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Markov Chain Analysis Of Rainfall Sum In A Small Hungarian Municipality

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Abstract: *The sequential process of rainy and rainless days as a physical system has been studied with Markov chains since the 1960s. However, in recent years there have been several publications in which the Markov chain analysis of annual precipitation sum has also been applied. This is a rather novel approach to the statistical analysis of precipitation sums, although the process itself is already known in several other hydrological processes. In this paper, the annual precipitation sums in a small municipality of Hungary and its region applying Markov chain were analysed based on 30-year data series. The data set was subjected to a normality test, and each year was classified into five categories (states) based on the amount of rainfall that year. With the help of a transition probability matrix, the invariant distributions of each category (state) were calculated. As a result, one specific year can be characterized in terms of precipitation over long term, which is a useful source of information for agriculture, as there is a correlation between annual rainfall and crop yields, especially in very droughty years. From the 30-year series, the return time of a very drought year could be also estimated applying stochastic method. The result obtained is then compared with the solution provided by the theoretical distribution function that best fits the traditionally used empirical distribution.*

Keywords: *invariant distribution, Markov chain, rainfall sum, return interval, transition matrix*

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I. INTRODUCTION

Climatic factors and extreme weather events have a major influence on crop yields. In Hungary, the temperature has risen by 1°C over the last 100 years and is expected to rise by 2.6°C by 2050 [1]. The centre region of Hungary, the Great Plain will be particularly affected by temperature changes in the future, where the annual precipitation has been reduced from 640 mm to 560 mm per year over the past 40 years [1]. Based on regional climate models adapted to Carpathian basin the temperature increase is evident, but changes in the annual precipitation is not expected [2-3]. However, the distribution of precipitation is uneven over time; it has been increased in winter and decreased in summer months, when the drought periods are critical for agriculture. Furthermore, the mean length of wet spell, the number of wet days, and the number of precipitation days exceeding 5 mm are projected to decrease in Hungary [4]. The average annual precipitation in Hungary in 1979 is shown in Figure 1 (a) [5] and in 2018 in Figure 1 (b) [6]. It can be seen that several areas have been affected by the decline in annual rainfall over the past 39 years.

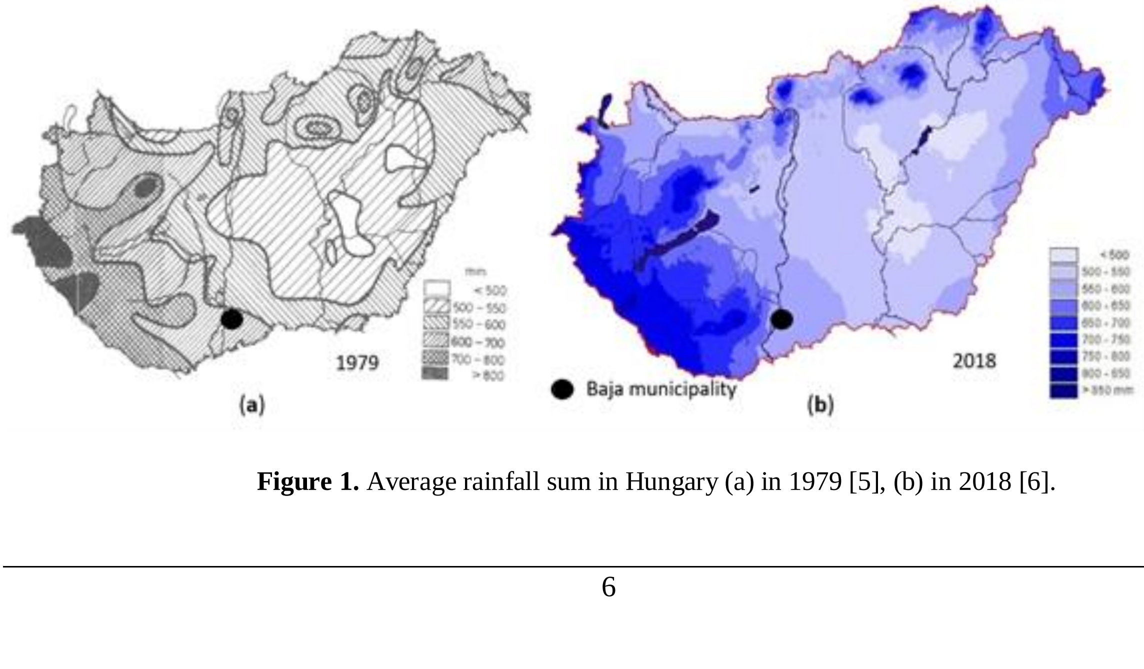

Figure 1. Average rainfall sum in Hungary (a) in 1979 [5], (b) in 2018 [6].

