

## Study of First & Second order Correlation Function for Two Composite State.

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### ABSTRACT

We theoretically analyze the steady state first and second order Correlation function for two level system. We consider a system Hamiltonian which interacts with a thermal environment. Our aim is to represent a composite state where these two state correlate. We observe the state goes to Coherent state (CS) to thermal State (TS) when plot as correlation function with delay time coordinate  $\tau$ . In first order Correlation there have much more distortion but in Second order Correlation. We see a linear correlation between these CS & TS states. So in our Study we represent a linear correlation between these two composite states.

**KEYWORDS-** Bunching, Coherence, Correlation.

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Date of Submission: 08-08-2020

Date of Acceptance: 21-08-2020

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### I. INTRODUCTION

In Quantum optics research there is some interaction between light and matter. Quantum optical principles correlate thermal and normal state. It is more important that it is possible to generate light of non-classical nature, as in the case of photon anti-bunching [1]. The two-level system is a simple model which has been applied to a great variety of physical problems. One application of growing importance is that of quantum computing [2]. Since any quantum two-level system has the potential to act as a quantum bit [3]. A photon light source has three components which depend on Excitation and Emission from one state to other. Coherent, bunched and anti-bunched light have been classified depending on second order correlation function. These points' help us to understand that bunched and coherence lights are composite state in classical Opto-mechanics, but not in anti-bunched light. It has no classical aspect. We know that photoelectric bunching can be explained by a classical interpretation [4]. Coherent light source can radiate highly bunched electron beams. The possible structure responsible for these enhancements are measured directly from the electron distributions using transition radiation techniques [5]. A radiation state of single Quantum mechanical system relates to certain correlation among each of the molecules. Spontaneous emission of radiation is a transition between two such levels leads to the emission of coherent radiation [6]. In extremely high brightness temperature a coherent source bunches and it is characterized by a multi-segment broken power law which breaks bunching properties and contributes to a coherent light source such as pulsars [7]. In Quantum optical point of view, it relates a single photons and the second-order correlation function is anti-bunched which obeys generally sub-Poisson photon statistics i.e.  $(g^2(0) < 1)$ . It indicates non-classical properties of light and termed as coherent states if  $g^2(0) = 1$  [8]. If two coherent states are statistically mixed which is purely classical then the second-order correlation function scales limitless [9]. If there is any state for which second order correlation function falls below 1/2 then it indicates Fock state. However the study of intermodal anti-bunching effect has not been done yet. It is more interesting effect for different coupling strengths in case of two level system [10]. This effect is also observed in nonlinear photonic molecules [11], Squeezed light generation [12], Frank Hertz Light, single trapped atom laser [13], Plasma-Cascade micro-bunching amplifier and coherent electron cooling of a Hadron Beams [14]. The first and second order correlation functions relevant for this work are successively discussed initially for the case of a simple two-level system and then for the eight level system of a Ba<sup>+</sup> ion [15]. The aim of this present paper is to compare with first order and second order correlation function for a certain Hamiltonian for two level CS and TS in thermal environment. For this purpose we choose a Hamiltonian [16].

$$H = \omega_c a^\dagger_k a_k + \omega_m b^\dagger_k b + G (a^\dagger_k a_k + b^\dagger_k b) \text{-----} (1)$$

Here we take  $\omega_c$  and  $\omega_m$  as state frequency and  $G$  as coupling strength, in our stimulation we choose the value of these parameters as 1 and 0.06 respectively for simplicity [17]. We set the creation and annihilation operator for two level Hamiltonian. Here  $a$  for CS state and  $b$  for TS state,  $a, a^\dagger$  &  $b, b^\dagger$  operators satisfying the commutation relation.

$$[a, a^\dagger] = 1.$$

$$[b, b^\dagger] = 1.$$

In this case, the interaction potential vanishes and, because the system is isolated.

