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Free Torsional Vibrations of Thin-Walled Open Section Beams Fixed at One End and Elastically Restrainedat the Other End using Spectral Dynamics Approach

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Abstract: *Free torsional vibration of doubly symmetric thin-walled beams of open section using spectral dynamics approach, is carried out in this paper. Frequency equation for the case of doubly symmetric thin-walled beam fixed at one end and the other end elastically restrained is derived in this paper. The derived Frequency equation with appropriate boundary conditions is solved for varying values of warping parameter and the elastic restraint parameters. The effects of restraint parameters and the warping parameter on the free torsional vibration frequencies are investigated in detail. A computer program using MATLAB is developed to solve the spectral frequency equation derived. Natural frequencies for various values of linear and rotational restraint parameters for different values of warping parameter are obtained.Results are presented in both tabular as well as graphical form showing the influence of these parameters on the values of fundamental torsional frequency parameters clearly.*

Keywords: *Beam, Open Section, Torsion, Dynamic Stiffness, Warping*

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I. Introduction

In many practical situations, by using elastically restrained edges against the rotation and translation, one can simulate the complex boundary conditions of structural members.

The problem of vibrations of generally restrained beams with various combinations of boundary conditions has been investigated by many researchers in the available literature Computation of natural frequencies and mode shapes of cantilever beams with flexible roots has been studied well [3, 5, 9-11]. Kameswara Rao and Mirza[12] derived the transcendental frequency equation and mode shape expressions for the case of generally restrained Euler-Bernoulli beams and presented extensive numerical results for various values of linear and rotational restraint parameters.

Strangely, there are quite a good number of publications on flexural vibrations of elastically restrained cantilever beams, the literature on torsional vibrations of doubly symmetric thin-walled beams of open section is surprisingly scarce. Including elastic torsional and warping restraints, Carr [15] and Christino and Salmela[13] presented numerical results using approximate methods for the calculation of natural frequencies. Torsional vibration frequencies for beams of open thin-walled sections, subjected to several combinations of classical boundary conditions were first derived by Gere [14].

Burlon et al [15] proposed an exact approach to coupled bending and torsional free vibration analysis of beams with mono-symmetric cross section, featuring an arbitrary number of in-span elastic supports and attached masses. The proposed method relies on the elementary coupled bending-torsion theory and makes use of the theory of generalized functions to handle the discontinuities of the response variables. In another paper, Burlon et al [16]investigated the stochastic response of a coupled bending–torsion beam, carrying an arbitrary number of supports/masses. Using the theory of generalized functions in conjunction with the Euler–St.Venant coupled bending–torsion beam theory, exact analytical solutions under stationary inputs are obtained based on frequency response functions derived by two different closed-form expressions.

The review presented by Sapountzakis[19], clearly shows that the problem of free torsional vibration analysis of doubly-symmetric thin-walled I-beams or Z-beams subjected to partial warping restraint is not being addressed till now in the available literature. In view of the same, an attempt has been made in this paper to present a spectral dynamic analysis of free torsional vibration of doubly-symmetric thin-walled beams of open section with fixity at one end and elastically restrained at the other end.

The resulting spectral frequency equation is solved for varying values of warping parameter and the restraint parameters. The influence of linear and rotational restraint parameters along with warping parameter on the free torsional vibration frequencies is investigated in detail by utilizing a computer program developed using MATLAB, to solve the spectral frequency equation derived in this paper. Numerical results for natural frequencies for various values of restraint parameters are obtained and presented in this paper.

II. Formulation and Analysis

Consider a long doubly-symmetric thin-walled beam of open cross section of length L and the beam as undergoing free torsional vibrations. The corresponding differential equation of motion can be written as:

$$EC_W\frac{\partial^4\varphi}{\partial z^4} - GC_S\frac{\partial^2\varphi}{\partial z^2} + \rho I_P\frac{\partial^2\varphi}{\partial t^2} = 0 \quad (1)$$

where,

E= young's modulus, C_W=warping constant,G =shear modulus,C_S = torsion constant, ρ =mass density of the material of the beam, I_P =polar moment of inertia, φ = angle of twist, z= distance along the length of the beam.

