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A New Analytical Approach for the Outage Probability and Ergodic Capacity of Cloud Radio Access Networks Using Generalized Nakagami-n Fading Channel Model*

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ABSTRACT: *In this paper, we introduce a new analyses of the Outage Probability (Pout) and the Ergodic Capacity (EC) of uplink transmission channel in Cloud Radio Access Networks (C-RAN) with the adoption of the Nakagami-n fading channel. The mathematical derivation is done by considering two random variables for the Rician factor (k) and thermal noise is considered as non-identically distributed function. Moreover, arbitrary path loss exponent* α *with the association of a single Remote Radio Head (RRH) of two antennas (L=2) is also considered. Furthermore, the user with a single antenna is assumed to be at the center of a disk D. The results show that the EC increases from 25 bps/Hz to 25.5 bps/Hz when the Rice factors increase from 1 to 3 while α decreases from 4 to 3. However, when* $k_1=k_2=8$*, the EC increases from 25.5 bps/Hz to 27.3bps/Hz at α=2.*

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I. INTRODUCTION

The current decade drives network operators to invest more resources to provide reliable, secure, and efficient networks for several markets including the Internet of Things (IoT) and other intensive data applications. In 5G, C-RAN is among many approaches that promote investors in the world of wireless communication to curtail the operating expenditures (Opex) and the capital expenses (Capex) since it possesses the property of providing energy-efficient networks, providing load balancing to ensure the quality of service (QoS), and high throughput to increase network capacity [1], [2].

For such networks to have a considerable amount of contribution to the incoming 5G, researchers conducted many studies on such networks taking into account the number of associated RRHs, the value of path loss exponent α and the fading channel type. In [3] for example, the authors studied the effect of the path loss exponent and the number of RRHs on both EC and the outage probability using gamma fading channel. In [4], the authors analyzed the performance of C-RAN using the Nakagami-m fading channel with a single RRH and arbitrary path loss exponent. In [5], the authors investigated the EC model based on single nearest RRH association with Nakagami-m fading. In the current work, we extend our knowledge of C-RAN to include a more sophisticated fading channel model. The higher diversity gain can be achieved by selecting the optimal number of the associated RRHs among all RRHs in the space. In other words, the number of Remote Radio Heads (N-RRHs) are those who have the maximum instantaneous received power taking all fading parameters into account. To make the work feasible, and for simplicity, the analysis herein focuses on only 2-RRH for the user to meet QoS. Moreover, the N-RRH study under the same scenario might need comprehensive examination in the future since the derivation of the Probability Density Function (PDF) expression of the signal to noise ratio (SNR) is quite challenging. In particular, in this study, we aim to introduce a compact closed-form formula for EC with Nakagami-m fading using arbitrary parameters with 2-RRH. The following sections of this manuscript are organized as follows: In Section II, the system model is described in detail to reflect the general approach, while in Section III, the 2-RRH association strategy has been discussed, however in Section IV, the derivation of the EC formula is presented. In Section V, results and discussions are given. In Section VI, we conclude the work, and finally, in the appendix, we finalize the work by introducing the solution for the E.C equation.

II. Fading Channel Model

The channel fading amplitude follows the Rician distribution as [6, 7-8]:

$$f_{\alpha}(\alpha) = \frac{2(1+n^2)\, e^{-n^2}\alpha}{\Omega} \exp\left(-\frac{(1+n^2)\alpha^2}{\Omega}\right) I_0\left(2n\alpha\sqrt{\frac{(1+n^2)}{\Omega}}\right) \ldots\ldots\ \alpha \geq 0, \quad k \geq 0, \quad \Omega \geq 0 \tag{1}$$

where:

α : fading amplitude (R.V)

Ω : mean square value $\Omega = \overline{\alpha}^2$

$I_n(\bullet)$: is the nth-order modified Bessel function of the first kind

Using the equation defined below [20]:

$$P_{\gamma}(\gamma) = \frac{P_{\alpha}\left(\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}\right)}{2\sqrt{\frac{\gamma\bar{\gamma}}{\Omega}}} \tag{2}$$

where:

γ : Instantaneous SNR

$\bar{\gamma}$: Average SNR

$\gamma = \alpha^2 \frac{E_s}{N_o}$

$\bar{\gamma} = \Omega \frac{E_s}{N_o}$

$$f_{\gamma}(\gamma) = \frac{2(1+n^2)\, e^{-n^2}\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}}{\Omega} \exp\left(-\frac{(1+n^2)\frac{\Omega\gamma}{\bar{\gamma}}}{\Omega}\right) I_0\left(2n\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}\sqrt{\frac{(1+n^2)}{\Omega}}\right) \Bigg/ 2\sqrt{\frac{\gamma\bar{\gamma}}{\Omega}} \tag{3}$$