**First order Correlation function:-**

The first order Coherence Expressed in the form of  $\langle \psi(\tau)\Omega(0) \rangle$  for non-steady state is the first order correlation function defined as  $g^1(\tau)$ . Hence it gives the degree of first-order temporal coherence between the two composite system at time  $t$  and  $(t + \tau)$  [18] and takes the values  $0 \leq |g^1(\tau)| \leq 1$  for all light sources. According to the Wiener-Khinchine theorem for a correlation function  $\langle \psi(\tau)\Omega(0) \rangle$  we expressed it as

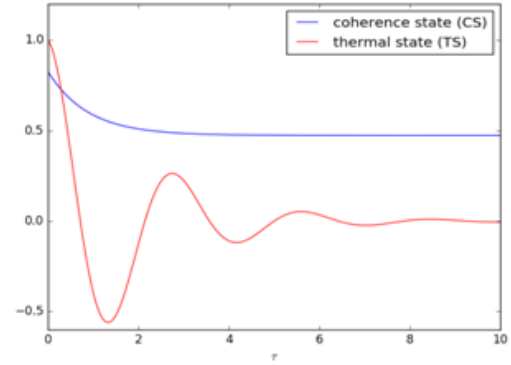
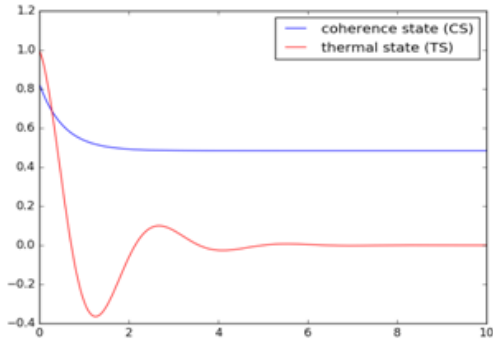
$$F(\omega) = \int \langle \psi(\tau)\Omega(0) \rangle e^{i\omega\tau} d\tau$$

In our investigation we plotted correlation function with respect to delay time and interestingly observed that when delay time and also the normalization parameter changes then the CS abruptly goes to TS. In that context the state vector jumps from initial state 0 to some another time state  $\tau$ . The normalized first order correlation function is

$$g_1(\tau) = \frac{\langle b^\dagger(\tau) b(0) \rangle}{\sqrt{\langle a^\dagger(\tau) a(\tau) \rangle \langle a^\dagger(0) a(0) \rangle}}$$

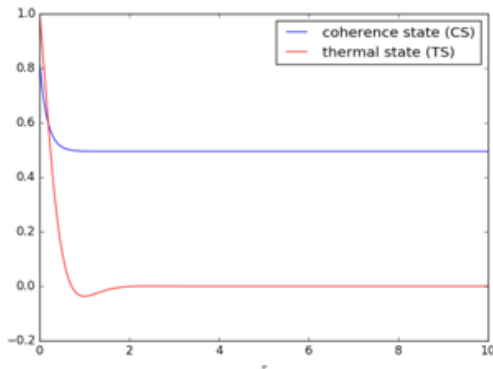
For the CS state,  $|g_1(\tau)| = 1$  and TS state  $|g_1(\tau)| = 0$ .

In our theoretical stimulation we have shown different Situation by graphically in figures-1(a, b, c, d, e, f) along x axis represent delay time, y axis represent Correlation function.

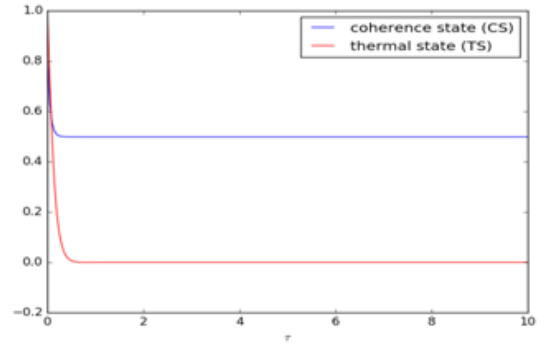


(fig1\_a)

(fig1\_b)



(fig1\_c)

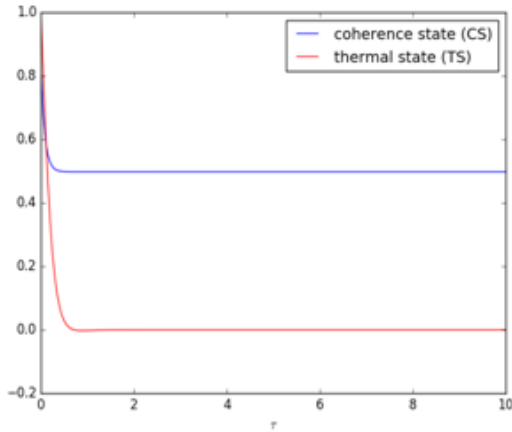


(fig1\_d)