For free torsional vibrations, the angle of twist $\varphi(z,t)$ can be expressed in the form.

$$\varphi(z,t) = x(z)e^{i\omega t} \quad (2a)$$

$$x(z) = Ce^{mz} \quad (2b)$$

In which $x(z)$ is the modal shape function corresponding to each beam torsional natural frequency ω.

The expression for $x(z)$which satisfies Eqn. (1) can be written as:

$$x(z) = Ae^{+\alpha z} + Be^{-\alpha z} + Ce^{+i\beta z} + De^{-i\beta z} \quad (3)$$

in which,

$$\beta L, \alpha L = \sqrt{\frac{\mp K^2 + \sqrt{K^4 + 4\lambda^2}}{2}} \quad (4)$$

where,

$K^2 = \left(\frac{GC_S L^2}{EC_W}\right)$; Non- dimensional warping parameter

$\lambda^2 = \left(\frac{\rho I_P \omega^2 L^4}{EC_W}\right)$;Non- dimensional frequency parameter

From Eqn. (4),we have the following relation between αL and βL

$$(\alpha L)^2 = (\beta L)^2 + K^2 \quad (5)$$

Knowingα and β, the frequency parameter λ can be evaluated using the following equation:

$$\lambda^2 = (\alpha L)(\beta L) \quad (6)$$

The four arbitrary constants A, B, C and D in Eqn. (3) can be determined from the boundary conditions of the beam. For any single-span beam, there will be two boundary conditions at each end and these four conditions then determine the corresponding frequency and mode shape expressions.

III. Derivation of Spectral Frequency Equation

Consider a thin-walled doubly symmetric I-beam with one end rotationally restrained and the other end transversely restrained as shown in figure 1, undergoing free torsional vibrations. In order to derive the spectral frequency equation for this case, let us first introduce the related nomenclature.

The variation of angle of twist φ with respect to z is denoted byθ*(z).* The flange bending moment and the total twisting moment are given by *M(z)*and *T(z).* Considering clockwise rotations and moments to be positive, we have

$$\theta(z) = \frac{d\varphi}{dz};\ hM(z) = -EC_W\frac{d^2\varphi}{dz^2} \quad (7)$$

$$T(z) = -EC_W\frac{d^3\varphi}{dz^3} + GC_S\frac{d\varphi}{dz} \quad (8)$$

where $EC_W = \frac{I_f h^2}{2}$. $I_f$being the flange moment of inertia and h is the distance between the center lines of the flanges of a thin-walled I-beam.

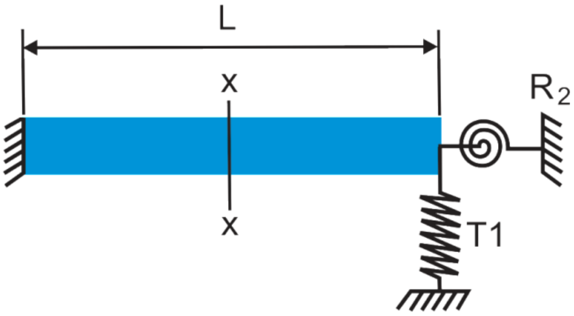


Figure 1 (a). A cantilever thin-walled open section I-beam with clamped edge at one end and Elastically Restrained at the Other End.