After mathematical manipulations, we can find the PDF of SNR as below:

$$f_{\gamma}(\gamma) = \frac{(1+n^2)\, e^{-n^2}}{\bar{\gamma}} \exp\left(-\frac{(1+n^2)\gamma}{\bar{\gamma}}\right) I_0\left(2n\sqrt{\frac{(1+n^2)\gamma}{\bar{\gamma}}}\right) \tag{4}$$

Also, the Ricain function can be defined with the help of the "k" parameter as follows:

$k = n^2 = \frac{a^2}{2\sigma^2}$ where "k" is the Rice factor. It is defined as the ratio of the power in the LoS component to the power in the weaker component in the propagation channel [9], while σ^2 the average power in the diffuse components is the Gaussian distribution "m" parameters. Also, we can define average SNR as:

$$\bar{\gamma} = a^2 + 2\sigma^2 = 2k\sigma^2 + 2\sigma^2 = 2\sigma^2(1+k) \tag{5}$$

Now, the PDF of SNR in eq. (5.4) can be re-written in terms of the "k" parameter as follows:

$$f_{\gamma}(\gamma) = \frac{(1+k)\, e^{-k}}{2\sigma^2(1+k)} \exp\left(-\frac{(1+k)\gamma}{2\sigma^2(1+k)}\right) I_0\left(2\sqrt{\frac{k(1+k)\gamma}{2\sigma^2(1+k)}}\right),$$

$$f_{\gamma}(\gamma) = \frac{(1+k)\exp\left(-\frac{a^2}{2\sigma^2}\right)}{2\sigma^2(1+k)} \exp\left(-\frac{(1+k)\gamma}{2\sigma^2(1+k)}\right) I_0\left(2\sqrt{\frac{k(1+k)\gamma}{2\sigma^2(1+k)}}\right),$$

$$f_{\gamma}(\gamma) = \frac{1}{2\sigma^2} \exp\left(-\frac{a^2+\gamma}{2\sigma^2}\right) I_0\left(2\sqrt{\frac{a^2\gamma}{2\sigma^2 2\sigma^2}}\right),$$

$$f_{\gamma}(\gamma)=\frac{1}{2\sigma^{2}}\exp\left(-\frac{\gamma+a^{2}}{2\sigma^{2}}\right)I_{0}\left(\frac{a\sqrt{\gamma}}{\sigma^{2}}\right) \quad (6)$$

Equation (6) represents the PDF of the SNR in Ricain fading propagation given in [10, 11], assuming that the two Gaussian R.Vs. are of different means $m_1 \neq m_2$ and the same variance σ^2. For Maximum Ratio Combining (M.R.C), the PDF of random variable Z at the RRH is the sum of two/many random variables.
To this point, we can distinguish three different scenarios:

Scenario 1

Two independent-identically distributed functions (i.i.d.) where:

$$\sigma_1^2=\sigma_2^2=\sigma^2, \qquad a_1^2=a_2^2=a^2 \quad (7)$$

Scenario 2

Two independent-non-identically distributed functions (i.n.i.d) where:

$$\sigma_1^2=\sigma_2^2=\sigma^2, \qquad a_1^2\neq a_2^2 \quad (8)$$

Scenario 3

Two independent-non-identically distributed functions (i.n.i.d) where:

$$\sigma_1^2\neq\sigma_2^2, \qquad a_1^2\neq a_2^2 \quad (9)$$

The third scenario is a generic case and will be adopted to find the ergodic capacity. Now, suppose that we need to find the PDF of a R.V. Z, where Z represents the sum of two random variables (diverse channels) from the user to the associated RRH. Mathematically speaking:

$$Z=U+W \quad (10)$$

where:

$$f_U(u)=\frac{1}{2\sigma_1^2}\exp\left(-\frac{u+a_1^2}{2\sigma_1^2}\right)I_0\left(\frac{a_1\sqrt{u}}{\sigma_1^2}\right) \qquad u\geq 0 \quad (11)$$

$$f_W(w)=\frac{1}{2\sigma_2^2}\exp\left(-\frac{w+a_2^2}{2\sigma_2^2}\right)I_0\left(\frac{a_2\sqrt{w}}{\sigma_2^2}\right) \qquad w\geq 0 \quad (12)$$

where: $\sigma_1^2\neq\sigma_2^2, \quad a_1^2\neq a_2^2$

We can further simplify eqs. (11) and (12) into the following forms:
In eq. (11), assume:

