# Imperfect Quality And Repairable Items Inventory Model With Different Deterioration Rates Under Price And Time Dependent Demand

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**ABSTRACT:** Generally items received or produced are not of 100% good quality. Some of them are defective and some them may be repairable. A deterministic inventory model with imperfect quality and repairable items is developed when deterioration rate is different during a cycle. Here it is assumed that demand is a function of price and time. Numerical example is taken to support the model. Sensitivity analysis is also carried out for parameters.

**KEY WORDS:** Inventory model, Varying Deterioration, Time dependent demand, Price dependent demand, Defective items

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# I. INTRODUCTION

Most of the time it is assumed that items can be stored indefinitely to meet the future demand. But many items are such that they either deteriorate or become obsolete in the course of time. Therefore, if the rate of deterioration is not sufficiently low, its impact on modeling of such an inventory system cannot be ignored. An inventory model with constant rate of deterioration was considered by Ghare and Schrader [4]. Covert and Philip [3] extended the model by considering variable rate of deterioration. Aggarwal and Goel [1] discussed an inventory model with weibull rate of decay with selling price dependent demand. Patra et al. [15] developed a deterministic inventory model when deterioration rate was time proportional. Demand rate was taken as a nonlinear function of selling price, deterioration rate, inventory holding cost and ordering cost were all functions of time. The related works are found in (Nahmias [10], Raffat [16], Ruxian, et al [17]).

Product quality is one of the important factor in every business. Generally we assume that all received items are of good quality. But all produced/ received items are not of good quality so they are said to be defective items. Some of the defective items are such that they can be repairable. Many models have been developed for defective items. Lee and Rosenblatt [9] considered an optimal ordering policy for defective items. Cheng [2] developed a model of imperfect production quantity by establishing relationship between demand dependent unit production cost and imperfect production process. A simple approach for determining economic production quantity model for imperfect quality items was developed by Goyal and Barron [6]. Papachristos. and Konstantaras [12] have examined models without shortages, probabilistic proportional imperfect quality and withdrawing at the end of planning horizon. Jaber et al. [7] considered an EPQ model for items with imperfect quality shipments. Patel and Patel [13] developed an EOQ model for deteriorating items with imperfect quality items. Patel and Sheikh [14] developed an inventory model with different deterioration rates under stock and price dependent demand.

A joint EOQ and EPQ model in which the stationary demand can be satisfied by recycled products and newly purchased products is developed by Koh et al.[8]. An inventory model for repairable items with linear demand was developed by Yadav and Kumar [19]. An inventory model for repairable items with exponential deterioration and linear demand was considered by Gothi et al. [5]. Uthayakumar and Sekar [18] considered a multiple production setups inventory model for imperfect items with salvage value.

Generally the products are such that initially there is no deterioration. Deterioration starts after certain time and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.

In this paper we have developed an inventory model for imperfect quality and repairable items with different deterioration rates under price and time dependent demand. Shortages are not allowed. To illustrate the model, numerical example is provided. Sensitivity analysis for major parameters is also carried out.

# **II. ASSUMPTIONS AND NOTATIONS**

**NOTATIONS:** The following notations are used for the development of the model:

- D(t) : Demand rate is a function of time and price (a+bt- $\rho$ p, a>0, 0<b<1,  $\rho$ >0)
- c : Purchasing cost per unit
- p : Selling price per unit
- d : defective items (%)
- 1-d : good items (%)
- d<sub>1</sub> : repairable items (%)
- $\lambda$  : Screening rate
- SR : Sales revenue
- A : Replenishment cost per order for
- z : Screening cost per unit
- p<sub>d</sub> : Price of defective items per unit
- h(t) : Variable Holding cost (x + yt)
- m : Transportation cost per unit of repairable items
- t<sub>1</sub> : Screening time
- T : Length of inventory cycle
- I(t) : Inventory level at any instant of time t,  $0 \le t \le T$
- Q : Order quantity
- $\theta \qquad : \text{Deterioration rate during } \mu_1 \leq t \leq \mu_2, \, 0 < \theta_1 < 1$
- $\theta t$   $\quad$  : Deterioration rate during ,  $\mu_2 \leq t \leq T, \, 0 {<} \, \theta_2 {<} 1$
- $\pi$  : Total relevant profit per unit time.

# **ASSUMPTIONS:**

The following assumptions are considered for the development of the model.