Baja municipality is situated in the mid-southern part of Hungary and is one of the most important agricultural areas in Hungary. The natural environment provides favourable conditions for agricultural production, but the agricultural peak season is unpredictable, which could manifest itself as drought, high precipitation rate or frost. There is a correlation between drought severity and yield averages [7]. A droughty year in the Baja region has a detrimental effect on crop yields, since the sandy soils have low water retention capacity and greater variation in groundwater levels.

Crop yields and average annual precipitation sums were analysed for the last decade and it was revealed that in extreme droughty years of 2011 and 2012 dropped from average 5.2 tons to 3.9 tons due to the 30% less annual precipitation [8].

Water management and spatial planning face conceptual challenges to prevent and mitigate the damages of drought. Various indexes were introduced to describe the drought tolerance and its effect; e.g. stress susceptibility index, stress tolerance index [9]. Others run drought scenarios and demonstrated that for crops exposed to the same drought - stress pattern, under current climates will have a similar impact on yield as that expected in the future, even though the probabilities of extreme drought stress will increase in the future [10]. Relation between the rainfall and drought was extensively examined in the past decades; empirical formula were introduced to predict the duration of droughts [11] and crop production [12]. However, rainfall sum analysis is not only for agriculture use, but it is an effective tool for landslide prediction [13] or estimating atmospheric pollen concentration [14].

Rainfall data processing vary with the field of application; trend analysis with linear regression or fitted with Gaussian function [15], but least square type calculations have also critics especially when the effect of climate change has to be considered. Replacing the sum-of-squared errors by sum-of-absolute errors could predict better the multi-year droughts [16]. Deep learning with neural network for rainfall prediction is also widely used, where extensive datasets are available for the learning stage of the system [17-19].

Since the 1960s the sequential process of rainy and rainless days of different periods with Markov chains have been modelled by several authors [20-23] and the likelihood of the rainy and drought periods were determined by the application of the transition matrix. We are drawn that, the likelihood of this physical system would be in a rainy or non-rainy state applying the transition matrix. In addition, the annual precipitation sums of different areas have been studied with Markov chains, which has been the subject of several publications [24-25]. These gave us the idea of using a stochastic method to examine the annual precipitation amounts in the Baja region, to give an estimate of the amount of precipitation for a given year.

II. MATERIAL AND METHODS

States of a system are denoted by the $X_0, X_1, X_2, \dots, X_n, \dots$ random variables at $t_0, t_1, t_2, \dots, t_n, \dots$ time, respectively. Assume that $X_0 = x_0$ at t_0, $X_n = i$ at t_n and $X_{n+1} = j$ at t_{n+1}. One-step transition probability is defined as X_{n+1} is at state *j* only if X_n is at state *i*. This condition can be formulated as follows:

$$P_{ij}^{n,n+1} := P(X_{n+1} = j \mid X_n = i).$$

This formula reflects that the transition probability is in function with initial states, end states and time. If the one-step transition probability is independent of the time step of *n*, then the transition probabilities of Markov chain are stationary or in other words, the Markov chain is homogeneous [26]. In this case the

$$P_{ij}^{n,n+1} := P_{ij}.$$

Markov property states that the calculated probability of a random process transitioning to the next possible state is only dependent on the current state and it is independent of the series of states that preceded it. Formally:

$$P(X_{n+1} = j \mid X_n = i, X_{n-1} = x_{n-1}, \dots X_1 = x_1, X_0 = x_0) = P(X_{n+1} = j \mid X_n = i) = P_{ij}.$$

In a Markov chain, we use a matrix to represent the transition probabilities from one state to another. This matrix is called the transition probability matrix. It is denoted by **P**=(P_{ij}), in detail

$$\mathbf{P} = \begin{bmatrix} P_{00} & P_{01} \dots & P_{0k} \\ P_{10} & P_{11} \dots & P_{1k} \\ \vdots & & \vdots \\ P_{k0} & P_{k1} \dots & P_{kk} \end{bmatrix}.$$

