

The Mathematics Of Modeling Directional Wells – A Computer Approach

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ABSTRACT: The main objective of a successful drilling operation lies in the construction of safe and economically efficient wells, success of which depends on hitting the target (pay) zone. Although, drilling oil/gas well is a very complex operation yet a good, efficient and safe drilling operation depends on a good well plan.

Well trajectory planning requires a mixture of different parameter of interest; the offset data, the contour mapping, computer technical know-how, core data, safety report, etc., but in the end, it drip down to optimum identification of the well path, which is a function of exact mathematical model used to precisely calculate the wellbore trajectory.

This paper investigates the concept of directional drilling, its application, and derivation of mathematical model for the coding of adamuegwu (AMG - 02) well profile, using MS-Excel. A mathematical model is also presented that can be used to estimate the tangent angle and evaluate build up rate if not known for a specific well.

Index terms: offset data, wellbore trajectory, core data, whip-stock, simulator,

SYMBOLS

	Notation
KOP	Kick off Point
D	Total vertical Depth
r^o	Build up rate
d	Horizontal Displacement
N	North
S	South
W	West
E	East
R	Radius of curvature
L	Length of Arc
α	Angle of inclination
ROP	Rate of Penetration
Db	vertical depth of build up section

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I INTRODUCTION

Introduction of rotary drilling techniques in early 20th, pace way for increase in drilling deviated wellbore in the oil and gas industry. Deviation of wellbore from its normal path is a complex scenario which can be achieve with the application of certain forces, alone or in combination with adherent geological condition. Until 1970s downhole survey were primarily used to monitor lateral deviation of a wellbore. Preventive measures were put in place to maintain vertically drilled well in other to annul whip-stocking which is considered illegal during those days. Successful application of directional drilling has no bound in history and in later days.

According to Faraq (2013), he stated that in 1930California driller John Eastman had a patent for the techniques of conducting downhole surveys and intentionally deviate wellbore. He later founded the EASTMAN OIL WELL SURVEY COMPANY after which he became famous four years after for drilling a directional relief well to intercept and extinguish another that was burning near Conroe, Texas.

Modern directional drilling drill into large areas of reservoir containing oil/gas from one surface location, since a single large platform can support nothing less than 50 wells. This has tremendously reduced drilling cost over the years. Directional drilling increases coverage substantially, depending on the angle of inclination of the well (short, 1993).

According to HussainRabia (2015), planning a directional well requires a lot of concentration and commitment couple with require available parameter of interest, such as; surface and Target coordinates, size and shape of the target, local reference co-ordinates, required well inclination when entering the target zone, prognoses Lithology, offset well bit and BHA data, casing program and drilling fluid type, details of all potential hole problem, off set data of all nearby well. In recent time, objective of a directional drilling can be achieved with proper, effective and executable planning are put in place to minimize loss. In planning a directional well, the type of well to be drill(fig 1.4), the available parameter, and technical know-how of the personnel involve are of utmost importance in other to achieve desire objectives.

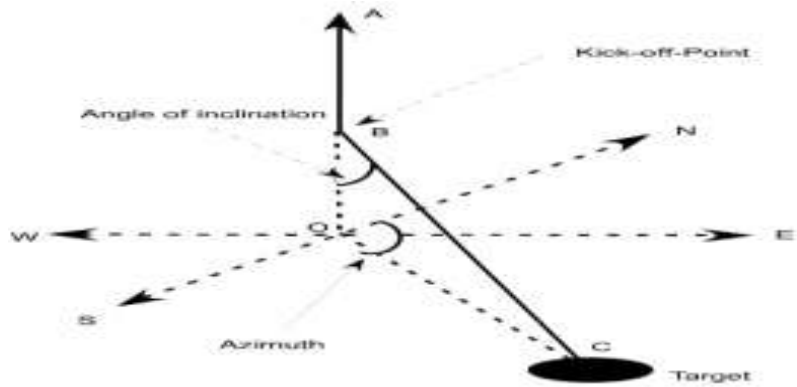


Figure1.1: Directional well parameter (Modified picture From Gabolde And Nguyen, 1991)

