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Analytical solution of bioheat transfer equation with variable thermal conductivity in skin

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ABSTRACT: Temperature dependent thermal properties of tissue is an important factor to achieve realistic models in thermal treatments. Tissue temperature dependent thermal conductivity is used to model the Pennes' bioheat transfer equation in skin with one relaxation time. Three different time-dependent surface heat flux, namely, continuous, periodic and ramp type are applied on skin surface without heat loss at the bottom. Laplace transform method is applied to solve the problem and the significant effect of variable conductivity parameter k_1 is observed on the tissue temperature distribution.
Keywords: Bioheat transfer; Skin tissue; Variable thermal Conductivity; Laplace transform.

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I. INTRODUCTION

The transfer of heat in skin is mainly a heat conduction process coupled to complicated physiological processes. The thermal properties of skin vary across different layers; even within the same layer. These properties of skin are influenced by temperature, damage, pressure, and age, etc. A variety of thermal methods have been developed and applied to the treatment of disease or injury involve either a raising or lowering of temperature in targeted skin area to kill or thermally denaturize necrotic cells. Pennes' model [1] is widely used to model such problems due to its simplicity. Thermal wave model was proposed by Cattaneo [2] and Vernotte [3] to capture the effect of microstructural interaction in heat flux. Various studies have been done to cure disease involving skin tissue without affecting the surrounding healthy tissue. Xu et al. [4] and Rossmann et al. [5] reviewed the biothermomechanical behavior and temperature dependence of electrical and thermal properties of tissue respectively. Poor et al. [6] studied temperature response of skin tissue due to time-dependent surface heat fluxes.

Later, Tzou [7-8] introduced dual-phase-lag (DPL) model of heat conduction. Askarizadeh et al. [9] utilized DPL model in treating the transient heat transfer problem in skin tissue. Liu et al. [10-12] employed the DPL model to analyze the heat transfer problem in hyperthermia treatment, skin and eliminated the inconsistence in theory discovered in the literature. Das et al. [13] estimated the breast tumor characteristics with known skin surface temperature by using finite volume method. Agrawal et al. [14] proposed a finite element model to study temperature distribution in skin and deep tissue of elliptical tapered shape human limb. In thermal ablation treatment. Kumar et al. [15] investigated the thermal behavior in living tissue using time fractional dual-phase-lag bioheat transfer (DPLBHT) model subjected to dirichlet boundary condition during thermal therapy. Shaoa et al. [16] developed a Radiofrequency ablation (RFA) mathematical model to study the influence of injected nanoparticals in irregularly shaped liver tumors.

It is well established that physical property of engineering and bio-physical materials vary significantly with temperature. The temperature dependence of material properties like thermal conductivity k and specific heat c affect biothermomechanical behavior of various materials. Therefore, to achieve efficient solution of temperature change problems, the temperature dependence of material properties should be taken into account, which require additional modeling, experimentation and computational efforts. A comprehensive work has been done by various researchers for variable thermal conductivity in continuum mechanics, notable among them are [17-28]. Li et al. [29-30] studied the transient responses for a half-space with variable thermal conductivity and diffusivity in the context of the generalized thermoelastic diffusion theory. In spite of all these studies no work has been done to investigate the effect of variable conductivity on Pennes' bioheat transfer equation. In this work, the analytical solution of the thermal wave model of bioheat transfer with temperature dependent variable thermal conductivity is obtained. The results shows that the tissue thermal conductivity parameter k_1 affects the tissue temperature distribution. T tissue temperature, $^\circ C$ T_0 initial temperature of body, $^\circ C$ T_a arterial blood temperature, $^\circ C$ q heat flux, Wm^{-2} c specific heat of tissue, $Jkg^{-1}K^{-1}$ c_b specific heat of blood,

$Jkg^{-1}K^{-1}$ k variable thermal conductivity of tissue, $Wm^{-1}K^{-1}$ k_0 constant thermal conductivity of tissue k_1 variable conductivity parameter q_0 incident heat flux amplitude (Wm^{-2}) q_1 dimensionless incident heat flux amplitude ρ tissue density, kgm^{-3} ρ_b blood mass density, kgm^{-3} ω_b blood perfusion rate, s^{-1} q_m heat source due to metabolic heat generation in the tissue, Wm^{-3} t time, s τ_q heat flux relaxation time, s κ tissue thermal diffusivity, m^2s^{-1} s Laplace domain parameter l bromwich contour integration line L tissue slab length, m x coordinate variable, m x' dimensionless coordinate ϑ' dimensionless tissue temperature τ_1 dimensionless heat flux relaxation time H Heaviside function δ Dirac delta function ω dimensionless heat flux frequency τ_i dimensionless duration time of pulse train heat flux

