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Analysis of a Queueing Model with Balking for Buffer Sharing in Atm

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ABSTRACT:- In this paper, we study a Markovian queueing model with balking for buffer sharing in ATM. It is assumed that server provides service with priorities termed as first, second and third. As soon as system accumulates L+1, M+1 customers third and second priority customers balk with balking function b_n. We have obtained the steady state probabilities for the system by recursive method. Some performance measures have also been obtained. A numerical example is also provided.

Keywords:- Markovian Queueing Model, Balking, Buffer sharing, Asynchronous Transfer Mode (ATM).

I. INTRODUCTION

In many queueing systems the arriving customers may not join the queue due to impatience if the number of customers is greater than the predetermined threshold value, that is, the customers may balk if system size is greater than a threshold value. Clearly balking of customers is a loss to the system and needs attention to improve the same. The improvement is possible only when full analysis of the system is available. Keeping this factor such queueing systems have been studied by many researchers in queueing literature. Ancker and Gafarian (1963) investigated some queueing problems with balking and reneging. Doshi and Jangerman (1986) studied an M/G/1 queue with class dependent balking; they obtained some performance measures for the system.

A related concept may be seen in asynchronous transfer mode (ATM) where loss priority control concerned with reducing the loss probability of customers in buffer sharing scheme. Bae and Suda (1991) provided a survey of traffic control scheme in protocols in ATM networks. Abou and Hariri (1992) analysed the M/M/C/N queueing system with balking and reneging. They also obtained some performance measures and numerical results for the system. Suri et al. (1994) evaluated space priority strategies in ATM network. Lee and Ahn (1998) analysed a queueing model for optimal control of partial buffer sharing in ATM. They gave an algorithm to obtain the steady state probabilities and performance measures. In this paper we model M_1, M_2, M_3/G/1/N+1 queue with balking for buffer sharing in ATM. We assume that first, second and third priority customers arrive according to poison with rates λ_1, λ_2 and λ_3 respectively. The service times are identical for all three types and generally distributed. As soon as the number of customers becomes L+1 and M+1, the third and second priority customers balk with balking function b_n. If the number of customers in the system is M+1 only first and second priority customers are allowed to join the queue. Since the buffer size is N, the number of customers in the system can not exceed N+1 including one in service.

II. NOTATIONS

λ_1 Arrival rate of first priority customers.
λ_2 Arrival rate of second priority customers.
λ_3 Arrival rate of third priority customers.
λ Total customers arrival rate = $\lambda_1 + \lambda_2 + \lambda_3$
L,M Thresholds
N Buffer capacity
S(x) Service time distribution function
s(x) Service time probability function
$S^*(\theta)$ Laplace transform of s(x)
E(s) Mean service time
P_{Lost1} Lost probability of first priority customers

P_{Lost2} Lost probability of second priority customers
P_{Lost3} Lost probability of third priority customers
$\begin{cases} a\ ; & \text{offered load} = \lambda\, E(S) \\ a^1\ ; & \text{carried load} = [\lambda_1(1-P_{Lost1}) + \lambda_2(1-P_{Lost2}) + \lambda_3(1-P_{Lost3})]E(S) \end{cases}$
R(t) Remaining service time of a customer in service at time t.
N(t) System size at time t.
$P_0(t)$ $pr[N(t) = 0]$
$P_n(xt)\Delta x$ $Pr[N(t) = n, \quad x \leq R(t) \leq x + \Delta x] \quad (n = 1,2,..N+1)$
$p_0 = \lim_{t\to\infty} p_0(t)$
$p_n(x) = \lim_{t\to\infty} p_n(x,t)$
L_s Average number of customers in the system (for all three types)
L_q Expected number of customers in the queue (for all three types)
W Expected time spent by a customer in the system
W_q Expected waiting time of a customer in the queue
λ^1 Effective total arrival rate
λ_1^1 Effective arrival rate of first priority customers
λ_2^1 Effective arrival rate of second priority customers
λ_3^1 Effective arrival rate of third priority customers