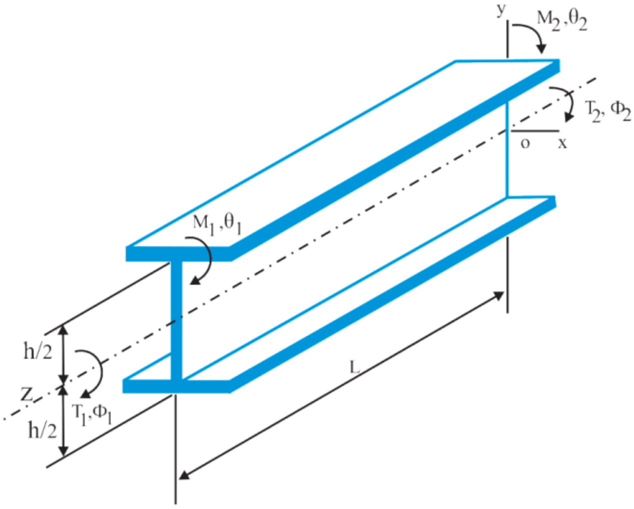


Figure 1 (b). *Cross-section of the beam at x-x.*

Taking S1 and S2 as the stiffnesses of the rotational springssituated at both endsand R_1 = (S_1L/EC_W) and R_2= (S_2L/EC_W)as the non-dimensional rotational spring stiffness parameters and Z=(z/L) as the non-dimensional length of the beam, the boundary conditions can be easily identified as follows:

$$\text{At } Z = 0, \varphi = 0, \frac{d\varphi}{dZ} = 0 \quad (9)$$

$$\text{And at } Z = L, \frac{d^3\varphi}{dZ^3} - K^2\frac{d\varphi}{dZ} = T\varphi, \frac{d^2\varphi}{dZ^2} = -R\frac{d\varphi}{dZ} \quad (10)$$

The spectral frequency equation obtained is as given below:

$$S_1 + \frac{RT}{(\alpha^2\beta^2)}S_2 - RF_3S_3 - TF_4S_4 = 0 \quad (11)$$

where

$$F_1 = \frac{(\alpha^2-\beta^2)}{(\alpha\beta)}; F_2 = \frac{(\alpha^2+\beta^2)}{(\alpha\beta)}; F_3 = \frac{(\alpha^2+\beta^2)}{(\alpha^2\beta^2)}; F_4 = \frac{(\alpha^2+\beta^2)}{(\alpha^3\beta^3)}; F_5 = \frac{(\alpha^4+\beta^4)}{(\alpha^2\beta^2)} \quad (12)$$

$$Q_1 = \frac{1+e^{2L(\alpha+i\beta)}}{4e^{L(\alpha+i\beta)}}; Q_2 = \frac{1+e^{2L(\alpha-i\beta)}}{4e^{L(\alpha-i\beta)}}; Q_3 = \frac{1-e^{2L(\alpha+i\beta)}}{4ie^{L(\alpha+i\beta)}}; Q_4 = \frac{1-e^{2L(\alpha-i\beta)}}{4ie^{L(\alpha-i\beta)}} \quad (13)$$

$$Q_{1p2} = (Q_1 + Q_2), Q_{1m2} = (Q_1 - Q_2), Q_{3p4} = (Q_3 + Q_4), Q_{3m4} = (Q_3 - Q_4) \quad (14)$$

$$S_1 = (2 + F_5Q_{1p2} + F_2Q_{1m2}); S_2 = [2(1 - Q_{1p2}) + F_1Q_{1m2}] \quad (15)$$

$$S_3 = (\alpha Q_{3p4} + \beta Q_{3m4}); S_4 = (\alpha Q_{3m4} - \beta Q_{3p4}) \quad (16)$$

Four degenerate cases spectral frequency equations can be easily obtained from Equation (11)as follows:

(1) For $R = 0$and $T = 0$, we get the case of cantilever beam for which we obtain

$$(F_5Q_{1p2} + F_2Q_{1m2} + 2) = 0 \quad (17)$$

(2) For $R = 0$and $T = \infty$, we get the case ofabeamclamped at one end and simply-supported at the other end for which we obtain

$$(\alpha Q_{3m4} - \beta Q_{3p4}) = 0 \quad (18)$$

(3) For $R = \infty$and $T = \infty$, we get the clamped at both ends of the beam case for which we obtain

$$[2(1 - Q_{1p2}) + F_1Q_{1m2}] = 0 \quad (19)$$

(4) For $R = \infty$and $T = 0$, we get the case of a beam clamped at one end and guided at the other end of the beam for which we obtain

