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Properties Of Wgr-Closed Sets In Topological Spaces

[1]Govindappa Navalagi And [*2]R G Charantimath
[1]Department of Mathematics KIT Tiptur 572202 ,Karnataka India
*[*2]Department of Mathematics KIT Tiptur 572202 ,Karnataka India*
Corresponding Author: R G Charantimath

ABSTRACT: In this paper , we define and study the concept of weakly generalized regular closed (brifly.wgr-closed) sets, wgr-open functions, wgr-closed functions ,wgr-Homeomorphism and wgr-Hausdorff spaces.
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I. INTRODUCTION

For the first time , N.Levine [9] has introduced the notion g-closed sets and g-open sets in topology. In 1993, N.Palaniappan [15] has defined and studied the notions of rg-closed sets ,rg-continuity and rg-irresoluteness in topological spaces.In 1995,1996 ,1997, 1998, 2009, 2011 and 2014 , resp., Dontchev [6] , Dontchev et al [5] , Gnanambal [7] , Noiri et al [14] , Al-Omari et al [1] , S.Bhattacharya [4] and K.Indirani et al [8] have defined and studied the concepts of gsp-closed sets , δg-closed sets, gpr-closed sets , gp-closed sets ,gb-closed sets ,gr-closed sets and gr*-closed sets in toplogical spaces.In this paper , , we define and study the concept of weakly generalized regular closed (brifly.wgr-closed) sets, wgr-open functions, wgr-closed functions ,wgr-Homeomorphism and wgr-Hausdorff spaces.

II. PRELIMINARIES

Throughout this paper (X , τ) and (Y, σ) (or simply X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated . If A be a subset of X, the Closure of A and Interior of A denoted by Cl(A) and Int(A)respectivly.

We give the following define are useful in the sequel :

Definition 2.1: The subset of A of X is said to be :
(i) regular open (in brief,r-open) if A = IntCl(A).
(ii) regular closed (in brief,r-closed) if A = ClInt(A).
Definition 2.2: The subset of A of X is said to be.
(i) semi-open [10] set, if $A \subset Cl(Int(A))$
(ii) pre-open [11]set, if $A \subset Int(Cl(A))$
(iii) semi-pre open[2]set , if $A \subset Cl(Int(Cl(A)))$
(iv) b-open [3] if $A \subset ClInt(A) \cup IntCl(A)$.
(v) δ-closed [16] if A = δCl(A) , where $\delta Cl(A) = \{x \in X : IntCl(U) \cap A \neq \emptyset$, U is open set and $x \in U\}$

The complement of semipre-open(resp.b-open ,δ-closed) set is called semipre-closed [2] (resp.b-closed [3] , δ-open [16]) set of a space X.
Definition 2.3 [2]: The intersection of all semipre-closed sets of X containing subset A is called the semipre-closure of A and is denoted by spCl(A).
Definition 2.4 [14]: The intersection of all pre-closed sets of X containing subset A is called the semipre-closure of A and is denoted by pCl(A).
Definition 2.5 [15]: The intersection of all regular closed sets containing set A is called the regular closure of A and is denoted by rCl(A) /δCl(A).

Definition 2.6 [3]: The intersection of all b-closed sets containing set A is called the b- closure of A and is denoted by bCl(A).

Similarly , spInt(A) ,pInt(A), rInt(A) ,bInt(A),δInt(A) can be defined.

Definition 2. 7: A subset A of a space (X, τ) is called:

(i) generalized closed (briefly, g- closed) **[9]** set if Cl(A)$\subseteq U$ whenever A$\subseteq U$ and U is open set in X

(ii) generalized regular -closed (briefly, gr- closed) **[4]** set if rCl(A) $\subseteq U$ whenever A$\subseteq U$ and U is semi-open set in X

(iii) regular generalized (briefly, rg- closed) [**15]** set if Cl(A)$\subseteq U$ whenever A$\subseteq U$ and U is r-open set in X

(iv) generalized semi-preclosed (briefly, gsp- closed) [**6**] set if spCl(A)$\subseteq U$ whenever A$\subseteq U$ and U is open in X