III. SYSTEM EQUATIONS

Following Lee and Ahn (1998) the equations in our case are as follows:

$$0 = \lambda p_0 + p_1(0) \qquad \text{……(1)}$$

$$-\frac{dp_1(x)}{dx} = -\lambda p_1(x) + p_2(0)s(x) + \lambda p_0 s(x) \qquad \text{……(2)}$$

$$-\frac{dp_n(x)}{dx} = -\lambda p_n(x) + p_{n+1}(0)s(x) + \lambda p_{n-1}(x);\ (n = 1,2,..L) \qquad \text{……(3)}$$

$$-\frac{dp_{L+1}(x)}{dx} = -(\lambda_1 + \lambda_2 + b_{L+1}\lambda_3)p_{L+1}(x) + p_{L+2}(0)s(x) + \lambda p_L(x) \qquad \text{……(4)}$$

$$-\frac{dp_n(x)}{dx} = -(\lambda_1 + \lambda_2 + b_n\lambda_3)p_n(x) + p_{n+1}(0)s(x) + (\lambda_1 + \lambda_2 + b_{n-1}\lambda_3)p_{n-1}(x);$$
$$(n = L+2, \dots \dots \dots M) \qquad \text{……(5)}$$

$$-\frac{dp_{M+1}(x)}{dx} = -(\lambda_1 + \lambda_2\ b_{M+1})p_{M+1}(x) + p_{M+2}(0)s(x) + (\lambda_1 + \lambda_2 + b_M\lambda_3)p_M(x) \qquad \text{……(6)}$$

$$-\frac{dp_n(x)}{dx} = -(\lambda_1 + b_n\lambda_2)p_n(x) + p_{n+1}(0)s(x) + (\lambda_1 + b_{n-1}\lambda_2)p_{n-1}(x);\quad (n = M+2, \dots N) \qquad \text{……(7)}$$

$$-\frac{dp_{N+1}(x)}{dx} = (\lambda_1 + b_n\lambda_2)p_N(x) \quad ......(8)$$

Assuming $(\lambda_1 + \lambda_2 + b_n\lambda_3) = \alpha_n$ and $(\lambda_1 + \lambda_2 b_n) = \beta_n$ in the above equations and taking the Laplace transform of equations (2) - (8);
We have

$$(\lambda - \theta)p_1^*(\theta) = S^*(\theta)[\lambda p_0 + p_2(0)] - p_1(0) \quad ......(9)$$

$$(\lambda - \theta)p_n^*(\theta) = \lambda p_{n-1}^*(\theta) + S^*(\theta)p_{n+1}(0) - p_n(0); \quad (n = 2, \ldots, L) \quad ......(10)$$

$$(\alpha_{L+1} - \theta)p_{L+1}^*(\theta) = \lambda p_L^*(\theta) + S^*(\theta)p_{L+2}(0) - p_{L+1}(0) \quad ......(11)$$

$$(\alpha_n - \theta)p_n^*(\theta) = \alpha_{n-1}p_{n-1}^*(\theta) + S^*(\theta)p_{n+1}(0) - p_n(0); \quad (n = L+2, \ldots, M) \quad ......(12)$$

$$(\beta_{M+1} - \theta)p_{M+1}^*(\theta) = \alpha_M p_M^*(\theta) + S^*(\theta)p_{M+2}(0) - p_{M+1}(0) \quad ......(13)$$

$$(\beta_n - \theta)p_n^*(\theta) = \beta_{n-1}p_{n-1}^*(\theta) + S^*(\theta)p_{n+1}(0) - p_n(0); \quad (n = m+2, \ldots, N) \quad ......(14)$$

$$-p_{N+1}^*(\theta) = \beta_N p_N^*(\theta) - p_{N+1}(0) \quad ......(15)$$

Using Equation (1) in (9) and adding (9) - (15) we have,

$$\sum_{n=1}^{N+1} p_n^*(\theta) = \left[\frac{1-S^*(\theta)}{\theta}\right] \sum_{n=1}^{N+1} p_n(0) \quad ......(16)$$

Setting $\theta = 0$ in (16), we have

$$\sum_{n=1}^{N+1} p_n^*(0) = E(S) \sum_{n=1}^{N+1} p_n(0) \quad ......(17)$$

Using (1) in (9) and setting $\theta = \lambda \; and \; \theta = 0$ respectively, we obtain

$$p_2^*(0) = \lambda \left(\frac{1-S^*(\lambda)}{S^*(\lambda)}\right) p_0 \quad ......(18)$$

$$p_1^*(0) = \frac{1}{\lambda} p_2(0) \quad ......(19)$$

Where $S^*(\lambda)$ is the probability that no customer arrives during a service time. Now setting $\theta = \lambda$ in equation (10), we have

$$p_{n+1}(0) = \frac{p_n(0) - \lambda p_{n-1}^*(\lambda)}{S^*(\lambda)} (n = 2,3, \ldots .L) \quad ......(20)$$