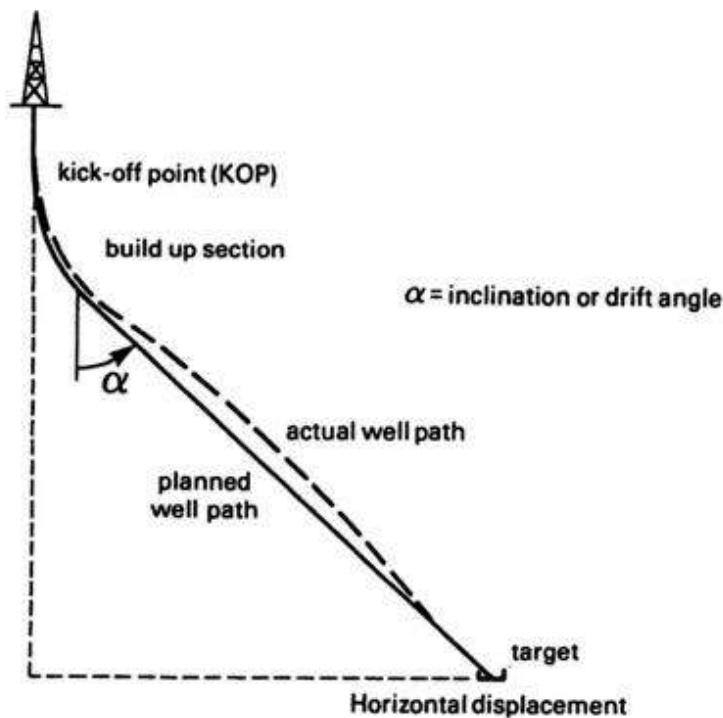


Figure 1.2: Vertical View of Directional well(Inglis, PETROLEUM ENGINEERING AND DEVELOPMENT STUDIES, VOLUME 2, DIRECTIONAL DRILLING, 1987)

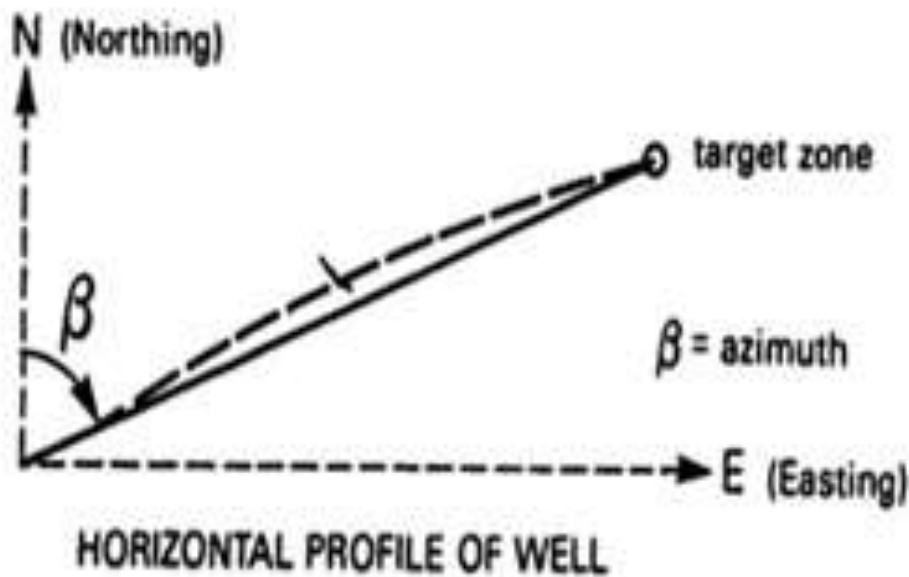


Figure 1.3: Horizontal View of Direction (Inglis, PETROLEUM ENGINEERING AND DEVELOPMENT STUDIES, VOLUME 2, DIRECTIONAL DRILLING, 1987)

From all enumerated and review, discovery of directional drilling has tremendously become a great breakthrough in oil gas field development both onshore and offshore, as target zone which could not be reach those days are drill to production now under accurate supervision.