- The demand of the product is declining as a function of time and price.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are not allowed.
- The screening process and demand proceeds simultaneously but screening rate ( $\lambda$ ) is greater than the demand rate i.e.  $\lambda > (a+bt-\rho p)$ .
- The defective items are independent of deterioration.
- Deteriorated units can neither be repaired nor replaced during the cycle time.
- A single product is considered.
- Holding cost is time dependent.
- The screening rate  $(\lambda)$  is sufficiently large. In general, this assumption should be acceptable since the automatic screening machine usually takes only little time to inspect all items produced or purchased.

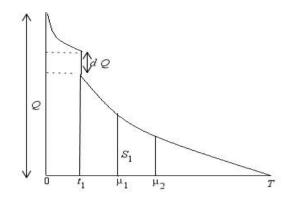
# III. THE MATHEMATICAL MODEL AND ANALYSIS

In the following situation, Q items are received at the beginning of the period. Each lot having a d % defective items out of which  $d_1$ % are repairable items. The nature of the inventory level is shown in the given figure, where screening process is done for all the received items at the rate of  $\lambda$  units per unit time which is greater than demand rate for the time period 0 to  $t_1$ . During the screening process the demand occurs parallel to the screening process and is fulfilled from the goods which are found to be of perfect quality by screening process. The defective items (excluding repairable) are sold immediately after the screening process at time  $t_1$  as a single batch at a discounted price and repairable items are sent back to manufacturer for repair. After the screening process at time  $t_1$  the inventory level will be I(t) and at time T, inventory level will become zero due to demand and partially due to deterioration.

Also here 
$$t_1 = \frac{Q}{\lambda}$$
 (1)

and defective percentage (d) is restricted to 
$$d \le 1 - \frac{(a+bt-\rho p)}{\lambda}$$
 (2)

Let I(t) be the inventory at time t ( $0 \le t \le T$ ) as shown in figure.





The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by 11(1)

$$\frac{dl(t)}{dt} = -(a + bt + \rho p), \qquad \qquad 0 \le t \le \mu_1 \qquad (3)$$

$$\frac{\mathrm{dI}(t)}{\mathrm{dt}} + \theta \mathrm{I}(t) = -(a + bt + \rho p), \qquad \qquad \mu_1 \le t \le \mu_2 \qquad (4)$$

$$\frac{\mathrm{dI}(t)}{\mathrm{dt}} + \theta t \mathrm{I}(t) = -(a + bt + \rho p), \qquad \qquad \mu_2 \le t \le T \qquad (5)$$

with initial conditions I(0) = Q,  $I(\mu_1) = S_1$  and I(T) = 0. Solutions of these equations are given by

$$I(t) = Q - (at - \rho pt + \frac{1}{2}bt^{2}),$$
(6)

$$I(t) = \begin{vmatrix} a(\mu_{1} - t) - \rho p(\mu_{1} - t) + \frac{1}{2} a \theta(\mu_{1}^{2} - t^{2}) - \frac{1}{2} \rho p \theta(\mu_{1}^{2} - t^{2}) + \frac{1}{2} b(\mu_{1}^{2} - t^{2}) \\ + \frac{1}{3} b \theta(\mu_{1}^{3} - t^{3}) - a \theta t(\mu_{1} - t) + \rho p \theta t(\mu_{1} - t) - \frac{1}{2} b \theta t(\mu_{1}^{2} - t^{2}) \end{vmatrix} + S_{1} [1 + \theta(\mu_{1} - t)]$$
(7)

$$I(t) = \begin{bmatrix} a(T-t) - \rho p(T-t) + \frac{1}{2}b(T^{2}-t^{2}) + \frac{1}{6}a\theta(T^{3}-t^{3}) - \frac{1}{6}\rho p\theta(T^{3}-t^{3}) \\ + \frac{1}{8}b\theta(T^{4}-t^{4}) - \frac{1}{2}a\theta t^{2}(T-t) + \frac{1}{2}\rho p\theta t^{2}(T-t) - \frac{1}{4}b\theta t^{2}(T^{2}-t^{2}) \end{bmatrix}.$$
(8)

(by neglecting higher powers of  $\theta$ )