On differentiating equations (9) and (10), (j+1) times at $\theta = \lambda$, we get

$$p_1^{*(j)}(\lambda) = \frac{1}{j+1} S^{*(j+1)}(\lambda)[p_1(0) + p_2(0)], (j = 0,1,2 \ldots .L-2) \quad ......(21)$$

$$p_1^{*(j)}(\lambda) = -\frac{1}{j+1}\left[\lambda p_{n-1}^{*(j+1)}(\lambda) + p_{n+1}(0)S^{*(j+1)}(\lambda)\right],$$
$$(n = 2 \ldots .L, j = 0,1 \ldots .L-n) \quad ......(22)$$

Where $p_1^{*(0)}(\lambda) = p_1^*(\lambda), p_n(0)\ (3 \le n \le L+1)$ can be obtained in terms of p_0 recursively from equations (20), (21) and (22)
Setting $\theta = \alpha_{L+1}$ in (11) we obtain

$$p_{L+2}(0) = \frac{p_{L+1}(0) - \lambda p_L^*(\alpha_{L+1})}{S^*(\alpha_{L+1})} \qquad \text{……(23)}$$

From equations (9) and (10), we have

$$p_1^*(\alpha_{L+1}) = \frac{S^*(\alpha_{L+1})(p_1(0)+p_2(0)) - p_1(0)}{\lambda_3(1-b_{L+1})} \qquad \text{……(24)}$$

$$p_n^*(\alpha_{L+1}) = \frac{\lambda p_{n-1}^* a_{n+1} + S^*(a_{L+1})p_{n+1}(0) - p_n(o)}{\lambda_3(1-b_{n+1})}, \quad (1 = 2,3 \dots . L) \qquad \text{……(25)}$$

Thus $p_{L+2}(0)$ can be obtained in terms of p_0 recursively from equations (23), (24) and (25). Now substituting $\theta = \alpha_n$ in (12), we obtain,

$$p_{n+1}(0) = \frac{p_n(0) - \alpha_{n-1} p_{n-1}^*(\alpha_n)}{S^*(\alpha_n)}; \quad (n = L+2, \dots \dots \dots M) \qquad \text{……(26)}$$

From equations (9), (10), (11) and (12), we have

$$p_1^{*(j)}(\alpha_n) = \frac{s^{*(j)}(\alpha_n)[p_1(0)+p_2(0)] + j p_1^{*(j-1)}(\alpha_n)}{\lambda_3(1-b_n)}; \text{ (j= 1,2,…….,M-L)} \qquad \text{……(27)}$$

$$p_n^{*(j)}(\alpha_n) = \frac{\left[\lambda p_{n-1}^{*(j)}(\alpha_n) + S^*(\alpha_n)p_{n+1}(0)\right] + j p_n^{*(j-1)}(\alpha_n)}{\lambda_3(1-b_n)}; \text{ (n=2,…L,j=1...,M-n-1)} \qquad \text{……(28)}$$

$$p_{L+1}^{*(j)}(\alpha_n) = -\frac{1}{j+1}\left[\lambda p_L^{*(j+1)}(\alpha_n) + S^{*(j+1)}(\alpha_n)p_{L+2}(0)\right]; \quad (j = 0, \dots M-L-2) \qquad \text{……(29)}$$

$$p_n^{*(j)}(\alpha_n) = -\frac{1}{j+1}\left[\alpha_{n-1} p_{n-1}^{*(j+1)}(\alpha_n) + S^{*(j+1)}(\alpha_n)p_{n+1}(0)\right]; \quad (n = L+2, \dots M; J = 0,1, M-n-1) \qquad \text{……(30)}$$

Thus $p_n(o)(L+3 \le n \le M+1)$ can be obtained in terms of P_0 recursively from (26) – (30). Now setting $\theta = \beta_{M+1}$ in equation (13), we get

$$p_{M+1}(0) = \frac{p_{M+2}(0) - \alpha_M p_M^*(\beta_{M+1})}{S^*(\beta_{M+1})} \qquad \text{……(31)}$$

From equations (9) – (14), we have

$$p_1^*(\beta_{M+1}) = \frac{S^*(\beta_{M+1})[p_1(0)+p_2(0)] - p_1(0)}{\lambda_3 + \lambda_2(1-b_{M+1})} \qquad \text{……(32)}$$

$$p_n^*(\beta_{M+1}) = \frac{\lambda p_{n-1}^*(\beta_{M+1}) + S^*(\beta_{M+1})p_{n+1}(0) - p_n(0)}{\lambda_3 + \lambda_2(1-b_{M+1})}; (n = 2, \dots \dots . L) \qquad \text{……(33)}$$

$$p_{L+1}^*(\beta_{M+1}) = \frac{\lambda p_1^*(\beta_{M+1}) + S^*(\beta_{M+1})p_{L+2}(0) - p_n(0)}{\lambda_3 b_{L+1} + \lambda_2(1-b_{M+1})} \qquad \text{……(34)}$$

$$p_n^*(\beta_{M+1}) = \frac{\alpha_{n-1} p_{n-1}^*(\beta_{M+1}) + S^*(\beta_{M+1})p_{n+1}(0) - p_n(0)}{\lambda_3 b_n + \lambda_2(1-b_{M+1})} \quad (n = L+2, \dots M) \qquad \text{……(35)}$$