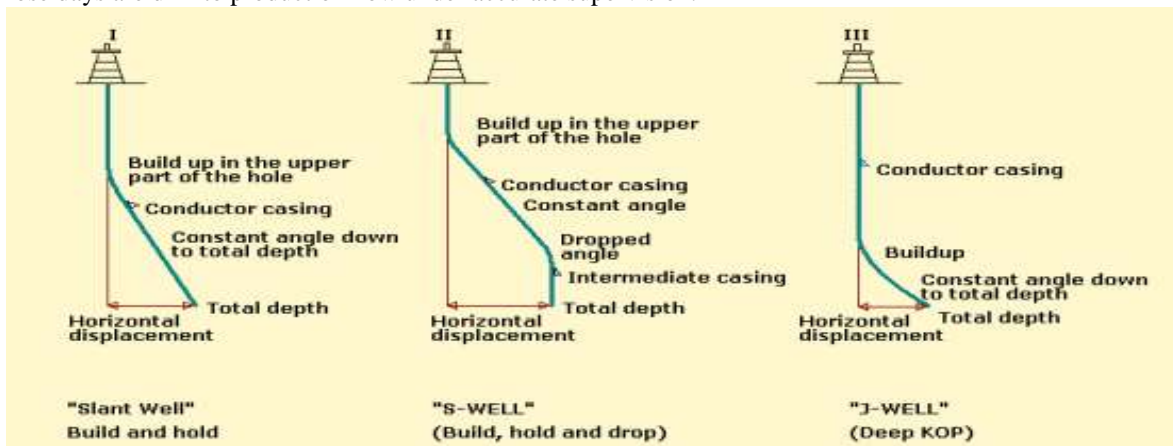


Fig.1.4: Directional Well Trajectories (<http://www.itcas.no/>)

MS-Excel

Microsoft Excel is a type of spreadsheet developed by Microsoft for used in all android, ios and windows system. It is multitask and provide use in the area of a macro programming language, pivot tables, and graphing tools. Since 1993, MS excel has been a very widely applied spreadsheet replacing lotus 1-2-3- as the most preferred industrial spreadsheet. Excel is attached as one other aspect of Microsoft. Microsoft- excel contain some principal features of a spreadsheets. It uses a cell grid which is arranged in column and rows. The columns are arranged and letter named while the rows are number named data which allow all aspect of arithmetic manipulation operations. It can be used by engineers, scientist, and financial enterprises in keeping, storing, and proper orientation of data for display and analysis. Sectioning of data for dependency factor evaluation can be carried out for different perspectives. (https://en.wikipedia.org/wiki/Microsoft_Excel, 2017)

II METHODOLOGY

MS-Excel, a simple, quick and easy to master Microsoft programming framework that is engineered for productively building type-safe and object-oriented applications was used in the planning the layout of the OMG-02 well profile. The mathematical model for calculating tangent angle was developed and incorporate into EXCEL spreadsheet to develop a simulator software that calculate the well trajectory path when other

parameters are known. The sensitivity of the data generated was plotted against accurate parameter to establish data eccentricity.

III MODELING OF AMG-02 WELL

The build and hold profile (fig 1.5) is the most common deviated well trajectory, and simple to achieve when drilling, it is employed in this well profile project. The important stage in planning the geometry profile of the well is to reach the target. Planning build and hold profile has three sections: vertical, build, and hold and tangent.

A potential build and hold well profile looks thus (Fig 1.5) and the prototype will be used for the well analysis in view.