II. FORMULATION OF THE PROBLEM

A one dimensional finite length tissue is considered whose bottom boundary is insulated. Three different type heat flux, namely, instantaneous, periodic and pulse train are applied on skin surface. One dimensional Pennes' bioheat equation is

$$\rho c(T)\frac{\partial T}{\partial t} = -\frac{\partial q}{\partial x} + \omega_b \rho_b c_b (T_a - T) + q_m \cdot \quad (1)$$

$$q = -k(T)\frac{\partial T}{\partial x}, (2)$$

$k(T)$ is the thermal conductivity with variable temperature such that

$$\rho c(T) = \frac{k(T)}{\kappa}, \quad (3)$$

Equation (1) with the use of equation (2) and equation (3) yield, the following form of Pennes' equation

$$\frac{\partial}{\partial x}(k(T)\frac{\partial T}{\partial x}) = [\frac{k(T)}{\kappa}\frac{\partial T}{\partial t} - \omega_b \rho_b c_b (T_a - T) - q_m] \cdot \quad (4)$$

In case of thermal wave model

$$(1+\tau_q \frac{\partial}{\partial t})q = -k(T)\frac{\partial T}{\partial x}. \quad (5)$$

Using equation (5), in equation (4) gives

$$(1+\tau_q \frac{\partial}{\partial t})[\frac{k(T)}{\kappa}\frac{\partial T}{\partial t} - \omega_b \rho_b c_b (T_a - T) - q_m] = \frac{\partial}{\partial x}[k(T)\frac{\partial T}{\partial x}] \cdot \quad (6)$$

Initial conditions

$$T(x,0) = 0, T_t(x,0) = 0. \quad (7)$$

Boundary conditions

$$T(0,t) = q_0 f(t), \frac{\partial T}{\partial t}(1,t) = 0. \quad (8)$$

For the theory of heat and mass transfer in living tissue one of the central issues is how to create good models that describe these transport phenomena. An important factor to achieve realistic models is the use of mathematical functions to describe the temperature dependence of thermal properties of tissue.

A commonly used approach for modeling the temperature dependence of thermal and electrical properties for temperature below $100^\circ C$ is based on linear equations and employs constant temperature coefficient such as:

$$k(T) = k_0(1+k_1 T). \quad (9)$$

Considering the mapping (Kirchhoff's transformation) [31] as

$$\vartheta = \frac{1}{k_0}\int_0^T k(\xi)d\xi, \quad (10)$$

with

$$\vartheta = T + \frac{k_1}{2}T^2. \quad (11)$$

After obtaining ϑ, the tissue temperature T can be obtained by solving equation (11) which gives

$$T = \frac{-1+\sqrt{1+2k_1\vartheta}}{k_1}. \quad (12)$$

Eq. (6) with the use of Eq. (9) and Eq. (10), yield the following equation in linear form

$$(1+\tau_q \frac{\partial}{\partial t})[\frac{\partial \vartheta}{\partial t}+\omega_b \rho_b c_b \frac{\kappa}{k_0}(\vartheta - T_a)-\frac{\kappa}{k_0} q_m]=\kappa \frac{\partial^2 \vartheta}{\partial x^2}. \qquad (13)$$

III. SOLUTION OF THE PROBLEM

Introducing the dimensionless variables and similarity criteria

$$x' = \frac{x}{L},\ t' = \omega_0 t,\ \vartheta' = \frac{\vartheta - T_0}{T_0},\ T' = \frac{T - T_0}{T_0},\ \vartheta_{a'} = \frac{T_a - T_0}{T_0},$$
$$\tau_1 = \omega_0 \tau_q,\ a_1 = \frac{W_b c_b \kappa}{k_0 \omega_0},\ a_2 = \frac{\kappa q_m}{k_0 \omega_0 T_0},\ W_b = \omega_b \rho_b,\ \omega_0 = \frac{k_0}{\rho c L^2}, \qquad (14)$$
$$b_1 = a_2 + a_1 \vartheta_{a'}$$