The elements P_{ij} are non-negative numbers and the sum of the elements on each row yields 1. The Markov chain can be determined completely by the transition probability matrix **P** and the initial distribution at time t_0 denoted by φ_0. After n step the distribution is $\varphi_n = \varphi_0 \cdot P^n$. It can be proved that this distribution φ_n approaches a limit as the number of steps approaches infinity. This limit is the equilibrium distribution of the Markov chain. On the other hand, high powers of the transition matrix **P** approach a matrix having all rows identical; these identical rows have as entries the entries of the equilibrium distribution. This agrees with the statement that the initial state is not significant. The matrix

$$\mathbf{P}^* = \lim_{n\to\infty} \mathbf{P}^n = \begin{bmatrix} P_0 & P_1 \dots & P_k \\ P_0 & P_1 \dots & P_k \\ \vdots & & \vdots \\ P_0 & P_1 \dots & P_k \end{bmatrix}$$

is the limiting matrix. The probabilities $P_0, P_1, \dots, P_k$ express the situation that after long change of states the system is at the state 0, 1, … , *k*. We can apply two methods to get the equilibrium distribution; (i) **P** is exponentiated until the rows of the matrix are identical or applying the idempotence of $\mathrm{P}^*\mathrm{P} = \mathrm{P}^*$, which leads to a linear equation system.

Annual precipitation of the past 30 years in the Baja region were examined, and then the years were classified into five categories based on the amount of precipitation: very droughty, droughty, average, rainy, very rainy [27]. The one-step transition probability matrix of these states was calculated, and the invariant distribution was determined. Applying this process, the likelihood of a given year having rainfall characteristics and the return time of a very drought year could be estimated. The result obtained was then compared with the solution provided by the theoretical distribution function that best fits the traditionally used empirical distribution.

In the database of the National Institute of Meteorology (NIM) the annual precipitation data at Baja station was available. The base period is from 1971 to 2000 and the mean annual precipitation was 648 mm and standard deviation was 131 mm. Annual precipitation sums between 1989 and 2018 are summarized in Figure 2.

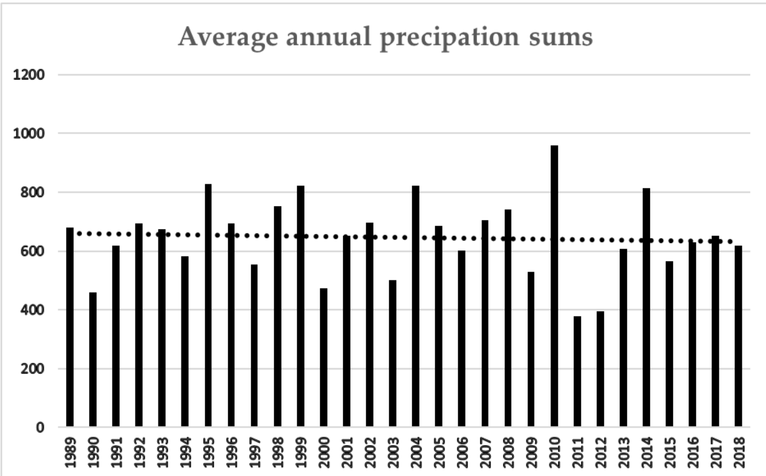

Figure 2. Annual precipitation sum (1989-2018)

Figure 2 shows that 2010 was an extremely wet year and in the following year there was drought. According to climatologists, the two record years are not clearly attributable to the impact of climate change, but indicate that the weather in Hungary will be more extreme. In 2010 the weather of the country was reminiscent of the rainy season of the savannah and in 2011 of the desert. In May 2010, unprecedented floods occurred on the smaller rivers, but in June floods of River Bodva, Sajó, Hernád, Ipoly and Danube also occurred. The year 2011 brought opposite fluctuations, extreme drought, and there was a region where the precipitation did not reach half of the average and only the wine producers could take advantage of the drought. The national rainfall in 2011 was 404.4 millimetres, which is 4.3 millimetres less than the lowest ever recorded by the National Institute of Meteorology in the warm and dry year 2000. According to the NIM, annual precipitation has declined over the past 100 years, meaning that extreme dry years are becoming more common. This is also confirmed by the trend line in Figure 2, which shows that the amount of rainfall during the year is decreasing in Baja municipality as well.