(v) generalized pre-closed (briefly, gp- closed) [**14**] set if pCl(A)$\subseteq U$ whenever A$\subseteq U$ and U is open in X

(vi) generalized b-closed (brifly, gb-closed) [1] if bCl(A) ⊂U whenever A ⊂ U and U is open in X

(vii) δ- generalized closed (brifly , δg-closed) [5] if δCl(A) ⊂U whenever A ⊂ U and U is open in X

The complement of a g-closed (resp, sg-closed, gs-closed, αg-closed, gα-closed, gsp- closed, gp-closed) set in X is called g-open (resp. sg-open, gs- open, αg- open, gα- open, gsp- open and gp- open) set in X.

III. PROPERTIES OF WGR-CLOSED SETS

We, define the following

Definition 3.1: A subset A of space X is called weakly generaralized regular closed (brifly,wgr-closed) set if rCl(A)⊆U whenever A⊆U and U is semipreopen in X

The complement of a wgr-closed set of X is called wgr-open set in X. The family of all wgr-open (resp,wgr-closed) sets a space X is denoted by WGRO(X) (resp, WGRF(X)).

Clearly, in view of Def.2.7 and Def. 3.1 , we have the following .

Lemma 3.2 :

(i) Every r-closed set is wgr-closed set.
(ii) Every gr-closed set is wgr-closed set.
(iii)Every rb-closed set is wgr –closed set.
(iv)Every wgr-closed set is rg-closed set.
(v)Every wgr-closed set is g-closed set
(vi) Every wgr-closed set is gp-closed set.
(vii)Every wgr-closed set is gsp-closed set.
(viii) Every wgr-closed set is gpr-closed set .
(ix) Every wgr-closed set is δg-closed set.
(x) Every wgr-closed set is gb-closed set.

Lemma 3.3: A subset A of space X is called wgr-open set if U ⊆ rInt(A) whenever U⊆A and U is semipre-closed set in X

We , define the following

Definition 3.4 : The union of all wgr-open sets which contained in A is called the wgr-interior of A and is denoted by wgrInt(A)

Definition 3.5: The intersection of all wgr-closed set containing set A is called the wgr-closure of A and is denoted wgrCl(A)

Lemma 3.6: Let x∊X, then x∊ wgr-Cl(A) if and only if V∩A$\neq \phi$ for every wgr-open set V containing x

Properties of wgr-closure and wgr –interior operators

Theorem 3.7 : If A is wgr-closed set then, rCl(A) -A does not contain a non empty semipre-closed set.

Proof: Suppose that A is wgr-closed . Let F be a semipre-closed subset of rCl(A) -A. Then F⊆rCl(A)∩(X-A) ⊆ X-A and so A⊆(X-F). but A is wgr-closed. Since X-F is semipre-open, rCl(A) ⊆(X-F) that implies F⊆X- rCl(A). As we have already F⊆ rCl(A), it follows that

F$\subseteq$ rCl(A) $\cap$(X-rCl(A)) =ϕ. Thus F = ϕ. Therefore rCl(A) -A does not contain a non empty semipre-closed set.

Theorem 3.8: Let A be wgr-closed. Then A is regular closed if and only if rCl(A) \A is semipre-closed

Proof: If A regular closed then rCl(A)= A and so rCl(A)) =ϕ which semipre-closed

Conversely, suppose that rCl(A)\ is semipre-closed. Since A is wgr-closed, by Theorem 3.6, rCl(A) -A = ϕ That is rCl(A) = A and hence A is regular closed

Theorem 3.9 : If A is wgr-closed and if A$\subseteq$B$\subseteq$rCl(A) then

(i) B is wgr-closed

(ii) rCl(B) - B contains no non empty semipre-closed set.

Proof: A$\subseteq$B$\subseteq$rCl(A) $\Longrightarrow$rCl(B) = rCl(A). Now suppose B$\subseteq$U and U is semipre-open. Since A is wgr-closed and since A$\subseteq$B$\subseteq$U, rCl(A) $\subseteq$U that implies rCl(B) $\subseteq$U. This proves (i). Since B is wgr-closed, (ii) follows from Theorem 3.6.