Thus $p_{M+2}(0)$ can be obtain in terms of p_0 recursively from equations (31) – (35).
Setting $\theta = \beta_n$ in equation (14), we have,

$$p_{n+1}(0) = \frac{p_n(0)-\beta_{n-1}p^*_{n-1}(\beta_n)}{S^*(\beta_n)}; (n = M+2, \dots \dots N) \quad \dots\dots(36)$$

From equation (9) – (14), we get

$$p_1^{*(j)}(\beta_n) = \frac{S^{*(j)}(\beta_n)[p_1(0)+p_2(0)] + jp_1^{*(j-1)}(\beta_n)}{\lambda_3 + \lambda_2(1-b_n)}; (j = 1,2, \dots M-L) \quad \dots\dots(37)$$

$$p_n^{*(j)}(\beta_n) = \frac{\lambda p_{n-1}^{*(j)}(\beta_n) S^{*(j)}(\beta_n) p_{n+1}(0) + jp_n^{*(j-1)}(\beta_n)}{\lambda_3 + \lambda_2(1-b_n)}; (n = 2, \dots L, j = 1, \dots \dots, M-n-1) \quad \dots\dots(38)$$

$$p_{L+1}^{*(j)}(\beta_n) = \frac{\lambda p_L^{*(j)}(\beta_n) + S^{*(j)}(\beta_n) p_{L+2}(0) + jp_{L+1}^{*(j-1)}(\beta_n)}{\lambda_3 b_{L+1} + \lambda_2(1-b_n)} \quad \dots\dots(39)$$

$$p_n^{*(j)}(\beta_n) = \frac{\alpha_{n-1} p_{n-1}^{*(j)}(\beta_n) S^{*(j)}(\beta_n) p_{n+1}(0) + jp_n^{*(j-1)}(\beta_n)}{\lambda_3 b_n + \lambda_2(1-b_n)}; (n = L+2, \dots . M, J = 1,2, \dots, N-n-1) \quad \dots\dots(40)$$

$$p_{M+1}^{*(j)}(\beta_n) = -\frac{1}{(j-1)}\left[\alpha_M p_m^{*(j+1)}(\beta_n) S^{*(j+1)}(\beta_n) p_{M+2}(0)\right]; (j = 0 \dots, N-M-2) \quad \dots\dots(41)$$

$$p_n^{*(j)}(\beta_n) = -\frac{1}{(j+1)}\left[\beta_{n-1} p_{n-1}^{*(j+1)}(\beta_n) + S^{*(j+1)}(\beta_n) p_{n+1}(0)\right]; n = M+2, \dots N, J = 0 \dots, N-n-1 \quad \dots\dots(42)$$

Thus $p_n(0)$ (n=M+3,…….N) can be obtained from equation (36) by using (37) – (42). Now substituting $\theta = 0$ in equations (10) – (14), we have

$$p_n^*(0) = \frac{1}{\lambda}[\lambda p_{n-1}^*(0) + p_{n+1}^*(0) - p_n(0)]; n = 2, \dots L \quad \dots\dots(43)$$

$$p_{L+1}^*(0) = \frac{1}{\alpha_{L+1}}[\lambda p_L^*(0) + p_{L+2}^*(0) - p_{L+1}(0)] \quad \dots\dots(44)$$

$$p_n^*(0) = \frac{1}{\alpha_n}[\alpha_{n-1} p_{n-1}^*(0) + p_{n+1}^*(0) - p_n(0)]; n = L+2, \dots, M \quad \dots\dots(45)$$

$$P_{M+1}^*(0) = \frac{1}{\beta_{M+1}}[\alpha_M p_M^*(0) + p_{M+2}(0) - p_{M-1}(0)] \quad \dots\dots(46)$$

$$p_n^*(0) = \frac{1}{\beta_n}[\beta_{n-1}p_{n-1}^*(0) + p_{n+1}^*(0) - p_n(0)]; n = M+2, \dots, N \qquad \text{.......(47)}$$