$$a_u=\frac{1}{2\sigma_2^2} \quad (13)$$

$$f_U(u)=a_u\exp\left(-a_u(u+a_1^2)\right)I_0\left(2a_ua_1\sqrt{u}\right), \quad (14)$$

$$f_U(u)=a_u\exp(-a_ua_1^2)\exp(-a_uu)\,I_0\left(2a_ua_1\sqrt{u}\right), \quad (15)$$

also,

Let $A_u=a_u\exp(-a_ua_1^2)$, and $b_u=2a_ua_1$, then eq. (15) will be reduced to the following simplified form:

$$f_U(u)=A_u\exp(-a_uu)\,I_0\left(b_u\sqrt{u}\right) \quad (16)$$

Similarly, for the random variable W, we have:

$$f_W(w)=A_w\exp(-a_ww)\,I_0\left(b_w\sqrt{w}\right) \quad (17)$$

where:

$A_w=a_w\exp(-a_wa_2^2)$, and $b_w=2a_wa_2$

From now and on, equations (16 and 17) are the two main ones that will be used to analyze the performance of C-RAN.
To find the PDF of Z, in literature, the density function is given as a convolution of two random variables as follows:

$$f_Z(z)=\int_{-\infty}^{+\infty}f_W(z-u)f_U(u)du, \quad (18)$$

Since both R.Vs. are non-negative R.Vs. i.e., $f_Z(z) = 0$ for $z < 0$ and for the positive number of z, $f_Z(z-u) = 0$ for $u > z$. In this case, the integrant above will be bounded between $u = 0$ and $u = z$, and will be simplified into:

$$f_Z(z) = \int_0^z f_W(z-u)\, f_U(u)\, du\,, \tag{19}$$

Substituting eq. (16) and eq. (17) into (19), we get:

$$\begin{aligned} f_Z(z) &= \int_0^z A_w \exp(-a_w(z-u))\; I_0\left(b_w\sqrt{(z-u)}\right) A_u \exp(-a_u u)\; I_0\left(b_u\sqrt{u}\right) du\,, \\ f_Z(z) &= A_w A_u e^{-a_w z} \int_0^z \exp(-u(a_u - a_w))\; I_0\left(b_w\sqrt{(z-u)}\right) I_0\left(b_u\sqrt{u}\right) du\,, \end{aligned} \tag{20}$$

Let:

$A = A_w A_u e^{-a_w z}$, $B = a_w - a_u$, and $\mu = a_u - a_w = -B$

equation (20) can be re-written as:

$$f_Z(z) = A\int_0^z e^{Bu}\; I_0\left(b_w\sqrt{(z-u)}\right) I_0\left(b_u\sqrt{u}\right) du\,. \tag{21}$$

In general, the modified Bessel function of the first kind and v order is given as:

$$I_v(z) = \sum_{k=0}^{\infty} \frac{1}{k!\,\Gamma(k+v+1)} \left(\frac{z}{2}\right)^{2k+v}. \tag{22}$$

Using eq. (22) above, the two Bessel terms in eq. (21) can be written as:

$$I_0\left(b_w\sqrt{(z-u)}\right) = \sum_{k=0}^{\infty} \left(\frac{\left(b_w\sqrt{(z-u)}\right)^k}{2^k\, k!}\right)^2 = \sum_{k=0}^{\infty} \frac{(b_w)^{2k}(z-u)^k}{(2^k\, k!)^2}$$

$$I_0\left(b_u\sqrt{u}\right) du = \sum_{k=0}^{\infty} \left(\frac{\left(b_u\sqrt{u}\right)^k}{2^k\, k!}\right)^2 = \sum_{k=0}^{\infty} \frac{{b_u}^{2k} u^k}{(2^k\, k!)^2}$$

Now, we can re-arrange eq. (22) as follows:

$$f_Z(z) = A\sum_{k=0}^{\infty} \frac{(b_w)^{2k}}{(2^k\, k!)^2} \sum_{k=0}^{\infty} \frac{{b_u}^{2k}}{(2^k\, k!)^2} \int_0^z u^k\, (z-u)^k\, e^{Bu}\, du\,. \tag{23}$$

according to [20], the integrant in (5.23) is:

$$\int_0^z u^k\, (z-u)^k\, e^{Bu}\, du\,. = \sqrt{\pi}\left(\frac{z}{B}\right)^{k+1/2} \exp\left(\frac{Bz}{2}\right) \Gamma(k+1)\, I_{k+1/2}\left(\frac{Bz}{2}\right) \tag{24}$$

It should be noted that the evaluation of this integral is also verified by using Maple software.