After screening process, the number of defective items at time  $t_1$  is dQ. So effective inventory level during  $t_1 \le t \le T$  is given by

$$I(t_1) = Q(1-d) - (at - \rho pt + \frac{1}{2}bt^2).$$
(9)  
From equation (6) putting  $t = u_1$ , we have

equation (6), putting  $t = \mu_1$ , we have

$$Q = S_1 + \left(a\mu_1 - \rho p\mu_1 + \frac{1}{2}b\mu_1^2\right).$$
 (10)

Moreover, we assume that  $d_1$ % of repairable items are sent to the manufacturer for repairs and after repairing become perfect or good items and received before the end of the cycle (i.e. during the time period  $\mu_2 \le t \le T$ ). So it can be sold at original price before the end of the cycle. Doing so there is increase in transportation cost for sending and receiving back the defective repaired items. So effective inventory level during  $\mu_2 \le t \le T$  is given by:

$$I(t) = \begin{bmatrix} a(T-t) - \rho p(T-t) + \frac{1}{2}b(T^{2}-t^{2}) + \frac{1}{6}a\theta(T^{3}-t^{3}) - \frac{1}{6}\rho p\theta(T^{3}-t^{3}) \\ + \frac{1}{8}b\theta(T^{4}-t^{4}) - \frac{1}{2}a\theta t^{2}(T-t) + \frac{1}{2}\rho p\theta t^{2}(T-t) - \frac{1}{4}b\theta t^{2}(T^{2}-t^{2}) \end{bmatrix} + d_{1}Q.$$
(11)

From equations (7) and (11), putting  $t = \mu_2$ , we have

$$I(\mu_{2}) = \begin{bmatrix} a(\mu_{1}-\mu_{2})-\rho p(\mu_{1}-\mu_{2})+\frac{1}{2}a\theta(\mu_{1}^{2}-\mu_{2}^{2})-\frac{1}{2}\rho p\theta(\mu_{1}^{2}-\mu_{2}^{2})+\frac{1}{2}b(\mu_{1}^{2}-\mu_{2}^{2})\\ +\frac{1}{3}b\theta(\mu_{1}^{3}-\mu_{2}^{3})-a\theta\mu_{2}(\mu_{1}-\mu_{2})+\rho p\theta\mu_{2}(\mu_{1}-\mu_{2})-\frac{1}{2}b\theta\mu_{2}(\mu_{1}^{2}-\mu_{2}^{2})\end{bmatrix} + S_{1}\left[1+\theta(\mu_{1}-\mu_{2})\right]$$
(12)  
$$I(\mu_{2}) = \begin{bmatrix} a(T-\mu_{2})-\rho p(T-\mu_{2})+\frac{1}{2}b(T^{2}-\mu_{2}^{2})+\frac{1}{6}a\theta(T^{3}-\mu_{2}^{3})-\frac{1}{6}\rho p\theta(T^{3}-\mu_{2}^{3})\\ +\frac{1}{8}b\theta(T^{4}-\mu_{2}^{4})-\frac{1}{2}a\theta\mu_{2}^{2}(T-\mu_{2})+\frac{1}{2}\rho p\theta\mu_{2}^{2}(T-\mu_{2})-\frac{1}{4}b\theta\mu_{2}^{2}(T^{2}-\mu_{2}^{2})\end{bmatrix} + d_{1}Q.$$
(13)

$$\left[ +\frac{1}{8}b\theta(T^{4}-\mu_{2}^{4}) - \frac{1}{2}a\theta\mu_{2}^{2}(T-\mu_{2}) + \frac{1}{2}\rho p\theta\mu_{2}^{2}(T-\mu_{2}) - \frac{1}{4}b\theta\mu_{2}^{2}(T-\mu_{2}) - \frac{1}{4}b\theta\mu_{2}^{2}(T-\mu_{2}) + \frac{1}{2}\rho p\theta\mu_{2}^{2}(T-\mu_{2}) - \frac{1}{4}b\theta\mu_{2}^{2}(T-\mu_{2}) + \frac{1}{4}b\theta\mu_{2}^{2}(T-\mu_{2})$$