Based on the data available, the years have been categorized into five categories for precipitation (denoting the future abbreviations of these scenarios in brackets), with a percentage difference from the mean:

1. average (S1): -10% <annual precipitation difference <10%
2. drought (S2): -30% <annual precipitation difference <-10%
3. rainfall (S3): 10% <annual precipitation difference <30%
4. very drought (S4): annual precipitation difference <-30%
5. very rainy (S5): annual precipitation difference> 30%>

III. RESULTS AND DISCUSSION

3.1. Normality test

The statistical matching test for normal distribution was performed by χ^2 test and graphically as well. Let the probability variable ξ denotes the annual precipitation sum. Then our H_0 hypothesis is that ξ is normally distributed. Based on the 30 years' series of data the expected value is 646 mm and the deviation is 133 mm. Table 1. shows the actual years and expected (calculated) years in each scenario.

Table 1. Values of the transition probability

mm	< 400	401 – 499	500-599	600-699	700-799
Nr. of actual years	2	2	5	13	3
Nr. of expected years	1	3	7	9	6

The first scenario is detailed as follows: the probability of less than 400 mm precipitation follows normal distribution:

$P(\xi<400) = F(400) = \phi\left(\frac{400-646}{133}\right) = \phi(-1.85) = 1 - \phi(1.85) = 1 - 0.9678 = 0.0322$.

If this result is multiplied by 30 and is rounded, then the first element number of expected years shown in Table 1 can be obtained. Similar procedure could be applied to determine the entire 3rd row in Table 1.

The value of the χ^2 test statistics is: $\chi^2 = \frac{(2-1)^2}{1} + \frac{(2-3)^2}{3} + \frac{(5-7)^2}{7} + \frac{(13-9)^2}{9} + \frac{(3-6)^2}{6} + \frac{(5-4)^2}{4} = 5.4325$.

The degree of freedom (DOF) is n=6-1=5. Confidence limit of χ^2 is 11.07 assuming 5% probability and DOF of 5. The value of the test statistics is less than this value, therefore there is no need to refuse the H_0 hypothesis, the data series is considered to be normally distributed.

Furthermore, statistical matching test was presented graphically. Let $x_1 < x_2 < \cdots < x_r$ be the boards of classes and let n be the number of elements in the sample realization and k_i be the number of elements in the sample realization less x_i. Then the value of the empirical distribution function for x_i is: $F_n^*(x_i) = \frac{k_i}{n}$. If the examined probability variable is normally distributed with expected value m and a standard deviation σ, then

$$\frac{k_i}{n} \cong \Phi\left(\frac{x_i - m}{\sigma}\right), i = 1, 2, \dots, r,$$

so

$$\Phi^{-1}\left(\frac{k_i}{n}\right) \cong \frac{1}{\sigma}x_i - \frac{m}{\sigma}, i = 1, 2, \dots, r.$$

Thus, with notation $y_i := \Phi^{-1}\left(\frac{k_i}{n}\right)$, the points with coordinates $(x_i, y_i)\ (i = 1, 2, \dots, r)$ fall on a line whose slope is $\frac{1}{\sigma}$ and intersects the vertical axis at the value $-\frac{m}{\sigma}$.

The classification of annual precipitation sums are from 400 mm to 1000 mm with 50 mm intervals. The Figure 3. shows the regression line and its equation:

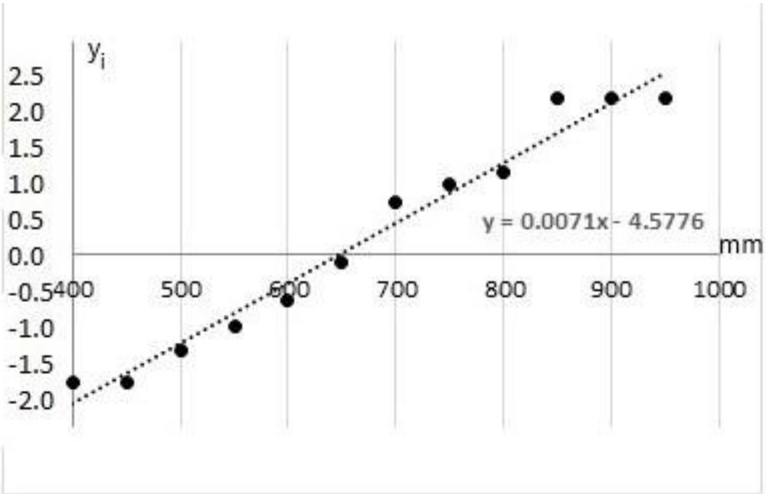

Figure 3. Normality test based on dataset from 1989-2018 average annual precipitation sums

From the regression line the standard deviation is 1/0.0071=140.845, whereas the expected value is 4.5776·140.845=644.732. This supports the hypothesis that the annual precipitation sum follows normal distribution.