Finally, the PDF of MRC of two (i.n.i.d) Ricain function is reduced as follows:

$$H_i = f_Z(z) = \sqrt{\pi}\, A \sum_{k=0}^{\infty} \frac{(b_w)^{2k}}{(2^k\, k!)^2} \sum_{k=0}^{\infty} \frac{{b_u}^{2k}}{(2^k\, k!)^2} \left(\frac{z}{B}\right)^{k+1/2} \exp\left(\frac{Bz}{2}\right) \Gamma(k+1)\, I_{k+1/2}\left(\frac{Bz}{2}\right) \tag{25}$$

III. Outage Probability Analytical Derivation

In literature, the number of various channels will be two (L=2). Equation (20) represents the fading gain at the *ith* RRH. The next step is to find the overall SNR with MRC according to:

$$SNR_{N-RRH} = \sum_{i=1}^{N} \frac{P_u H_i}{\sigma^2 r_i^{\alpha}} \tag{26}$$

Where P_u is the power transmitted by the user, and σ^2 is the thermal noise power. The CDF of SNR is equal to the outage probability given that z is a random variable and the SNR to access the nearest RRH is smaller than the threshold T. Mathematically speaking, we have:

$$\begin{aligned} P_{out} &= F_Z(Z) = E\{\Pr(SNR < T)\} \\ &= E\left\{ \Pr\left(\sum_{i=1}^{2} \frac{P_u H_i}{\sigma^2 r_i^{\alpha}} < T \right) \middle| r_1, r_2 \right\} \\ &= E\left\{ \Pr\left(\frac{P_u H_1}{\sigma^2 r_1^{\alpha}} + \frac{P_u H_2}{\sigma^2 r_2^{\alpha}} \right) \middle| r_1, r_2 \right\}, \end{aligned} \tag{27}$$

The solution of equation (26) is given below:

$$P_{out} = \int_a^b \int_c^d 4\lambda^2 \pi^2 r_1 r_2 \exp(-\lambda\pi r_2^2)\, dr_1 dr_2, \tag{28}$$

Where

$$a = \left(\frac{E\{\rho H_1 + \rho H_2}{T} \right)^{1/\alpha}, \tag{29}$$

$$b = \infty, \tag{30}$$

$$c = \left(\frac{E\{\rho H_1\} r_2^{\alpha}}{T r_2^{\alpha} - E\{\rho H_1\}} \right)^{1/\alpha}, \tag{31}$$

$$d = r_2. \tag{32}$$

and H_1 and H_2 are given in equation (25).

As stated earlier in equation (10), the random variable Z represents the MRC of the two random variables channels from user U to the associated RRH. Substituting equation (26) into equation (28) and with the aid of [12], the solution for the outage probability is given below:

$$\begin{aligned} P_{out} = F_Z(z) = \left(\frac{\sigma_1}{\sigma_2} \right)^2 \exp\left(-\frac{\sigma_2^2}{2\sigma_2^2} \right) \times \sum_{i=0}^{\infty} \sum_{p=0}^{\infty} \frac{\Gamma(i+p+1)}{i!\,p!\,\Gamma(p+1)} \left(\frac{\sigma_2^2 \sigma_1^2}{2\sigma_2^2} \right)^p \\ \times \left(\frac{\sigma_2^2 - \sigma_1^2}{\sigma_2^2} \right)^i \left\{ 1 - Q_{i+p+2}\left(\frac{a_1}{\sigma_1}, \frac{\sqrt{z}}{\sigma_1} \right) \right\} \qquad z \ge 0 \end{aligned} \tag{33}$$

where $Q(x, y)$ is the generalized (m-the order) Marcum Q-function, defined in [13].

$$Q(A, B) = \frac{1}{A^{m-1}} \int_B^{\infty} x^m \exp\left(-\frac{x^2 + A^2}{2} \right) I_{m-1}(Ax)\, dx \tag{34}$$

Where $I_{m-1}(\bullet)$ is the *modified Bessel function of the first kind of order m-1.*