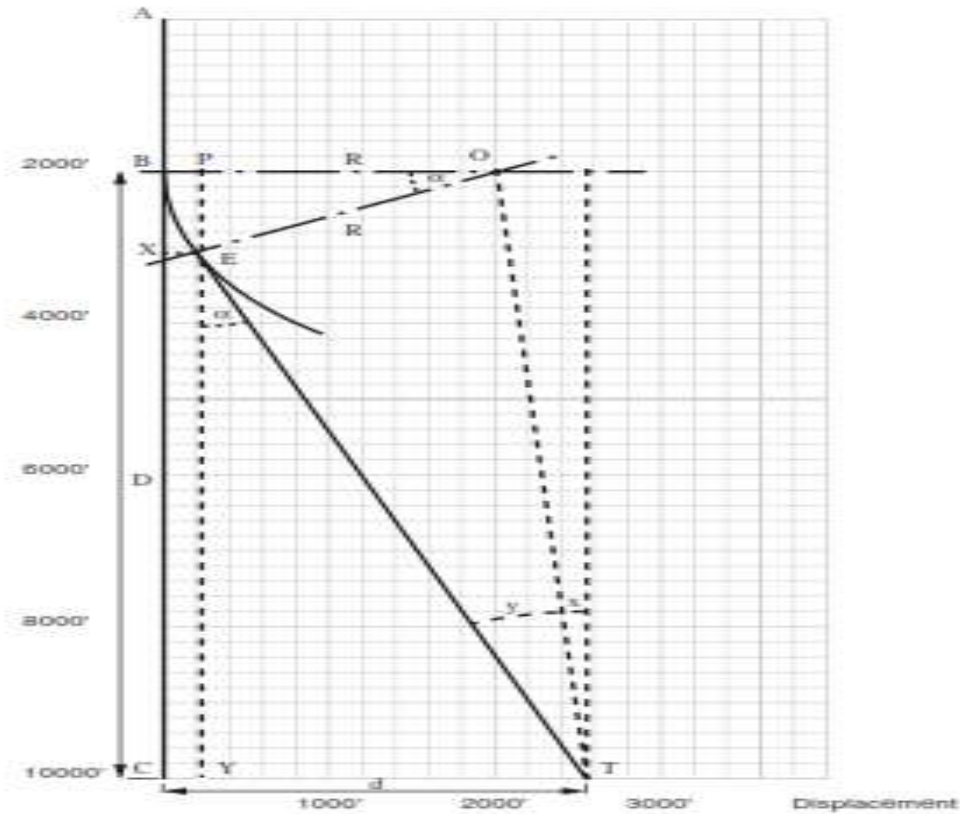


Fig 1.5: Draft Well Profile Trajectory

A proposed build and hold well profile takes two parts, the first part takes the situation where the proposed radius of curvature is greater than the total displacement of the well ($R > d$), fig 1.6 and the second where the radius of curvature is less than the total displacement of the well ($R < d$), fig 1.7.