$$\begin{split} \mathbf{S}_{1} &= \frac{1}{\left[1 + \theta(\mu_{1} - \mu_{2}) - d_{1}\right]} \\ & \left[a\left(T - \mu_{2}\right) - \rho p\left(T - \mu_{2}\right) + \frac{1}{2}b\left(T^{2} - \mu_{2}^{2}\right) + \frac{1}{6}a\theta\left(T^{3} - \mu_{2}^{3}\right) - \frac{1}{6}\rho p\theta\left(T^{3} - \mu_{2}^{3}\right) + \frac{1}{8}b\theta\left(T^{4} - \mu_{2}^{4}\right) \\ & - \frac{1}{2}a\theta\mu_{2}^{2}\left(T - \mu_{2}\right) + \frac{1}{2}\rho p\theta\mu_{2}^{2}\left(T - \mu_{2}\right) - \frac{1}{4}b\theta\mu_{2}^{2}\left(T^{2} - \mu_{2}^{2}\right) - a\left(\mu_{1} - \mu_{2}\right) + \rho p\left(\mu_{1} - \mu_{2}\right) \\ & - \frac{1}{2}a\theta\left(\mu_{1}^{2} - \mu_{2}^{2}\right) + \frac{1}{2}\rho p\theta\left(\mu_{1}^{2} - \mu_{2}^{2}\right) - \frac{1}{2}b\left(\mu_{1}^{2} - \mu_{2}^{2}\right) - \frac{1}{3}b\theta\left(\mu_{1}^{3} - \mu_{2}^{3}\right) + a\theta\mu_{2}\left(\mu_{1} - \mu_{2}\right) \\ & - \rho p\theta\mu_{2}\left(\mu_{1} - \mu_{2}\right) + \frac{1}{2}b\theta\mu_{2}\left(\mu_{1}^{2} - \mu_{2}^{2}\right) + d_{1}\left(a\mu_{1} - \rho p\mu_{1} + \frac{1}{2}b\mu_{1}^{2}\right) \end{split} \end{split}$$

Putting value of  $S_1$  from equation (14) into equation (10), we have

$$Q = \frac{1}{\left[1+\theta(\mu_{1}-\mu_{2})-d_{1}\right]} \begin{bmatrix} a(T-\mu_{2})-\rho p(T-\mu_{2})+\frac{1}{2}b(T^{2}-\mu_{2}^{2})+\frac{1}{6}a\theta(T^{3}-\mu_{2}^{3})-\frac{1}{6}\rho p\theta(T^{3}-\mu_{2}^{3})+\frac{1}{8}b\theta(T^{4}-\mu_{2}^{4}) \\ -\frac{1}{2}a\theta\mu_{2}^{2}(T-\mu_{2})+\frac{1}{2}\rho p\theta\mu_{2}^{2}(T-\mu_{2})-\frac{1}{4}b\theta\mu_{2}^{2}(T^{2}-\mu_{2}^{2})-a(\mu_{1}-\mu_{2})+\rho p(\mu_{1}-\mu_{2}) \\ -\frac{1}{2}a\theta(\mu_{1}^{2}-\mu_{2}^{2})+\frac{1}{2}\rho p\theta(\mu_{1}^{2}-\mu_{2}^{2})-\frac{1}{2}b(\mu_{1}^{2}-\mu_{2}^{2})-\frac{1}{3}b\theta(\mu_{1}^{3}-\mu_{2}^{3})+a\theta\mu_{2}(\mu_{1}-\mu_{2}) \\ -\rho p\theta\mu_{2}(\mu_{1}-\mu_{2})+\frac{1}{2}b\theta\mu_{2}(\mu_{1}^{2}-\mu_{2}^{2})+d_{1}\left(a\mu_{1}-\rho p\mu_{1}+\frac{1}{2}b\mu_{1}^{2}\right) \\ +\left(a\mu_{1}-\rho p\mu_{1}+\frac{1}{2}b\mu_{1}^{2}\right). \tag{15}$$