IV. Analytical Derivation of the Ergodic Capacity

Equation (33) represents Pout in the association of two nearest Remote Radio Head (2n-RRH). In order to derive the corresponding metric (EC), the PDF of CDF must be obtained first; then, by using equation (28), we can find the PDF as

$$PDF = \frac{\partial}{\partial z} F_Z(z) \tag{35}$$

$$PDF = \frac{1}{2\sigma_1^2}\left(\frac{\sigma_1}{\sigma_2}\right)^{2m_2}\left(\frac{z}{\sigma_1^2}\right)^{(m_1+m_2-1)/2} \times \exp\left(-\frac{z}{2\sigma_1^2}\right) \times \exp\frac{-1}{2}\left[\left(\frac{a_1^2}{\sigma_1^2}+\frac{a_2^2}{\sigma_2^2}\right)\right]$$
$$\times \sum_{i=0}^{\infty}\sum_{l=0}^{\infty}\frac{\Gamma(m_2+i+l)}{i!\,l!\,\Gamma(m_2+1)}\left(\frac{\sqrt{z}(\sigma_2^2-\sigma_1^2)}{a_1\sigma_2^2}\right)^l$$
$$\times \left(\frac{\sqrt{z}\sigma_2^2\sigma_1^2}{2a_1\sigma_2^4}\right)^i I_{m_1+m_2+l+1-1}\left(\frac{\sqrt{z}a_1}{\sigma_1^2}\right) \qquad z \geq 0$$

Now, the EC can be calculated using the following equation

$$EC_{2-RRH} = \int_0^{\infty} f_Z(z)\log_2(1+z)\,dz. \tag{36}$$

$$EC_{2-RRH} = \int_0^{\infty}\left\{\begin{array}{l} \frac{1}{2\sigma_1^2}\left(\frac{\sigma_1}{\sigma_2}\right)^{2m_2}\left(\frac{z}{\sigma_1^2}\right)^{(m_1+m_2-1)/2} \times \exp\left(-\frac{z}{2\sigma_1^2}\right) \times \exp\frac{-1}{2}\left[\left(\frac{a_1^2}{\sigma_1^2}+\frac{a_2^2}{\sigma_2^2}\right)\right] \\ \times \sum_{i=0}^{\infty}\sum_{l=0}^{\infty}\frac{\Gamma(m_2+i+l)}{i!\,l!\,\Gamma(m_2+1)}\left(\frac{\sqrt{z}(\sigma_2^2-\sigma_1^2)}{a_1\sigma_2^2}\right)^l \\ \times \left(\frac{\sqrt{z}\sigma_2^2\sigma_1^2}{2a_1\sigma_2^4}\right)^i I_{m_1+m_2+l+1-1}\left(\frac{\sqrt{z}a_1}{\sigma_1^2}\right) \end{array}\right\}\log_2(1+z)\,dz$$

(37)

V. RESULTS, DISCUSSION AND CONCLUSION

In this paper, the results obtained herein are represented by the general formulas for the outage probability and the ergodic capacity shown in equations (33) and (37), respectively. A numerical solution is provided to evaluate the network's performance with arbitrary values of rice factor. In the figure below, it is shown that the EC increases from 25 bps/Hz to 25.5 bps/Hz when the Rice factors increase from 1 to 3 while α decreases from 4 to 3. However, when $k_1=k_2=8$, the EC increases from 25.5 bps/Hz to 27.3bps/Hz at α=2.

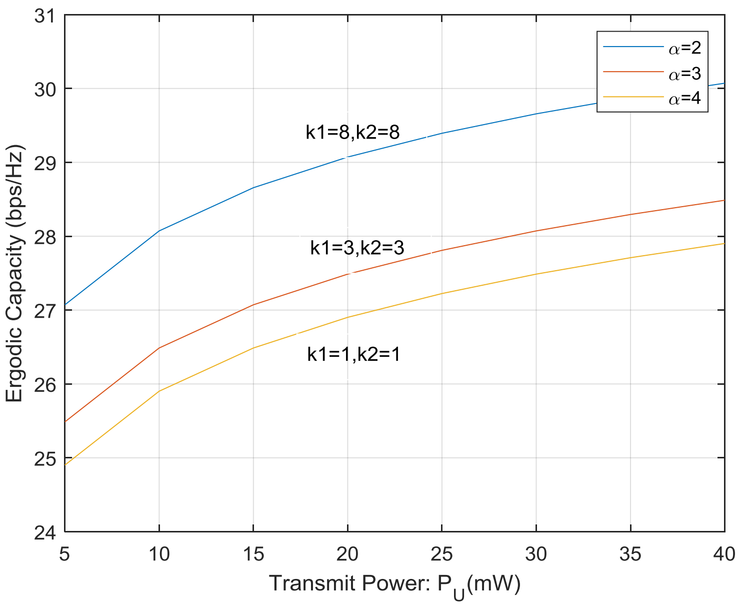

Figure1: Ergodic Capacity vs. transmitted power

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