The equations (43) – (47) can be rewritten as

$$p_n^*(0) = \frac{p_{n+1}(0)}{\lambda}; \ (n = 2, \dots, L) \qquad \text{.......(48)}$$

$$p_n^*(0) = \frac{p_{n+1}(0)}{\alpha_n}; (n = L+1, \dots M) \qquad \text{.......(49)}$$

$$p_n^*(0) = \frac{p_{n+1}(0)}{\beta_n}; (n = M+1, \dots N) \qquad \text{.......(50)}$$

Now differentiating equation (15) with respect to $\theta \ at \ \theta = 0$ to obtain the unknown quantity $p_{N+1}^*(0)$, we have

$$p_{N+1}^*(0) = -\beta_N p_N^{*(1)}(0) \qquad \text{.......(51)}$$

Where the quantity $p_N^{*(1)}(0)$ can be determined by differentiating equations (9) – (14) respectively with respect to $\theta \ at \ \theta = 0$, we have

$$p_1^{*(1)}(0) = \frac{\lambda S^{*(1)}(0)[p_1(0)+p_2(0)]+p_2(0)}{\lambda^2} \qquad \text{.......(52)}$$

$$p_n^{*(1)}(0) = \frac{\lambda p_{n-1}^{*(1)}(0)+S^{*(1)}(0)p_{n+1}(0)}{\lambda} + \frac{\lambda p_{n-1}^{*(1)}(0)+p_{n+1}(0)-p_n(0)}{\lambda^2}; (n = 2,3, \dots, L) \qquad \text{.......(53)}$$

$$p_{L+1}^{*(1)}(0) = \frac{\lambda p_L^{*(1)}(0)+S^{*(1)}(0)p_{L+2}(0)}{\alpha_{L+1}} + \frac{\lambda p_L^{*(1)}(0)+p_{L+2}^*(0)-p_{L+1}(0)}{\alpha_{L+1}^2} \qquad \text{.......(54)}$$

$$p_n^{*(1)}(0) = \frac{\alpha_{n-1}p_{n-1}^{*(1)}(0)+S^{*(1)}(0)p_{n+1}(0)}{\alpha_n} + \frac{\alpha_{n-1}p_{n-1}^{*(1)}(0)+p_{n+1}(0)-p_n(0)}{\alpha_n^2}; (n = L+2, \dots M) \qquad \text{.......(55)}$$

$$p_{M+1}^{*(1)}(0) = \frac{\alpha_M p_M^{*(1)}(0)+S^{*(1)}(0)p_{M+2}(0)}{\beta_{M+1}} + \frac{\alpha_M p_m^{*(1)}(0)+p_{M+2}(0)-p_{M+1}(0)}{\beta_{M+1}^2} \qquad \text{.......(56)}$$

$$p_n^{*(1)}(0) = \frac{\beta_{n-1}p_{n-1}^{*(1)}(0)+S^{*(1)}(0)p_{n+1}(0)}{\beta_n} + \frac{\beta_{n-1}p_{n-1}^*(0)+p_{n+1}(0)-p_n(0)}{\beta_n^2}; (n = M+2, \dots N) \qquad \text{.......(57)}$$

$p_1^{*(1)}(0)$ can be obtain from (52) and $p_n^{*(1)}(0); \ (2 \le n \le N)$ can be determined recursively from (53) – (57) hence $p_{N+1}^*(0)$ can be found from (51). Thus, $p_n^*(0); \ (1 \le n \le N+1)$ can be determined in terms of P_0. Now using normalizing condition, we have

$$p_o + \sum_{n=1}^{N+1} p_n^*(o) = 1 \qquad \text{.......(58)}$$

Let

$$p_n^*(o) = x_n(o) \, p_o \qquad \text{.......(59)}$$

Using (59), from (58) we have

$$p_o = [1 + \sum_{n=1}^{N+1} x_n(o)]^{-1}$$ …….(60)

The steady state probabilities at an arbitrary time are

$$p_n = \begin{cases} \frac{1}{1+\sum_{n=1}^{N+1} x_n(o)}, & n = o \\ \frac{x_n(o)}{1+\sum_{n=1}^{N+1} x_n(o)}, & n = 1,2, \dots \dots, N+1 \end{cases}$$

IV. PERFORMANCE MEASURES

Using equation (61), the performance measures for the system can be obtained as

$$L_s = \sum_{n=1}^{N+1} n\, p_n$$ …….(61)

$$L_q = \sum_{n=1}^{N+1} (n-1)\, p_n$$ …….(62)