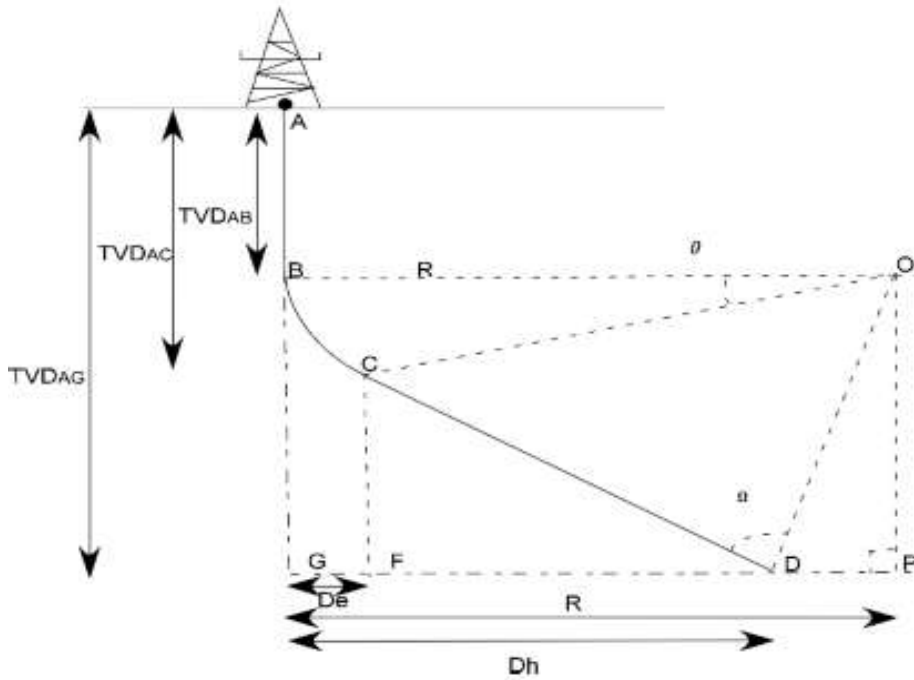


Fig. 1.6: Geometry of Build-and-Hold Type Well Path for $D_h > R$

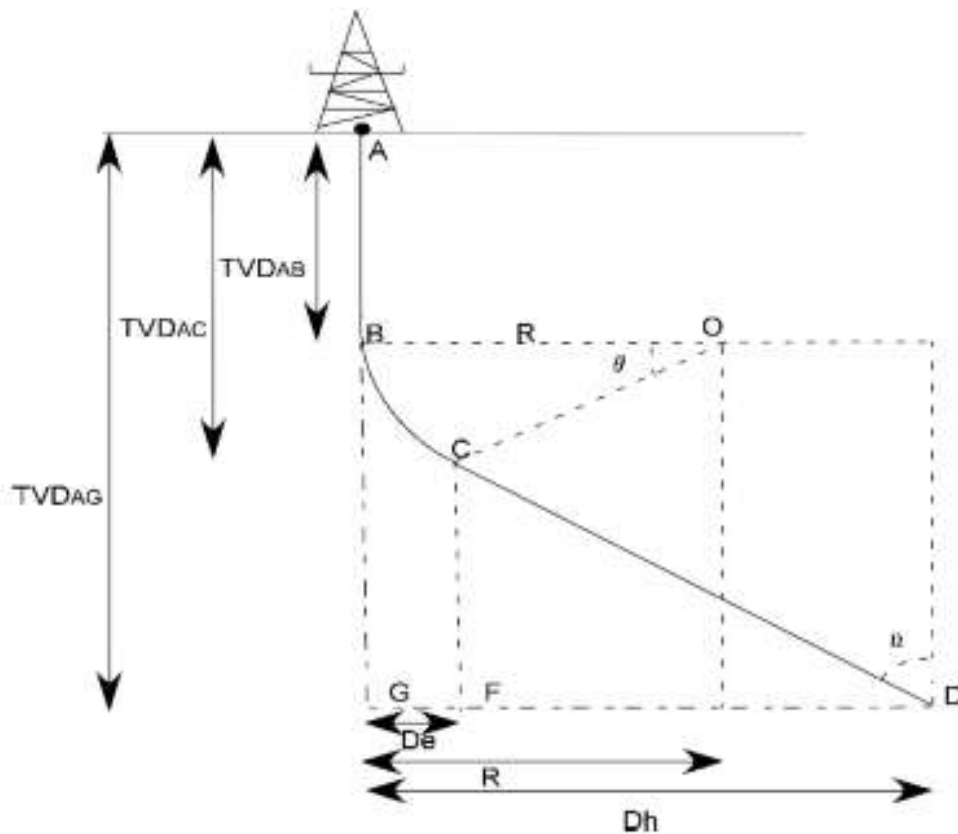


Fig. 1.7: Geometry of Build-and-Hold Type Well Path for $D_h > R$

For the well in view, the build and hold type well of path $d > R$ is chosen, since the planned value of horizontal displacement is greater than the radius of curvature of the well.