Using the non-dimensional variable and removing the dashes, equation (13) reduces to the following form

$$(1+\tau_1 \frac{\partial}{\partial t})[\frac{\partial \vartheta}{\partial t}+a_1 \vartheta - b_1]=\frac{\partial^2 \vartheta}{\partial x^2}. \qquad (15)$$

with initial conditions

$$\vartheta(x,0)=0,\ \vartheta_t(x,0)=0, \qquad (16)$$

and boundary conditions

$$\vartheta(0,t)=q_1 f(t)+q_1^2 \frac{k_1}{2} f(t)^2,\ \vartheta_x(1,t)=0, \qquad (17)$$

where $f(t)$ is a known function of dimensionless t as follows,

Case 1

$$f(t)=\delta(t), \qquad (18)$$

Case 2

$$f(t)=e^{i\omega t}, \qquad (19)$$

Case 3

$$f(t)=H(t)-H(t-\tau_i). \qquad (20)$$

Applying Laplace transform technique, equation (15) becomes,

$$\frac{d^2 \overline{\vartheta}}{dx^2}-\beta \overline{\vartheta} = -\frac{b_1}{s}, \qquad (21)$$

where

$$\beta = \tau_1 s^2 + (1+\tau_1 a_1)s + a_1, \qquad (22)$$

In Laplace domain, boundary conditions are

$$\overline{\vartheta}(0,s)=V_R,\ \overline{\vartheta}_x(1,s)=0, \qquad (23)$$

where $F(s)$ is Laplace transform of $f(t)$ and

$$V_R = q_1 F(s) + \frac{k_1}{2} q_1^2 F(s)^2. \qquad (24)$$

Solution of equation (21), by using the boundary conditions (23) is

$$\overline{\vartheta}(x,s)=(V_R - \frac{b_1}{\beta s})\frac{\cosh(\sqrt{\beta}(x-1))}{\cosh(\sqrt{\beta})}+\frac{b_1}{\beta s}. \qquad (25)$$

Inverse Laplace transform of $\overline{\vartheta}(x,s)$ can be obtained from the following Bromwich contour integration [32]

$$\vartheta(x,t)=\frac{1}{2\pi i}\lim_{l\to\infty}\int_{\gamma - il}^{\gamma + il} e^{ts}\overline{\vartheta}(x,s)ds. \qquad (26)$$

Using the inversion theorem, the following inverse Laplace transform of equation (25) is obtained for

Case 1

$$\vartheta = I_0 + \sum_{m=1}^{2}\sum_{n=1}^{\infty}\frac{2\lambda_n i l_1 \cos(\lambda_n (x-1))}{\sin(\lambda_n) n_3} e^{s_{nm} t}, \qquad (27)$$

Case 2

$$\begin{aligned}\vartheta(x,t) &= I_0 + q_1\frac{\cosh(n_1(x-1))}{\cosh(n_1)}e^{i\omega t} + q_1\sum_{m=1}^{2}\sum_{n=1}^{\infty}\frac{2\cos(\lambda_n(x-1))\lambda_n i}{(s_{nm}-i\omega)\sin(\lambda_n)n_3}e^{s_{nm}t} \\ &+ \frac{q_1^2k_1}{2}\left(\frac{t\cosh(n_1(x-1))}{\cosh(n_1)} + \frac{\sinh(n_1(x-1))(x-1)n_2}{2\cosh(n_1)n_1} - \frac{\sinh(n_1)\cosh(n_1(x-1))n_2}{2\cosh^2(n_1)n_1}\right)e^{i\omega t},\end{aligned} \quad (28)$$

Case 3

For $t < \tau_i$

$$\begin{aligned}\vartheta(x,t) &= I_0 + q_1\frac{\cosh(\sqrt{a_1}(x-1))}{\cosh(\sqrt{a_1})} + q_1\sum_{m=1}^{2}\sum_{n=1}^{\infty}\frac{2i\lambda_n\cos(\lambda_n(x-1))}{s_{nm}\sin(\lambda_n)n_3}e^{s_{nm}t} \\ &+ \frac{q_1^2k_1}{2}\sum_{m=1}^{2}\sum_{n=1}^{\infty}\frac{2i\lambda_n\cosh(\lambda_n(x-1))}{s_{nm}^2\sin(\lambda_n)n_3}e^{s_{nm}t} + \frac{q_1k_1^2}{2}[t\frac{\cosh(\sqrt{a_1})(x-1)}{\cosh(\sqrt{a_1})} \\ &+ \frac{\sinh(\sqrt{a_1}(x-1))n_4}{2\sqrt{a_1}\cosh(\sqrt{a_1})} - \frac{\sinh(\sqrt{a_1})\cosh(\sqrt{a_1}(x-1))n_4}{2\sqrt{a_1}\cosh^2(\sqrt{a_1})}],\end{aligned} \quad (29)$$