Remark 3.10: wgr-closure of a set A is not always wgr-closed

Lemma 3.11: Let A and B be subsets of X. Then

(i) wgrCl(ϕ**) =** ϕ and wgrCl(X) = X

(ii) If A$\subseteq$B, wgrCl(A) $\subseteq$ wgrCl(B).

(iii) A$\subseteq$wgrCl(A)

Lemma 3.12: Let x$\in$X. Then xwgrCl(A) if and only if V$\cap$**A**$\neq$ ϕ for every wgr-open set V containing x.

Lemma 3.13: Let A and B be subsets of X. Then

(i) wgrCl(A) = wgrCl(wgrCl(A))

(ii) wgrCl(A) $\cup$ wgrCl(B) $\subseteq$ wgrCl(A$\cup$B)

(iii) wgrCl(A$\cap$ B) $\subseteq$ wgrCl(A) $\cap$ wgrCl(B)

Theorem 3.14: A set A $\subseteq$X is wgr-open if and only if F$\subseteq$rInt(A) whenever F$\subseteq$A, F is semipre-closed.

Proof: Let **A** $\subseteq$X be wgr-open . Let F be semipre-closed and F$\subseteq$A. Then X- A $\subseteq$X - F where

X - F is semipre-open. Since X- A is wgr-closed,rCl(X-A) $\subseteq$X - F and hence X-rInt(A) $\subseteq$X - F that implies F$\subseteq$rInt(A).

Conversely, assume that F$\subseteq$rInt(A) whenever F$\subseteq$A,F is semipre-closed. Suppose X- A $\subseteq$U where U is semipre-open. Then X- U $\subseteq$A where X- U is semipre-closed. By assumption, X- U $\subseteq$rInt(A) that implies rCl(X-A) $\subseteq$U. This proves that X- A is wgr-closed and hence A is wgr-open.

Theorem3.15 : If rInt(A) $\subseteq$B$\subseteq$A and A is wgr-open, then B is wgr-open.

Proof: Let A be wgr-open and rInt(A) $\subseteq$B$\subseteq$A. Then X - A $\subseteq$ X- B$\subseteq$X - rInt(A) that implies X-A$\subseteq$ X-BrCl(X-A). Theorem 3(i), X-B is wgr-closed. This oroves that B is wgr-open

Theorem 3.16 : If A $\subseteq$ X is wgr-closed and let F be a semipre -closed set such that F$\subseteq$rCl(A)-A. Then by Theorem 1, F= ϕthat implies F$\subseteq$rInt(rCl(A)-A). This proves that rCl(A)-A is wgr-open.

We, define the following.

Definition 3.17: A space X is said to be wgr-Hausdroff if whenever x and y are distinct points of X there exist disjoint wgr-open sets U and V such that x$\in$U and y$\in$V.

Definition 3.18: A space X is said to be r-Hausdroff if whenever x and y are distinct points of X there exist disjoint r-open sets U and V such that x$\in$U and y$\in$V.

Definition 3.19: A space X is said to be semipre-Hausdroff if whenever x and y are distinct points of X there exist disjoint wgr-open sets U and V such that x$\in$U and y$\in$V.

We, recall the following from .

Definition 3.20[13]: A function f:X$\rightarrow$Y is called wgr-continuous if f^{-1} (V) is wgr -closed in X for every closed subset V of Y.

Definition 3.21[13]: A function f:X$\rightarrow$Y is called wgr-irresolute if f^{-1} (V) is wgr -closed in X for every wgr-closed subset V of Y.

Definition 3.22 [13]: A function f : X→Y is called strongly wgr-continuous if the inverse image of each wgr-open set of Y is open in X.
Definition 3.23[13] : A function f : X→Y is called spwgr-continuous if the inverse image of each semipreopen set of Y is wgr-open in X.