Using (15) in (6), we have

$$\begin{split} I(t) &= \frac{1}{\left[1 + \theta(\mu_{1} - \mu_{2}) - d_{1}\right]} \\ & \left[a\left(T - \mu_{2}\right) - \rho p\left(T - \mu_{2}\right) + \frac{1}{2}b\left(T^{2} - \mu_{2}^{2}\right) + \frac{1}{6}a\theta\left(T^{3} - \mu_{2}^{3}\right) - \frac{1}{6}\rho p\theta\left(T^{3} - \mu_{2}^{3}\right) + \frac{1}{8}b\theta\left(T^{4} - \mu_{2}^{4}\right)\right) \\ & - \frac{1}{2}a\theta\mu_{2}^{2}\left(T - \mu_{2}\right) + \frac{1}{2}\rho p\theta\mu_{2}^{2}\left(T - \mu_{2}\right) - \frac{1}{4}b\theta\mu_{2}^{2}\left(T^{2} - \mu_{2}^{2}\right) - a\left(\mu_{1} - \mu_{2}\right) + \rho p\left(\mu_{1} - \mu_{2}\right) \\ & - \frac{1}{2}a\theta\left(\mu_{1}^{2} - \mu_{2}^{2}\right) + \frac{1}{2}\rho p\theta\left(\mu_{1}^{2} - \mu_{2}^{2}\right) - \frac{1}{2}b\left(\mu_{1}^{2} - \mu_{2}^{2}\right) - \frac{1}{3}b\theta\left(\mu_{1}^{3} - \mu_{2}^{3}\right) + a\theta\mu_{2}\left(\mu_{1} - \mu_{2}\right) \\ & - \rho p\theta\mu_{2}\left(\mu_{1} - \mu_{2}\right) + \frac{1}{2}b\theta\mu_{2}\left(\mu_{1}^{2} - \mu_{2}^{2}\right) + d_{1}\left(a\mu_{1} - \rho p\mu_{1} + \frac{1}{2}b\mu_{1}^{2}\right) \\ & + \left(a\left(\mu_{1} - t\right) - \rho p\left(\mu_{1} - t\right) + \frac{1}{2}b\left(\mu_{1}^{2} - t^{2}\right)\right). \end{split}$$
(16)

Similarly, using (15) in (9), we have

$$\begin{split} \mathbf{I}(t) &= \frac{(1-d)}{\left[1+\theta(\mu_{1}-\mu_{2})-d_{1}\right]} \\ & \left[a\left(\mathbf{T}-\mu_{2}\right)-\rho p\left(\mathbf{T}-\mu_{2}\right)+\frac{1}{2}b\left(\mathbf{T}^{2}-\mu_{2}^{2}\right)+\frac{1}{6}a\theta\left(\mathbf{T}^{3}-\mu_{2}^{3}\right)-\frac{1}{6}\rho p\theta\left(\mathbf{T}^{3}-\mu_{2}^{3}\right)+\frac{1}{8}b\theta\left(\mathbf{T}^{4}-\mu_{2}^{4}\right)\right) \\ & -\frac{1}{2}a\theta\mu_{2}^{2}\left(\mathbf{T}-\mu_{2}\right)+\frac{1}{2}\rho p\theta\mu_{2}^{2}\left(\mathbf{T}-\mu_{2}\right)-\frac{1}{4}b\theta\mu_{2}^{2}\left(\mathbf{T}^{2}-\mu_{2}^{2}\right)-a\left(\mu_{1}-\mu_{2}\right)+\rho p\left(\mu_{1}-\mu_{2}\right) \\ & -\frac{1}{2}a\theta\left(\mu_{1}^{2}-\mu_{2}^{2}\right)+\frac{1}{2}\rho p\theta\left(\mu_{1}^{2}-\mu_{2}^{2}\right)-\frac{1}{2}b\left(\mu_{1}^{2}-\mu_{2}^{2}\right)-\frac{1}{3}b\theta\left(\mu_{1}^{3}-\mu_{2}^{3}\right)+a\theta\mu_{2}\left(\mu_{1}-\mu_{2}\right) \\ & -\rho p\theta\mu_{2}\left(\mu_{1}-\mu_{2}\right)+\frac{1}{2}b\theta\mu_{2}\left(\mu_{1}^{2}-\mu_{2}^{2}\right)+d_{1}\left(a\mu_{1}-\rho p\mu_{1}+\frac{1}{2}b\mu_{1}^{2}\right) \\ & +\left(1-d\right)\left(a\mu_{1}-\rho p\mu_{1}+\frac{1}{2}b\mu_{1}^{2}\right)-\left(at-\rho pt+\frac{1}{2}bt^{2}\right). \end{split}$$