for $t \geq \tau_i$

$$\begin{aligned}\vartheta(x,t) &= I_0 + \sum_{m=1}^{2}\sum_{n=1}^{\infty}\frac{2i\lambda_n q_1\cos(\lambda_n(x-1))}{s_{nm}\sin(\lambda_n)n_3}e^{s_{nm}t}(1-e^{-s_{nm}\tau_i}) \\ &+ \frac{q_1^2k_1}{2}\sum_{m=1}^{2}\sum_{n=1}^{\infty}\frac{2i\lambda_n\cos(\lambda_n(x-1))}{s_{nm}^2\sin(\lambda_n)n_3}e^{s_{nm}}(1+e^{-2s_{nm}\tau_i} - e^{-s_{nm}\tau_i}),\end{aligned} \quad (30)$$

where

$\lambda_n = (2n-1)\frac{\pi}{2}$ and $v_n = \lambda_n^2$, $n = 1,2...$

s_{nm} are roots of equation (31)

$$\tau_1 s_{nm}^2 + (1+\tau_1 a_1)s_{nm} + a_1 + v_n = 0. \quad (31)$$

Other constants are given in equation (32)

$$\begin{aligned}I_0 &= \frac{b_1}{a_1}(1-\frac{\cosh(\sqrt{a_1}(x-1))}{\cosh(\sqrt{a_1})}) - \sum_{m=1}^{2}\sum_{n=1}^{\infty}\frac{2b_1\cos(\lambda_n(x-1))}{i\lambda_n s_{nm}\sin(\lambda_n)n_3}e^{s_{nm}t}, \\ l_1 &= q_1 + \frac{q_1^2k_1}{2}, \\ n_1 &= \sqrt{(a_1+i\omega)(1+ia_1\tau_1)}, \\ n_2 &= 2i\tau_1\omega + a_1\tau_1 + 1, \\ n_3 &= 2\tau_1 s_{nm} + a_1\tau_1 + 1, \\ n_4 &= a_1\tau_1 + 1.\end{aligned} \quad (32)$$

IV. NUMERICAL RESULTS AND DISCUSSION

The following specified values of relevant parameters are considered similar to the parameters given in literature [33-34]. The thermal physical properties of skin tissue are $\rho_b = 1000\ kgm^{-3}$, $\rho = 1000\ kgm^{-3}$, $k_0 = 0.628\ Wm^{-1}K^{-1}$, $c_b = 4187\ Jkg^{-1}K^{-1}$, $c = 4187\ Jkg^{-1}K^{-1}$, $\omega_b = 1.87\times10^{-3}s^{-1}$, $q_m = 1.19\times10^3 Wm^{-3}$, $T_a = 37^\circ C$, $T_0 = 37^\circ C$, $L = 0.05m$, $q_0 = 5\times10^3\ Wm^{-2}$, $\tau_q = 16\ s$.

The study is done for limiting cases.

Figure 1 depicts the tissue temperature response with different values of variable thermal conductivity parameter k_1 for the case 1. It clearly shows that the value of k_1 significantly affecting the tissue temperature response. Temperature increases for $0 < t < .02$ and decreases for $0.02 < t < .04$ after that it becomes constant.

Figure 2 depicts the tissue temperature response with different values of variable thermal conductivity parameter k_1 for the case 2 i.e. periodic heat flux when $\omega = 1$. It is noticed that as the value of k_1 increases amplitude of temperature profile increases.

Figure 3 depicts the tissue temperature response with different values of variable thermal conductivity parameter k_1 for the case 3 i.e. pulse train heat flux.

Figure 4 depicts the tissue temperature response with dimensionless distance for case 1 and it shows that temperature profile is affected by the parameter k_1.

Figure 5 depicts the tissue temperature response subjected to periodic heat flux at three different depths under the skin surface when $\omega = 1$ and $k_1 = .5$. It is noticed that oscillatory amplitude becomes low for a large depth from the heating skin.