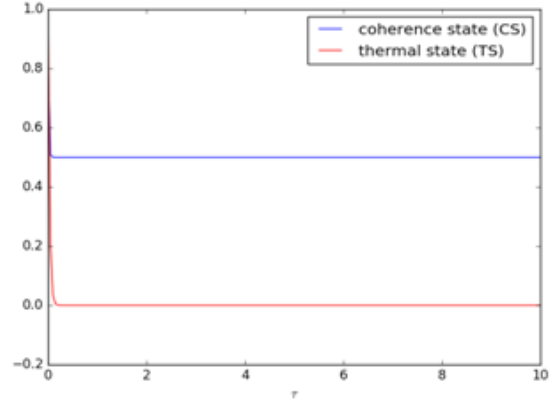
In these figures we plotted correlation function with the delay time and observe that the situation when we changed the intensity of thermal surroundings then CS State is just parallel to thermal state. There are abruptly changed from CS state to TS state. We have plotted using coupling constant as the value of 0.06, for two level correlation  $N=2$ , & intensity of thermal Surroundings with

$n_{th} = 9$  (fig1\_a)  
 $15$  (fig1\_b)

50 (fig1\_c)  
 100(fig1\_d)  
 600(fig1\_e)  
 800(fig1\_f)



(fig1\_e)



(fig1\_f)

**Second-Order Correlation function:-**

The second order correlation function for systematic studies of coherent , thermal light sources first carried out by Brown and R. Q. Twiss in 1956 [19,20]. We consider the second-order correlation function  $g^2(\tau)$ , where the normalized form is defined as [21] with delay time  $\tau$ .

$$g^2(\tau) = \frac{\langle a^\dagger(0) a^\dagger(\tau) a(0) a(\tau) \rangle}{[\langle a^\dagger(0) a(0) \rangle \langle a^\dagger(0) a(0) \rangle]}$$

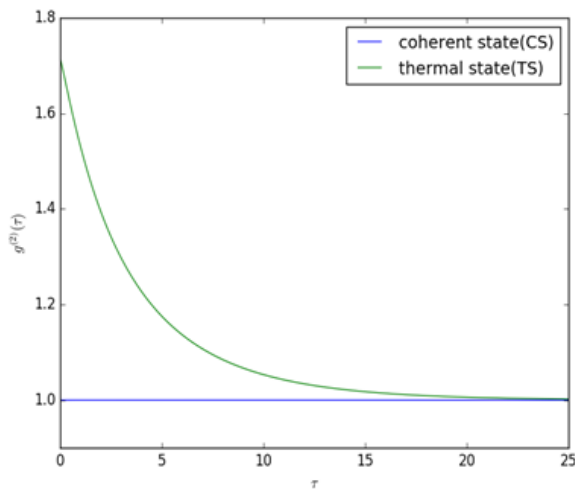
Hanbury-Brown and Twiss [22,23] first have shown that a continuous emitting coherent light source, such as thermal light, exhibits the effect of "photon bunching", i.e. after a detection of a photon it is more likely to detect another one shortly after wards. [12]

Second-order coherence of the classical source,

defining the intensity

$$X(t) = \langle X \rangle + \langle \delta X(t) \rangle \text{ with } \langle \delta X(t) \rangle = 0, \text{ as}$$

$1 \leq g^2(\tau) \leq \infty$ . The relation between first and second order correlation is [12]



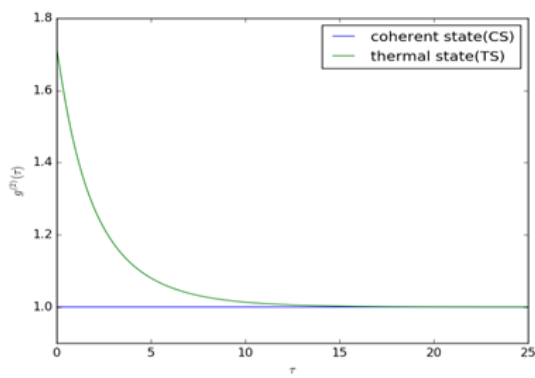
(fig2\_a)