Similarly putting value of  $S_1$  from equation (14) in equation (7), we have

$$\begin{split} I(t) &= \frac{\left[1 + \theta(\mu_{1} - t)\right]}{\left[1 + \theta(\mu_{1} - \mu_{2}) - d_{1}\right]} \\ &= \left[ a(T - \mu_{2}) - \rho p(T - \mu_{2}) + \frac{1}{2} b(T^{2} - \mu_{2}^{2}) + \frac{1}{6} a\theta(T^{3} - \mu_{2}^{3}) - \frac{1}{6} \rho p\theta(T^{3} - \mu_{2}^{3}) + \frac{1}{8} b\theta(T^{4} - \mu_{2}^{4}) \right] \\ &- \frac{1}{2} a\theta\mu_{2}^{2}(T - \mu_{2}) + \frac{1}{2} \rho p\theta\mu_{2}^{2}(T - \mu_{2}) - \frac{1}{4} b\theta\mu_{2}^{2}(T^{2} - \mu_{2}^{2}) - a(\mu_{1} - \mu_{2}) + \rho p(\mu_{1} - \mu_{2}) \\ &- \frac{1}{2} a\theta(\mu_{1}^{2} - \mu_{2}^{2}) + \frac{1}{2} \rho p\theta(\mu_{1}^{2} - \mu_{2}^{2}) - \frac{1}{2} b(\mu_{1}^{2} - \mu_{2}^{2}) - \frac{1}{3} b\theta(\mu_{1}^{3} - \mu_{2}^{3}) + a\theta\mu_{2}(\mu_{1} - \mu_{2}) \\ &- \rho p\theta\mu_{2}(\mu_{1} - \mu_{2}) + \frac{1}{2} b\theta\mu_{2}(\mu_{1}^{2} - \mu_{2}^{2}) + d_{1}\left(a\mu_{1} - \rho p\mu_{1} + \frac{1}{2} b\mu_{1}^{2}\right) \\ &+ \left[ a(\mu_{1} - t) - \rho p(\mu_{1} - t) + \frac{1}{2} a\theta(\mu_{1}^{2} - t^{2}) - \frac{1}{2} \rho p\theta(\mu_{1}^{2} - t^{2}) + \frac{1}{2} b(\mu_{1}^{2} - t^{2}) \\ &+ \frac{1}{3} b\theta(\mu_{1}^{3} - t^{3}) - a\theta t(\mu_{1} - t) + \rho p\theta t(\mu_{1} - t) - \frac{1}{2} b\theta t(\mu_{1}^{2} - t^{2}) \\ \end{bmatrix} . \end{split}$$

Based on the assumptions and descriptions of the model, the total annual relevant profit ( $\pi$ ), include the following elements:

(i) Ordering 
$$\cot(OC) = A$$
 (19)

(ii) Screening 
$$\cot(SrC) = zQ$$
 (20)

(iii) Transportation cost (TC) = 
$$md_1Q$$
 (21)

(iii) 
$$HC = \int_{0}^{t_{1}} (x+yt)I(t)dt$$
$$= \int_{0}^{t_{1}} (x+yt)I(t)dt + \int_{t_{1}}^{\mu_{1}} (x+yt)I(t)dt + \int_{\mu_{1}}^{T} (x+yt)I(t)dt + \int_{\mu_{2}}^{T} (x+yt)I(t)dt$$
(22)

(iv) 
$$DC = c \left( \int_{\mu_1}^{\mu_2} \theta I(t) dt + \int_{\mu_2}^{T} \theta t I(t) dt \right)$$
(23)

(vi) SR = Sum of sales revenue generated by demand meet during the period (0,T)

+ Sales of imperfect quality items

$$= p\left(\int_{0}^{T} (a+bt-\rho p)dt + p_{d}dQ + pd_{1}Q\right) = p\left(aT - \rho pT + \frac{1}{2}bT^{2} + p_{d}dQ + pd_{1}Q\right)$$
(24)

The total profit ( $\pi$ ) during a cycle consisted of the following:

$$\pi = \frac{1}{T} [SR - OC - SrC - TC - HC - DC]$$

(25)

Substituting values from equations (19) to (24) in equation (25), we get total profit per unit. Putting  $\mu_1 = v_1 T$  and  $\mu_2 = v_2 T$  and value of  $t_1$  and Q in equation (25), we get profit in terms of T and p. Differentiating equation (25) with respect to T and p and equate it to zero, we have

i.e. 
$$\frac{\partial \pi(T,p)}{\partial T} = 0, \ \frac{\partial \pi(T,p)}{\partial p} = 0$$
 (26)