$$\left(\alpha Q_{3p4} + \beta Q_{3m4}\right) = 0 \quad (20)$$

IV. Results and Discussions

CLAMPED –ELASTICALLY RESTRAINED THIN-WALLED BEAM

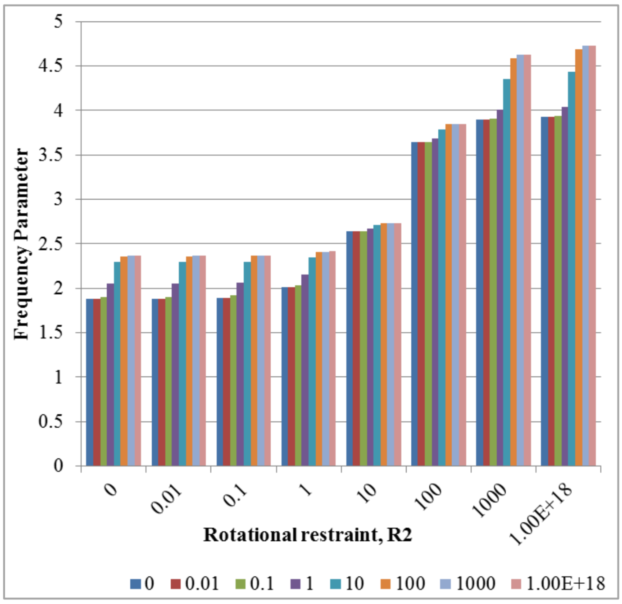
R1=10^18 and T1=10^18

Table1.First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter $K = 0.0$.

$R2$	$T2 = 0$	$T2 = 0.01$	$T2 = 0.1$	$T2 = 1$	$T2 = 10$	$T2 = 100$	$T2 = 1000$	$T2 = 10^{18}$
0	1.8751	1.8766	1.8901	2.0100	2.6389	3.6405	3.8978	3.9266
0.01	1.8780	1.8795	1.8928	2.0121	2.6393	3.6411	3.8990	3.9279
0.1	1.9022	1.9036	1.9163	2.0304	2.6426	3.6456	3.9101	3.9398
1	2.0540	2.0550	2.0641	2.1491	2.6662	3.6818	4.0042	4.0418
10	2.2912	2.2917	2.2970	2.3470	2.7147	3.7888	4.3562	4.4303
100	2.3564	2.3569	2.3613	2.4037	2.7310	3.8403	4.5845	4.6853
1000	2.3641	2.3646	2.3689	2.4104	2.7330	3.8475	4.6205	4.7253
10^{18}	2.3650	2.3655	2.3698	2.4112	2.7332	3.8483	4.6247	4.7300

Table2.First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter $K = 0.01$.

$R2$	$T2 = 0$	$T2 = 0.01$	$T2 = 0.1$	$T2 = 1$	$T2 = 10$	$T2 = 100$	$T2 = 1000$	$T2 = 10^{18}$
0	1.8751	1.8766	1.8901	2.0100	2.6389	3.6405	3.8978	3.9266
0.01	1.8780	1.8795	1.8929	2.0122	2.6393	3.6411	3.8991	3.9280
0.1	1.9022	1.9036	1.9163	2.0304	2.6426	3.6456	3.9101	3.9398
1	2.0540	2.0550	2.0642	2.1491	2.6662	3.6818	4.0042	4.0418
10	2.2912	2.2918	2.2970	2.3470	2.7147	3.7888	4.3562	4.4303
100	2.3564	2.3569	2.3689	2.4037	2.7310	3.8403	4.5845	4.6853
1000	2.3642	2.3646	2.3698	2.4104	2.7330	3.8475	4.6205	4.7253
10^{18}	2.3650	2.3655	2.3698	2.4112	2.7332	3.8483	4.6247	4.7300