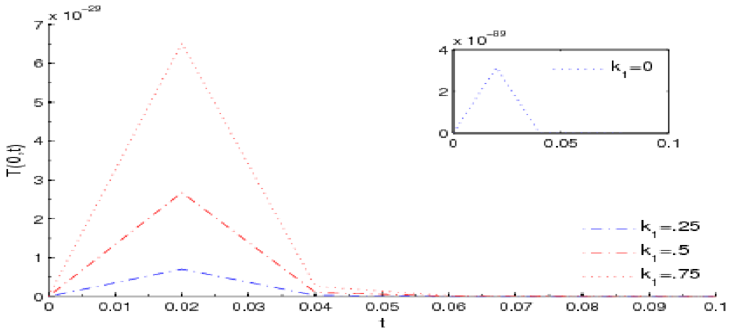

Figure 1: Temperature responses with time for the case 1 with different values of variable conductivity k_1

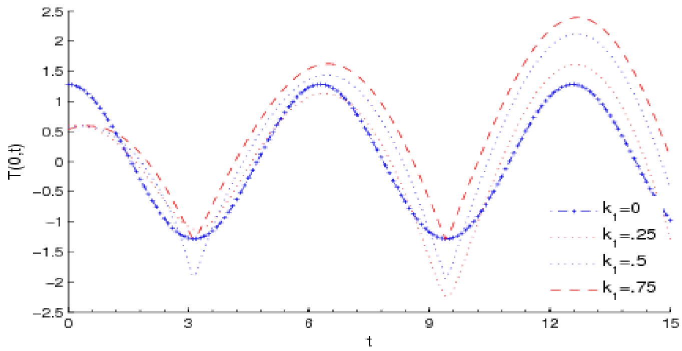

Figure 2: Temperature responses with time for the case 2 with different values of variable conductivity k_1

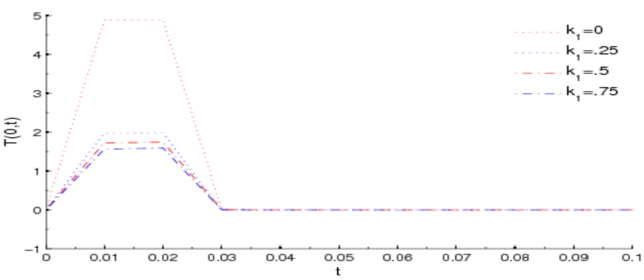

Figure 3: Temperature responses with time for the case 3 with different values of variable conductivity k_1

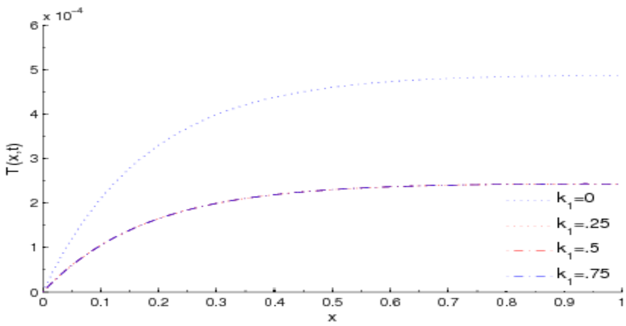

Figure 4: Temperature responses with dimensionless distance for the case 1 with different values of variable conductivity k_1

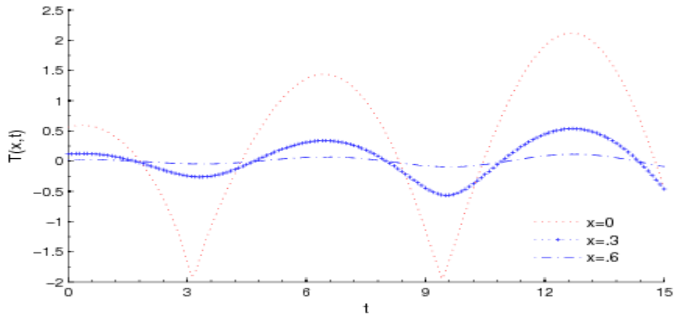

Figure 5: Temperature responses with time at different tissue depths for $k_1 = .5$

V. CONCLUSIONS

The lack of information on temperature-dependent thermal properties prevents us from accurately predicting the temperature distribution of the target tissue undergoing thermotherapy. Therefore, it is necessary to consider temperature-dependent thermal properties to achieve realistic models of thermal therapies in order to generate valid results. In this study, thermal wave model of bioheat transfer equation is modeled with variable conductivity of tissue to achieve realistic model. The problem is solved by using the Laplace transform method and subjected to three different boundary conditions, instantaneous, periodic and pulse train. Tissue temperature is obtained for all the three cases and it is observed that tissue thermal behavior is significantly affected by the variable thermal conductivity parameter k_1.

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