*=1303170,
When quantity incentive is not offered then the optimal values obtained from following parameters with
k = 0 and α = 1 are as follows:
T=150 days, λ_1= 250, λ_2 = 950, p = 800rs, b = 50, a = 100, A = 100, α = 1
h = 3.2 rs/unit/time, c = 200 rs, k = 0, η = 2.8, w = 3.8
t_1*= 37.5904 days, t_2*= 88.7358 days $F^*(t_1)$ =169961rs, PQ^*=1084970,
These results show that sometimes quantity incentive is profitable for inventory manager.

IV. CONCLUSION

When a new product is launched in the market or when its demand goes on decreasing continuously, the strategy of offering a quantity incentive (free bonus quantity) initially for some time is beneficial for a manufacturer as it enhances the demand of the product to a large extent and ultimate results in a good average profit. Applying this feature of customers and market, a manufacturing inventory model is developed in the present paper for non- deteriorating items with two production rates λ_1 and λ_2, price, time and quantity incentive dependent demand, quantity incentive (free bonus quantity) and without shortages. In the starting production rate λ_1 is kept low but greater than demand. When the demand of the product is established in the market and goes on increasing with time, the production rate is raised high up to $\lambda_2$to fulfill the increased demand. Numerical examples are presented with tables, graphs and sensitivity analysis to describe the model very well. Tables, graphs and sensitivity analysis reveal that results obtained are very much near to real practical situation. Lastly, a numerical example of case without incentive illustrates that usually the quantity incentive offered initially for some time is beneficial for a manufacturer in launching a new product. The present paper can be studied further by some other practical situations of manufacturing inventory system.

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