Using Little's formula, the time spent by a customer in the system, W, and in the queue Wq can be obtained by

$$W = {}^{L_o}/_{\lambda'}$$

and

$$W_q = {}^{L_q}/_{\lambda'}$$ …….(63)

Where

$$\lambda' = \lambda_1' + \lambda_2' + \lambda_3'$$

$$\lambda_1' = \lambda_1(1 - p_{Lost\,1}) = \lambda_1(1 - p_{N+1})$$

$$\lambda_2' = \sum_{n=o}^{M} \lambda_2 p_n + \sum_{n=M+1}^{N} \lambda_2\, b_n\, p_n$$

$$\lambda_3' = \sum_{n=o}^{L} \lambda_3 p_n + \sum_{n=L+1}^{M} \lambda_3\, b_n\, p_n$$

V. NUMERICAL EXAMPLE

We consider an example with $\lambda_1 = 1.0$, $\lambda_2 = 2.0$, $\lambda_3 = 3.0$. We assume an exponential service time with $E(S) = \frac{1}{\mu} = 0.33$ (approx.) as $\mu = 3$.

$$\lambda = \lambda_1 + \lambda_2 + \lambda_3 = 6.0, \qquad S^*(\theta) = \frac{\mu}{\mu+\theta}, b_n = \frac{1}{n+1}$$

Step 1: Compute $p_n(0), (n = 1,2), p_1^*(0)$

$$p_1(0) = \lambda p_0 = 6p_0$$

$$p_2(0) = \lambda p_0 \left[\frac{1-S^*(\lambda)}{S^*(\lambda)}\right] = 12.181818\ p_0$$

$$p_1^*(0) = \frac{p_2(0)}{\lambda} = 2.030303\ p_0$$

Step 2: Compute $p_n(0), (n = 2,3,4,5,6,7,8)$

$$p_1^*(\lambda) = -S^{*(1)}(\lambda)\,[p_1(0) + p_2(0)] = 0.6733999\ p_0$$

$$p_1^{*(1)}(\lambda) = \frac{-S^{*(2)}(\lambda)}{2}\ [p_1(0) + p_2(0)] = -0.0748218\ p_0$$

$$p_3(0) = \frac{p_2(0) - \lambda p_1^*(\lambda)}{S^*(\lambda)} = 24.670986\ p_0$$

$$p_2^{*(1)}(\lambda) = -\left[\lambda p_1^{*(1)}(\lambda) + p_3(0)\, S^{*(1)}(\lambda)\right] = 1.3626693\ p_0$$

$$p_4(0) = \frac{P_3(0) - \lambda p_2^*(\lambda)}{S^*(\lambda)} = 49.9847\ p_0$$

$$p_1^*(\alpha_{L+1}) = \frac{S^*(\alpha_{L+1})\,[p_1(0)+p_2(0)]}{\lambda_3(1-b_{L+1})} = 0.9435256\ p_0$$

$$p_2^*(\alpha_4) = \frac{\lambda p_1^*(\alpha_4) S^*(\alpha_4)\ p_3(0) + p_2(0)}{\lambda_3(1-b_4)} = 1.955587\ p_0$$

$$p_3^*(\alpha_4) = \frac{\lambda p_2^*(\alpha_4) + S^*(\alpha_4)\ p_4(0) - p_3(0)}{\lambda_3(1-b_4)} = 4.0761962\ p_0$$

$$p_5(0) = \frac{P_4(0) - \lambda p_3^*(\alpha_4)}{S^*(\alpha_4)} = 56.160557\ p_0$$

$$p_1^{*(1)}(\alpha_5) = \frac{S^{*(1)}(\alpha_5)\,[p_1(0)+p_2(0)] + p_1^*(\alpha_5)}{\lambda_3(1-b_5)} = -0.3343735\ p_0$$

$$p_1^{*(2)}(\alpha_5) = \frac{S^{*(2)}(\alpha_5)\,[p_1(0)+p_2(0)] + 2p_1^{*(1)}(\alpha_5)}{\lambda_3(1-b_5)} = -0.108605\ p_0$$

$$p_2^{*(1)}(\alpha_5) = \frac{\left[\lambda p_1^{*(1)}(\alpha_5) + S^*(\alpha_5)\ p_3(0)\right] + 2p_2^*(\alpha_5)}{\lambda_3(1-b_5)} = 5.3344264\ p_0$$

$$p_3^{*(1)}(\alpha_5) = \frac{\lambda p_2^{*(1)}(\alpha_5) + S^{*(1)}(\alpha_5)\ p_4(0) + p_3^*(\alpha_5)}{\lambda_3(1-b_5)} = 9.7403768\ p_0$$