Next , we prove the following.
Theorem 3.24 : Let X be a space and Y be Hausdorff . If f:X→Y be wgr-continuous injective,then X is wgr-Hausdorff .
Proof : Easy
Theorem 3.25: Let X be a space and Y be r-Hausdorff . If f:X→Y be r- wgr-continuous injective, then X is wgr-Hausdorff .
Proof : Easy
Theorem 3.26 : Let X be a space and Y be semipre-Hausdorff . If f:X→Y be spwgr-continuous injective, then X is wgr-Hausdorff .
Proof : Easy
Theorem 3.27 : Let X be a space and Y be wgr-Hausdorff . If f:X→Y be spwgr-continuous injective, then X is Hausdorff .
Proof : Easy
Theorem 3.28 : Let X be a space and Y be wgr-Hausdorff . If f:X→Y be wgr-irresolute injective, then X is wgr-Hausdorff .
Proof : Easy

We define the following.
Definition 3.29: A function f:X→Y is said to be wgr-open if the image of each open set of X is wgr-open in Y.
Definition 3.30: A function f:X→Y is said to be always -wgr-open if the image of each wgr-open set of X is wgr-open in Y.

Clearly , every always -wgr-open function is wgr-open.

Definition 3.31. A function f:X→Y is said to be wgr-closed if the image of each closed set of X is wgr-closed in Y
Definition 3.32: A function f:X→Y is said to be always -wgr-closed if the image of each wgr-closed set of X is wgr-closed in Y.
Clearly , every always -wgr-closed function is wgr-closed.

We ,prove the following.
Theorem 3.33: Let f:X→Y be a bijection. Then the following are equivalent.
(i) f is always- wgr-open.
(ii) f is always-wgr-closed.
(iii) f^{-1} is wgr-irresolute.
Proof: (i)→(ii).Suppose f is always wgr-open. Let F be wgr-closed in X. Then X-F is wgr-open. By definition 3.32, f(X-F) is wgr-open. Since f is a bijection. Y-f(F) is wgr-open in Y. Therefore f is always- wgr-closed.
(ii)→(iii).Let g = f^{-1}. Suppose f is wgr--closed. Let V be wgr-open in X. Then X-V is wgr-closed in X. Since f is always wgr-closed, f(X-V) is wgr-closed. Since f is a bijection, Y-f(V) is wgr-closed that implies f(V) is wgr-open in Y. Since g= f^{-1} and since g and f are bijection $g^{-1}(V)$= f(V) so that $g^{-1}(V)$ is wgr-open in Y. Therefore f^{-1} is wgr-irresolute.
(iii)→(i).Suppose f^{-1} is wgr-irresolute. Let V be wgr-open in X. Then X-V iswgr-closed in X. Since f^{-1} is wgr-irresolute and $(f^{-1})^{-1}$(X-V) = f(X-V) = Y-f(V) is wgr-closed in Y that implies f(V) is wgr-open in Y. Therefore f is wgr-open.
Theorem 3.34: Let f:X→Y and g : Y→Z be any two function. Suppose f and g are wgr-closed(resp, wgr-open). Then g.f is wgr-closed (resp,wgr-open).
Proof: Let U be any wgr-closed (resp,wgr-open) st in X. Since f is wgr-closed, using Definition 3.28, f(U) is wgr-closed (resp,wgr-open) in Y. again since g is wgr-closed (resp,wgr-open), using Definition 3.28, g(U) is wgr-closed (resp,wgr-open) in Z. This shows that g.f is wgr-closed (resp,wgr-open).
Definition 3.35 : A bijection f:X→Y is called regular weakly generalized regular homeomorphisms (brifly wgr- homeomorphisms) if f and f^{-1} are wgr-continuous
We say that the spaces x and y are wgr- homeomorphism from X onto Y
Theorem 3.36 : Every homeomorphism is an wgr- homeomorphism.
Proof: Let f:X→Y be a homeomorphism. Then f and f^{-1} are continuous and f is bijection.
Since every continuous function is wgr- continuouss, it follows that f is wgr- homeomorphism.

We , define the following.
Definition 3.37 : A bijection $f : X \to Y$ is said to be always wgr- homeomorphism if both f and f^{-1} are wgr-irresolute
Theorem 3.38: Every always wgr- homeomorphism is an wgr- homeomorphism.
Proof: Let $f : X \to Y$ be an always wgr- homeomorphism. Then f and f^{-1} are wgr-irresolute and f is bijection. By theorem 3.24, and f^{-1} are wgr-conyinuous. Therefore f is wgr- homeomorphism

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