TANGENT ANGLE

Therefore from fig. 1.5 we can establish that, the tangent angle of the projected well can be establish thus,

$$\tan x = (d-R)/D \quad \text{-----} 1.01$$

$$\sin y = (R \cos x)/D \quad \text{-----} 1.02$$

$$\alpha = x + y \quad \text{-----} 1.02$$

AHD at the end of build section: =AE:

$$AE = AB + BE \text{ (CURVED LENGTH)} \quad \text{-----} 1.1$$

$$\frac{BE}{2\pi R} = \frac{\alpha}{360} \quad \text{-----} 1.2$$

TVD at the end of the build section: AX

$$AX = AB + PE \quad \text{-----} 1.3$$

$$PE = R * \sin(\alpha) \quad \text{-----} 1.4$$

$$AX = AB + R * \sin(\alpha) \quad \text{-----} 1.5$$

Displacement at the end of the build section: .XE is

$$XE = OB - OP \quad \text{-----} 1.6$$

$$OB = R \quad \text{-----} 1.7$$

$$OP = R * (\alpha) \quad \text{-----} 1.8$$

$$XE = R - R * (\alpha) \quad \text{-----} 1.9$$

AHD to the tangent section , AT

$$AT = AE + ET \quad \text{-----} 1.10$$

IV THE MATHEMATICAL MODEL OF BUILD UP SECTION BUILD AND DROP TYPE OF WELL TRAJECTORY

The minimum permissible drop rate is a function of the well total depth, maximum torque and drag limitation, mechanical limitation of the drill string or casing, and mechanical limitation of logging tools and production casing. The optimum build up rate in conventional directional wells are in the range of 1.50 to 30 degree per 100ft, though much higher rate are used for horizontal and multilateral well. If the available well parameter is limited and there is need to estimate or predict the buildup rate of the well, then the mathematical equation below is thus presented for evaluation purpose if the parameter of interest is available. It can be established from the pictorial justification of Case I ($R < d$) as depicted by fig 1.5 that:

That horizontal displacement at the end of a buildup section in equation 1.9,

$$R = R * \cos(\alpha) + d \quad \text{-----} 1.11$$

Also, it can be established from the same fig. 1.5 that the different between vertical depths at the end of buildup is thus related to KOP and R as,

$$Db - KOP = R * \sin(\alpha) \quad \text{-----} 1.12$$

Dividing equation 1.11. by 1.12

$$\frac{d}{Db - KOP} = \frac{R(1 - \cos(\alpha))}{R * \sin(\alpha)} \quad \text{-----} 1.13$$

On simplification,

$$\frac{d}{Db - KOP} = \frac{1 - \cos(\alpha)}{\sin(\alpha)} \quad \text{-----} 1.14$$

Square both sides of equation 1.9 result to

$$\frac{(1 - \cos(\alpha))^2}{(\sin(\alpha))^2} = \frac{d^2}{(Db - KOP)^2} \quad \text{-----} 1.15$$

Since $(\sin x)^2 + (\cos x)^2 = 1$ ----- 1.16

Therefore, on further simplification of equation 1.15

$$1 - 2 * \cos(\alpha) + \cos(\alpha)^2 = (1 - \cos(\alpha))^2 * \frac{d^2}{(Db - KOP)^2} \text{----- 1.7}$$

On expansion, we obtained

$$1 - 2 * \cos(\alpha) + \cos(\alpha)^2 = \frac{d^2}{(Db - KOP)^2} \cos(\alpha)^2 * \frac{d^2}{(Db - KOP)^2} \text{----- 1.8}$$

Collecting like terms

$$\left(1 + \frac{d}{Db - KOP}\right) * \cos(\alpha)^2 - 2 * \cos(\alpha) + 1^2 - \left(\frac{d}{Db - KOP}\right)^2 = 0 \text{----- 1.9}$$

As noted earlier, completing the square method of solving quadratic equation will be applied in the analysis of the generating the mathematical model in equation 1.9 above

$$\left(1 + \left(\frac{d}{Db - KOP}\right)^2\right) * \cos(\alpha)^2 - 2 * \cos(\alpha) = \left(\left(\frac{d}{Db - KOP}\right)^2 - 1^2\right) \text{----- 1.10}$$

Divide both side by the coefficient of $\cos(\alpha)^2$

$$\cos(\alpha)^2 - \frac{2\cos(\alpha)}{1 + \left(\frac{d}{Db - KOP}\right)^2} = \left(\left(\frac{d}{Db - KOP}\right)^2 - 1\right) / \left(1 + \left(\frac{d}{Db - KOP}\right)^2\right) \text{----- 1.11}$$