$$p_4^*(\alpha_5) = -\left[\lambda p_3^{*(1)}(\alpha_5) + S^{*(1)}(\alpha_5)\ p_5(0)\right] = -54.45453\ p_0$$

$$p_6(0) = \frac{p_5(0) - \alpha_4 p_4^*(\alpha_5)}{S^*(\alpha_5)} = 546.42658\ p_0$$

$$p_1^*(\beta_6) = \frac{S^*(\beta_6)\,[p_1(0)+p_2(0)] - p_1(0)}{\lambda_3 + \lambda_2(1-b_6)} = 1.426997\ p_0$$

$$p_2^*(\beta_6) = \frac{\lambda p_1^*(\beta_6) + S^*(\beta_6)\ p_3(0) - p_2(0)}{\lambda_3 + \lambda_2(1-b_6)} = 2.8954206\ p_0$$

$$p_3^*(\beta_6) = \frac{\lambda p_2^*(\beta_6) + S^*(\beta_6)\ p_4(0) - p_3(0)}{\lambda_3 + \lambda_2(1-b_6)} = 5.8738155\ p_0$$

$$p_4^*(\beta_6) = \frac{\lambda p_3^*(\beta_6) + S^*(\beta_6)\ p_5(0) - p_4(0)}{\lambda_3 b_4 + \lambda_2(1-b_6)} = 10.616918\ p_0$$

$$p_5^*(\beta_6) = \frac{\alpha_4 p_4^*(\beta_6)+S^*(\beta_6)\, p_6(0)-p_5(0)}{\lambda_3 b_4+\lambda_2(1-b_6)} = 164.43952\ p_0$$

$$p_7(0) = \frac{p_6(0)-\alpha_5 p_5^*(\beta_6)}{S^*(\beta_6)} = 641.5882\ p_0$$

$$p_1^*(\beta_7) = \frac{S^*(\beta_7)\,[p_1(0)+p_2(0)]}{\lambda_3+\lambda_2(1-b_7)} = 2.7019416\ p_0$$

$$p_1^{*(1)}(\beta_7) = \frac{S^{*(1)}(\beta_7)\,[p_1(0)+p_2(0)]+p_1^*(\beta_7)}{\lambda_3+\lambda_2(1-b_7)} = 1.2045804\ p_0$$

$$p_1^{*(2)}(\beta_7) = \frac{S^{*(2)}(\beta_7)\,[p_1(0)+p_2(0)]+2p_1^*(\beta_7)}{\lambda_3+\lambda_2(1-b_7)} = 0.8063687\ p_0$$

$$\begin{cases} p_2^{*(1)}(\beta_7) = \dfrac{\lambda p_1^{*(1)}(\beta_7)+S^{*(1)}(\beta_7)\, p_3(0)+p_2^*(\beta_7)}{\lambda_3+\lambda_2(1-b_7)} = 3.642505\ p_0 \\ p_3^{*(1)}(\beta_7) = \dfrac{\lambda p_2^{*(1)}(\beta_7)+S^{*(1)}(\beta_7)\, p_4(0)+p_3^*(\beta_7)}{\lambda_3 b_4+\lambda_2(1-b_7)} = 33.199202\ p_0 \\ p_5^{*(1)}(\beta_7) = \dfrac{\alpha_4 p_4^{*(1)}(\beta_7)+S^*(\beta_7)\, p_6(0)+p_5^*(\beta_7)}{\lambda_3 b_5+\lambda_2(1-b_7)} = 135.69654\ p_0 \\ p_6^*(\beta_7) = -\left[\alpha_5 p_5^{*(1)}(\beta_7) + S^{*(1)}(\beta_7)\, p_7(0)\right] = -481.84526\ p_0 \end{cases}$$

$$p_8(0) = \frac{p_7(0)-\beta_6\, p_6^*(\beta_7)}{S^*(\beta_7)} = 818.73014\ p_0$$

$$\begin{cases} p_2^*(\beta_7) = \dfrac{\lambda p_1^*(\beta_7) + S^*(\beta_7)\, p_3(0)}{\lambda_3 + \lambda_2(1-b_7)} = 5.1878484\ p_0 \\ p_3^*(\beta_7) = \dfrac{\lambda p_2^*(\beta_7) + S^*(\beta_7)\, p_4(0)}{\lambda_3 + \lambda_2(1-b_7)} = 13.981137\ p_0 \\ p_4^*(\beta_7) = \dfrac{\lambda p_3^*(\beta_7) + S^*(\beta_7)\, p_5(0)}{\lambda_3 + \lambda_2(1-b_7)} = 52.56577\ p_0 \\ p_5^{*(1)}(\beta_7) = \dfrac{\alpha_4 p_4^*(\beta_7) + S^*(\beta_7)\, p_6(0)}{\lambda_3 b_5 + \lambda_2(1-b_7)} = 276.55946\ p_0 \end{cases}$$