$$g^2(\tau) = 1 + |g^1(\tau)|^2$$

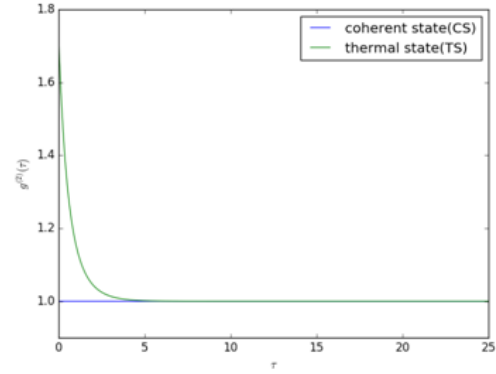
In our investigating the  $g^2(\tau)$  function of a second-order coherence is then given by [2]

$$g^2(t, t+\tau) = \langle \psi(t)\psi(t+\tau)\Omega(t+\tau)\Omega(t) \rangle$$

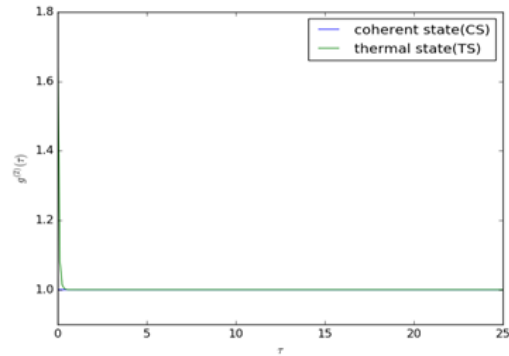
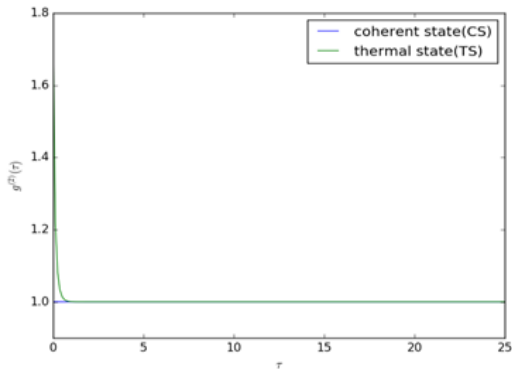
We plotted second order correlation function with delay time for the above Hamiltonian and are shown in figure (2-a, b, c d, e, f).



(fig2\_b)



(fig2\_c)



In our stimulation we have taken the coupling constant  $G=0.06$ , and  $N=10$ . We construct different situation for thermal environment by taking the intensity of thermal environment as

$n_{th}= 9$  (fig2\_a)

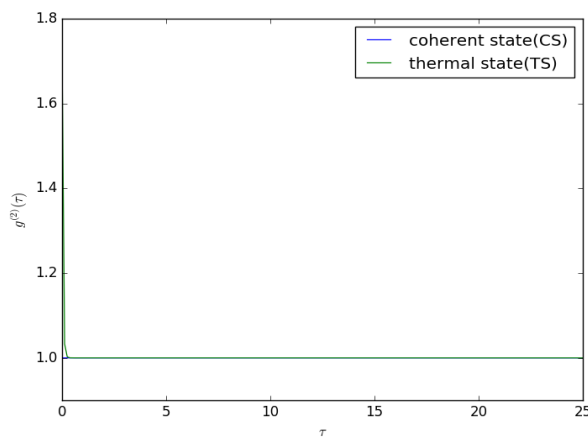
15 (fig2\_b)

50 (fig2\_c)

70 (fig2\_d)

400 (fig2\_e)

900 (fig2\_f)



(fig2\_f)

We observe when the intensity is nearly 70 then the CS state superimposes on TS, and at the value of 900 it almost superimposes.

## II. CONCLUSION:-

We simulate the profile of coherent and thermal state and when we change the delay time surprisingly the state CS goes to TS. We have already plotted the first order correlation function and also second order correlation function with delay time. It is clear that CS gradually decays to a thermal state in the above two situations. We next observed correlation function and have shown that it is not steady state but distorted at two different times. In the two level system it is evident that in first order correlation function there is much distortion in coherence and thermal states when intensity of thermal surroundings is increased. The result is much better in case of second order correlation when we increase the intensity. The CS and TS state are superimposed and for certain value of thermal intensity these two states just merge into one.

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Anjan Samanta, et. al. "Study of First & Second order Correlation Function for Two Composite State." *International Journal of Engineering Research And Development*, vol. 16(8), 2020, pp 13-17.