3.2. Calculation of invariant distribution

From the series of data, it can be determined how frequently the system changes from one state to another. For example, in the graph the frequency of the transition from S1 state to S2 state is 4. It can be seen that this value is closer to S2 on the line connecting S1 and S2. The frequency of the transition from S2 to S1 is 3 and this number is closer to S1 on the line between S1 and S2. The different states can be the same in the following year. The numbers in the closed loops next to the states show the transition frequencies going into themselves. The Table 1. can be one-to-one mapped to the graph.

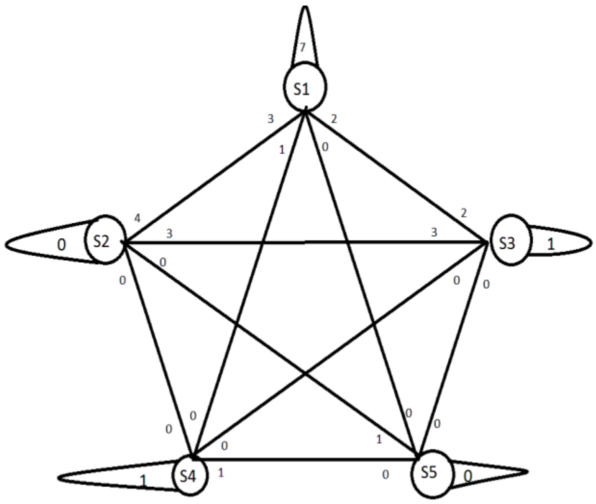

Figure 4. Graph of transitional matrix

Table 2. Values of the transition frequencies.

	S1	S2	S3	S4	S5
S1	7	4	2	0	0
S2	3	0	3	0	1
S3	2	3	1	0	0
S4	1	0	0	1	0
S5	0	0	0	1	0

From Table 2. the initial distribution can be determined. It means what the probabilities of the states are in the system. For example the probability of average precipitation S1 state is $\frac{13}{29}$. The initial distribution is $\varphi_0 = \left(\frac{13}{29}; \frac{7}{29}; \frac{6}{29}; \frac{2}{29}; \frac{1}{29}\right)$. The one step transition probability matrix can be given from the table of transition frequencies.

$$\boldsymbol{P} = \begin{pmatrix} 0.54 & 0.31 & 0.15 & 0.00 & 0.00 \\ 0.43 & 0.00 & 0.43 & 0.00 & 0.00 \\ 0.33 & 0.5 & 0.17 & 0.00 & 0.00 \\ 0.50 & 0.00 & 0.00 & 0.50 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 & 0.00 \end{pmatrix}$$

The above mentioned $P^*P = P^*$ property needs to calculate the limit matrix. Applying this:

$$\begin{pmatrix} P_1 & P_2 & P_3 & P_4 & P_5 \\ P_1 & P_2 & P_3 & P_4 & P_5 \\ P_1 & P_2 & P_3 & P_4 & P_5 \\ P_1 & P_2 & P_3 & P_4 & P_5 \\ P_1 & P_2 & P_3 & P_4 & P_5 \end{pmatrix} \begin{pmatrix} 0.54 & 0.31 & 0.15 & 0.00 & 0.00 \\ 0.43 & 0.00 & 0.43 & 0.00 & 0.00 \\ 0.33 & 0.5 & 0.17 & 0.00 & 0.00 \\ 0.50 & 0.00 & 0.00 & 0.50 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 & 0.00 \end{pmatrix} = \begin{pmatrix} P_1 & P_2 & P_3 & P_4 & P_5 \\ P_1 & P_2 & P_3 & P_4 & P_5 \\ P_1 & P_2 & P_3 & P_4 & P_5 \\ P_1 & P_2 & P_3 & P_4 & P_5 \\ P_1 & P_2 & P_3 & P_4 & P_5 \end{pmatrix}$$