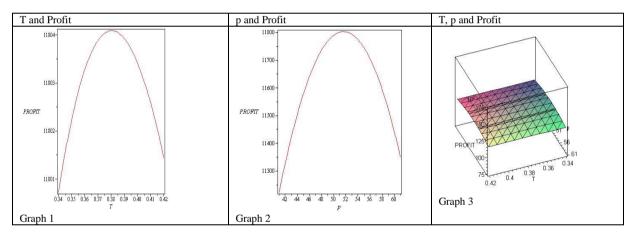
provided it satisfies the condition

$$\begin{bmatrix} \frac{\partial^2 \pi(\mathbf{T},\mathbf{p})}{\partial \mathbf{T}^2} & \frac{\partial^2 \pi(\mathbf{T},\mathbf{p})}{\partial \mathbf{p} \partial \mathbf{T}} \\ \frac{\partial^2 \pi(\mathbf{T},\mathbf{p})}{\partial \mathbf{T} \partial \mathbf{p}} & \frac{\partial^2 \pi(\mathbf{T},\mathbf{p})}{\partial \mathbf{p}^2} \end{bmatrix} > 0.$$
(27)

#### **IV. NUMERICAL EXAMPLE**

Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25,  $p_d = 15$ , d= 0.05,  $d_1 = 0.03$ , z = 0.40, m = 70,  $\lambda$ = 10000,  $\theta$ =0.05, x = Rs. 5, y=0.05,  $v_1$ =0.30,  $v_2 = 0.50$ , in appropriate units. The optimal value of T\*=0.3803, p\* = 51.6214, Profit\*= Rs. 11804.0937 and optimum order quantity Q\*=95.1266.

The second order conditions given in equation (27) are also satisfied. The graphical representation of the concavity of the profit function is also given.



## V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

## Table 1 Sensitivity Analysis

/ 515	- T			-	
Parameter	%	Т	р	Profit	Q
a	+20%	0.3468	61.5750	17310.2032	104.7272
	+10%	0.3624	56.5965	14427.6842	100.0471
	-10%	0.4012	46.6504	9439.6244	89.9412
	-20%	0.4261	41.6851	7334.5276	84.4504
θ	+20%	0.3768	51.6248	11800.1335	94.2976
	+10%	0.3786	51.6231	11802.1083	94.7249
	-10%	0.3821	51.6196	11806.0898	95.5530
	-20%	0.3839	51.6179	11808.0970	95.9788
x	+20%	0.3505	51.6680	11758.3903	87.5633
	+10%	0.3645	51.6452	11780.7690	91.1158
	-10%	0.3984	51.5965	11828.4922	99.7228
	-20%	0.4192	51.5703	11854.1239	105.0076
А	+20%	0.4160	51.6714	11753.8670	103.9851
	+10%	0.3986	51.6470	11778.4183	99.6690
	-10%	0.3611	51.5945	11831.0667	90.3576
	-20%	0.3407	51.5661	11859.5607	85.2867
ρ	+20%	0.3814	43.2893	9658.7607	94.7604

	+10%	0.3808	47.0765	10633.7884	94.9317
	-10%	0.3798	57.1763	13234.7686	95.3206
	-20%	0.3794	64.1203	15023.4513	95.5384
	+20%	0.3804	51.6211	11804.1928	95.1523
λ	+10%	0.3804	51.6212	11804.1478	95.1521
	-10%	0.3803	51.6215	11804.0276	95.1264
	-20%	0.3802	51.6218	11803.9449	95.1007

From the table we observe that as parameter a increases/ decreases average total profit and optimum order quantity also increases/ decreases.

Also, we observe that with increase and decrease in the value of  $\theta$ , x and  $\rho$ , there is corresponding decrease/increase in total profit and optimum order quantity.

From the table we observe that as parameter A increases/ decreases average total profit decreases/ increases and optimum order quantity increases/ decreases.

From the table we observe that as parameter  $\lambda$  increases/ decreases, there is almost no change in average total profit and optimum order quantity.

## VI. PARTICULAR CASE

(i) When d=0.02, m=0,  $d_1$ =0, we get T\* = 0.3855, p\* = 50.5668, and Profit = Rs. 11959.9902 which is same as Naik and Patel [11]

(ii) When  $p_d=0$ , d = 0, m = 0,  $d_1=0$ , we get  $T^* = 0.2712$ ,  $p^* = Rs. 50.5187$  and Profit = Rs. 11982.1661, which is same as Patel and Sheikh [14]

#### VII.CONCLUSION

In this paper, we have developed an inventory model for deteriorating items with price and time dependent demand with different deterioration rates. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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