Step 3: Compute $p_n^*(0); (n = 2,3,4,5,6,7)$

$$p_2^*(0) = {p_3(0)}/{\lambda} = 4.1118276\ p_0$$

$$p_3^*(0) = {p_4(0)}/{\lambda} = 8.3307833\ p_0$$

$$p_4^*(0) = {p_5(0)}/{\alpha_4} = 15.600154\ p_0$$

$$p_5^*(0) = {p_6(0)}/{\alpha_5} = 156.12188\ p_0$$

$$p_6^*(0) = \frac{p_7(0)}{\beta_6} = 499.01304\ p_0$$

$$p_7^*(0) = {p_8(0)}/{\beta_7} = 654.98411\ p_0$$

Step 4: Compute $p_8^*(0)$

$$p_1^{*(1)}(0) = \frac{\lambda S^{*(1)}(0)[p_1(0)+p_2(0)]+p_2(0)}{\lambda^2} = -0.6717171\ p_0$$

$$p_2^{*(1)}(0) = \frac{\lambda p_1^{*(1)}(0) + S^{*(1)}(0)p_3(0)}{\lambda} + \frac{\lambda p_1^*(0) + p_3(0) - p_2(0)}{\lambda^2} = -1.3570215\ p_0$$

$$p_3^{*(1)}(0) = \frac{\lambda p_2^{*(1)}(0) + S^{*(1)}(0)p_4(0)}{\lambda} + \frac{\lambda p_2^*(0) + p_4(0) - p_3(0)}{\lambda^2} = -2.7454853\ p_0$$

$$p_4^{*(1)}(0) = \frac{\lambda p_3^{*(1)}(0) + S^{*(1)}(0)p_5(0)}{\alpha_4} + \frac{\lambda p_3^*(0) + p_5(0) - p_4(0)}{\alpha_4^2} = -5.4424845\ p_0$$

$$p_5^{*(1)}(0) = \frac{\alpha_4 p_4^{*(1)}(0) + S^{*(1)}(0)p_6(0)}{\alpha_5} + \frac{\alpha_4 p_4^*(0) + p_6(0) - p_5(0)}{\alpha_5^2} = -10.011792\ p_0$$

$$p_6^{*(1)}(0) = \frac{\alpha_5 p_5^{*(1)}(0) + S^{*(1)}(0)p_7(0)}{\beta_6} + \frac{\alpha_5 p_5^*(0) + p_7(0) - p_6(0)}{\beta_6^2} = -41.630486\ p_0$$

$$p_7^{*(1)}(0) = \frac{\beta_6 p^{*(1)}(0) + S^*(0)p_8(0)}{\beta_7} + \frac{\beta_6 p_6^*(0) + p_8(0) - p_7(0)}{\beta_7^2} = -261.81077\ p_0$$

$$p_8^*(0) = -\beta_7 p_7^{*(1)}(0) = 336.61384\ p_0$$

Step 5: Compute p_n, (n=0,1,2,…..,8) using equation (61)

P_0= 0.000596, p_1 = 0.00121, p_2 = 0.0024506

P_3= 0.0049651, p_4 = 0.0092976, p_5 = 0.0930486

P_6= 0.2974117, p_7 = 3903705, p_8 = 0.2006218

Step 6: Compute Ls; Lq, W and Wq by using equations (62) and (63)

Ls = 6.6452435, Lq = 5.6461021

W = 4.9962723 Wq = 4.2450609

VI. DISCUSSION ON NUMERICAL RESULTS

We consider a numerical example with $\lambda_1 = 1.0,\ \lambda_2 = 2.0,\ \lambda_3 = 3.0$; exponential service time with $E(S) = \frac{1}{\mu} = 0.33$ (approx) when $\mu = 3, L = 3, M = 5, N = 7$. We have calculated $p_n(n = 0,1,2, \ldots .8)$ recursively and the performance measure Ls; Lq, W and Wq by using equations.

VII. CONCLUSIONS

In this paper, we have analysed M_1, M_2, M_3/G/1/N+1 queueing system with balking. The steady state probabilities have been obtained recursively. We provide a numerical example where we calculate the performance measure Ls; Lq, W and Wq.

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