This leads to the following equation system (it is known that $P_1 + P_2 + P_3 + P_4 + P_5 = 1$):

$$0.54P_1 + 0.43P_2 + 0.33P_3 + 0.5P_4 = P_1$$
$$0.31P_1 + 0.50P_3 = P_2$$
$$0.15P_1 + 0.43P_2 + 0.17P_3 = P_3$$
$$0.5P_4 + P_5 = P_4$$
$$0.14P_2 = P_5$$

Gauss elimination helped in solving the linear equation system:
$P_1 = 0{,}449$, $P_2 = 0{,}244$, $P_3 = 0{,}205$, $P_4 = 0{,}068$, $P_5 = 0{,}034$.
The invariant distribution of the different states are given by these results. It can be checked it by raising to the power of matrix **P**; by taking the power of 16 full identities could be achieved. Based on the equilibrium distribution, in a longer period the probability of the average precipitation year is 44.9%, the drought year is 24.4%, the rainy year is 20.5%, the very drought year is 6.8% while the very rainy year is 3.4%. As a result, the average return time of the drought year is $\frac{1}{0.244} = 4.1$ and for the average return time of the very drought year is $\frac{1}{0.068} = 14.7$. Results shows that very droughty year could be expected rarely, although it happened twice in the last 8 years.
As next step, it is analysed how the probabilities by Markov chain vary from the results calculated by normal distribution. The probability of very droughty year (the probability variable ξ denotes the annual precipitation sum):
$P(\xi<451) = F(451) = \phi\left(\frac{451-646}{133}\right) = \phi(-1.47) = 1 - \phi(1.47) = 1 - 0.9292 = 0.0708$,
while for the probability of the droughty year:
$P(451<\xi<480) = F(480)-F(451) = \phi\left(\frac{480-646}{133}\right) - \phi\left(\frac{451-646}{133}\right) = \phi(-0.5) - \phi(-1.47) = 1 - \phi(0.5) - 1 + \phi(1.47) = 0.9292-0.6915=0.2377$.

It is remarkable that the probability values calculated by the two different methods are really close to each other: in case of very droughty year the difference is 0.0028, while in case of droughty year it is only 0.0063.
Examination of annual precipitation at Baja over the past 30 years has supported the widespread hypothesis that desertification processes, drought and drought rates will continue to increase in the future in most parts of Hungary. The trend line in Figure 2 confirms that the annual precipitation is decreasing.
From the data set available Markov chain was used to calculate the invariant distribution of mean, drought, very drought, precipitation, and very rainy years. As a result, drought years are averaged every four years, while very drought years are averaged every 14 years, although two extremes, very drought years have occurred twice in the last eight years.

In recent decades, drought and water scarcity have become a significant risk factor, the incidence and severity of which have increased, and it is clear that global warming plays a major role in changing the water economy. These unfavourable developments are increasingly challenging the natural and economic environment. Based on the forecasting models and the lessons learned from the observations, it is expected that Hungary will become more involved, and quantitative and qualitative changes in water resources will require prevention, as well as planned use [28].

As the frequency of droughts increases, we must expect a decrease in agricultural areas providing food supplies (desertification), as well as a reduction in crop yields and production costs. This may result in higher food prices. The solution obviously is to increase the irrigated area in the Baja area as well. This could be done mainly through storage. The volume of storage could be increased by encouraging water retention by regions, municipalities, businesses and the general public. Local water storage for municipalities and populations should therefore be promoted in order to ensure non-drinking water needs.

One of the most important drought response programs could be the development of irrigated farming [28]. The proportion of irrigated areas is low in Hungary; only 100,000 hectares are irrigated compared to the previously established irrigation facilities of over 300,000 hectares. There is a significant demand for expanding agricultural water services and irrigation development investments.

IV. CONCLUSIONS

Agriculture has a major importance in Hungary's economy, but due to the changing environmental factors the future of this sector became unpredictable. Primarily rainfall sums determine the crop yields, therefore the prediction of the precipitation could give us indications on the agricultural performance. The applied Markov chain based method is quite novel in the analysis of annual precipitation sums, only in recent years this method has been used to investigate precipitation in different areas. The advantage of this method is that even without the best-fit probability distribution for the data series, the probabilities of individual conditions (average, drought, very drought, rainy, very wet years) can be calculated over long term. The correctness of this method was checked by comparing the probabilities of each condition with the values calculated from the normal distribution and the differences of the estimated and actual values were negligible. The test area was in the southern part of Hungary and the calculation has revealed that every fourth year has drought, and every 14 years faces severe drought.

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