Evaluation of equation 1.11 above gives

$$\cos \alpha^2 - \frac{2(Db - KOP)^2}{(Db - KOP)^2 + d^2} \cos \alpha = \frac{d^2 - (Db - KOP)^2}{d^2 + (Db - KOP)^2} \text{----- 1.12}$$

Further simplification result to

$$\left(\cos(\alpha) - \frac{(Db - KOP)^2}{(Db - KOP)^2 + d^2}\right)^2 = \frac{d^2 - (Db - KOP)^2}{d^2 + (Db - KOP)^2} + \left(-\frac{(Db - KOP)^2}{(Db - KOP)^2 + d^2}\right)^2 \text{----- 1.13}$$

On further simplification and evaluation we have

$$\cos(\alpha) = \frac{(Db - KOP)^2}{(Db - KOP)^2 + d^2} \pm \sqrt{\frac{d^2 - (Db - KOP)^2}{d^2 + (Db - KOP)^2} + \frac{(Db - KOP)^4}{(Db - KOP)^2 + 2d^2(Db - KOP)^2 + d^4}} \text{----- 1.14}$$

$$\alpha = \cos^{-1}\left(\frac{(Db - KOP)^2}{(Db - KOP)^2 + d^2} \pm \sqrt{\frac{d^2 - (Db - KOP)^2}{d + Db - KOP} + \frac{(Db - KOP)^4}{(Db - KOP)^2 + 2d^2(Db - KOP)^2 + d^4}}\right) \text{----- 1.15}$$

The above mathematical model are the generated mathematical model develop for estimating the tangent angle of a deviated well with proposing horizontal displacement greater than the radius of curvature.

RESULT AND DISCUSSION

The available data for Adamuegwu (AMG-02), were input into the excel spreadsheet develop (table 1.0) and the result obtain are presented on table 1.1. The values obtained were analyzed to established relationship between the vertical depth of the well and its horizontal distance at each end of section of the well profile. The result of the establish relationship were used to pictorially established AMG-02 well profile, as shown on fig. 1.8

Table1.0: available data for AMG-02 well

INPUT DATA		
COMPONENT	VALUE	UNIT
TVD	10000	feet
TOTAL DISPLACEMENT	3000	feet
KOP	2000	feet
BUILD UP RATE	2	degree/100ft

Table 1.1: result summary table for AMG-02 well

OUTPUT RESULT		
COMPONENT NAME	VALUE	UNITS
RADIUS OF BUILD UP SECTION	2864.78898	feet
TANGENT ANGLE	21.9486192	Degree/100ft
ALONG HOLE DEPTH TO BU SECTION	3097.43096	feet
VERTICAL DEPTH TO BU SECTION	3070.78644	feet
DISPLACEMENT AT THE END OF THE BU	207.64158	feet
ALONG HOLE DEPTH TO TANGENT SECTION	9470.69382	feet

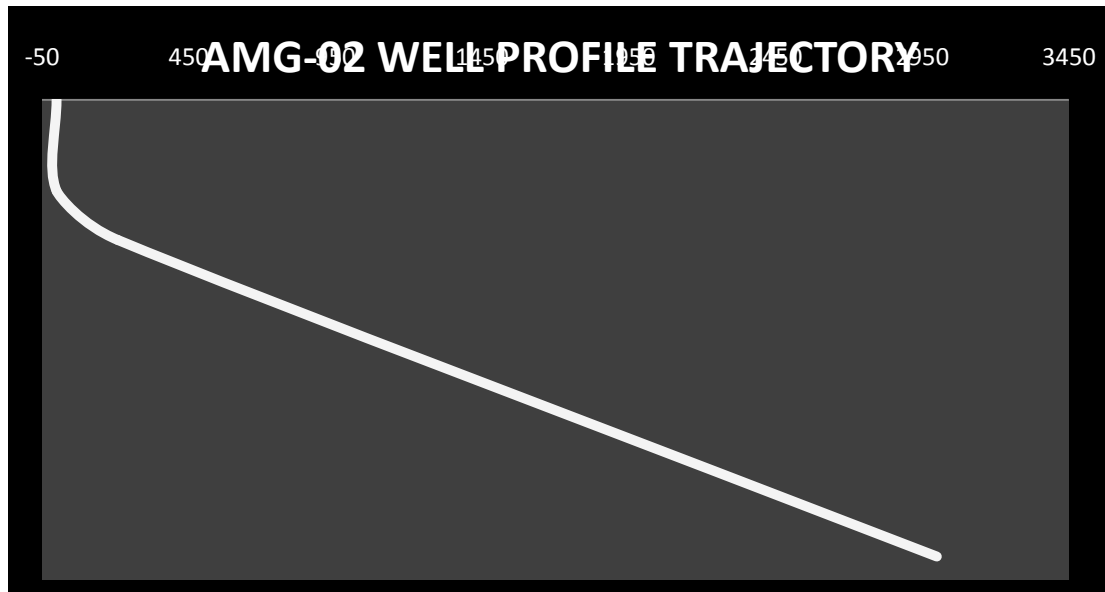


Fig 1.8: AMG- 02 plan well profile trajectory

The mathematical model developed for calculating estimated build up rate was established and presented against documentation on educational purpose. This model is open to further study and accessibility.

V CONCLUSION

The rational knowledge behind this paper is to develop an educational algorithm code, where other programmer can plan and modify on, as well as a new and improved user-friendly well planning program that can be used for educational purpose than the old graphical methods of well trajectory analysis due to unavailability of the industry standard software. In conclusion, this study focus only on the type I directional well (J well), but neglects the availability literature of surveying, well planning calculation, and the well planning program software's.

The analysis on the work done on the spreadsheet shows that, it has a good functions and attributes, but lacks some limitations when it comes to proper presentation of information works and explanation on the level of assumption made for the calculation in view.

REFERENCE

- [1]. Adams, N., 1985. Drilling Engineering a Complete Well Planning Approach. Tulsa (Oklahoma): Penn Well Publishing Company.
- [2]. Baihly, J., Grant D., Fan, L., and Bodwadkar, S. 2007. Horizontal Wells in Tight Gas Sands – A Methodology for Risk Management To maximize Success. Paper SPE 145 110067 presented at the SPE annual Conference and Exhibition, Anaheim, California, 11-14 November.
- [3]. Baker Hughes INTEQ, 1995. Drilling Engineering Workbook Training & Development, 80270H Rev. B, Houston, United States of America.
- [4]. Blankenship, D., Finger, J., 2010. Handbook of Best Practices for Geothermal Drilling, Prepared for the International Energy Agency, Geothermal Implementing Agreement Sandia National Laboratories. Albuquerque, New Mexico.
- [5]. Farah, F. O. (2013). DIRECTIONAL WELL DESIGN, TRAJECTORY AND SURVEY CALCULATIONS, WITH A CASE STUDY IN FIALE, ASAL RIFT, DJIBOUTI .Grensasvegur 9, IS-108 Reykjavik, Iceland : GEOTHERMAL TRAINING PROGRAMME Orkustofnun.
- [6]. Short, J. (1993). Introduction to directional and horizontal drilling. Pennwell Publishing Co.
- [7]. Sveinbjörnsson, B. (2010). Estimate of costs and uncertainties in high-temperature drilling in the Hengill area.
- [8]. Torres, D. E. & Anders, J. L. . (1995). Using MS Visual Basic to Write Engineering Applications. Society of Petroleum Engineers.
- [9]. Torres, D. E. (1995, January 1). In Using MS Visual Basic to Write Engineering Applications. Society of Petroleum Engineers. doi:10.2118/30215-MS.
- [10]. Vieira, J. (2009). In Controlled directional drilling (4th edition). Petroleum Extension Service, Austin TX (p. 130).
- [11]. Walstrom J.E, B. A. (1969). Directional Survey Models. Petr. Techn.
- [12]. Williamson, H. S. (1999). Accuracy Prediction for Directional MWD. Society of Petroleum Engineers.

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