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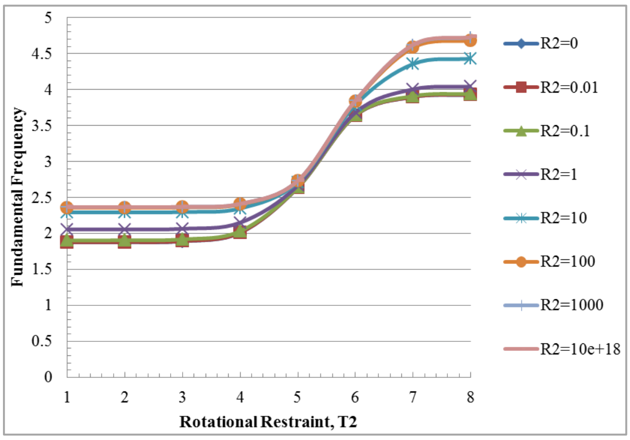

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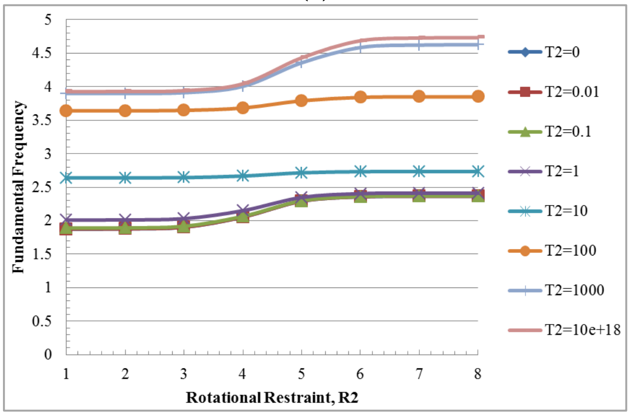

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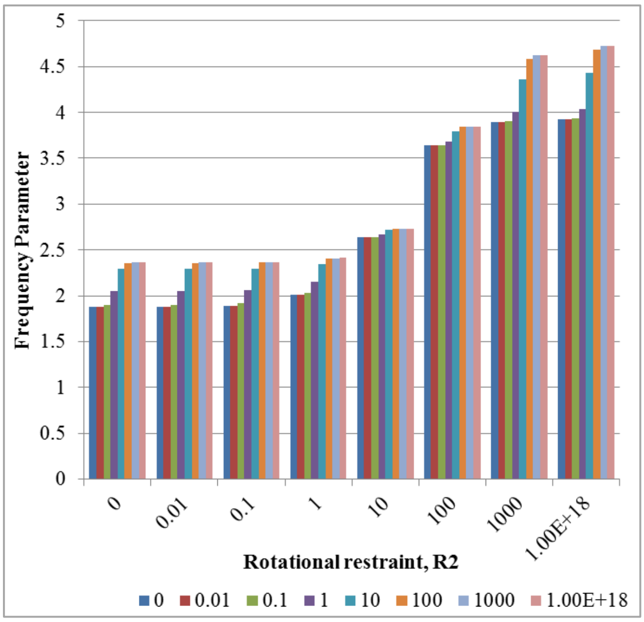
Figure 2. (a), (b) and (c). Variation of frequency parameter with elastic restraints (R2 &T2=0 to 10^{18}) for a given Warping parameter (K=0).



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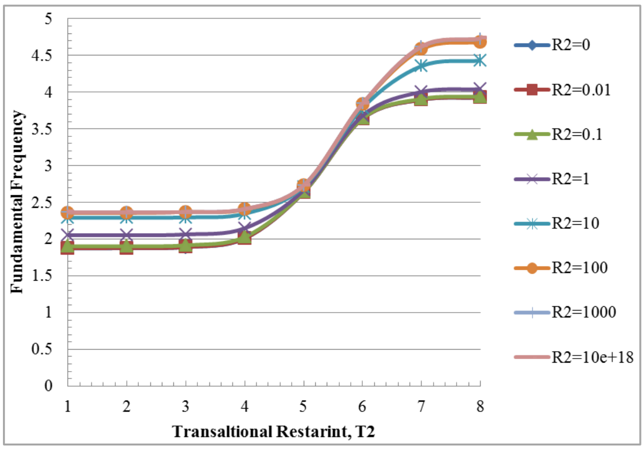

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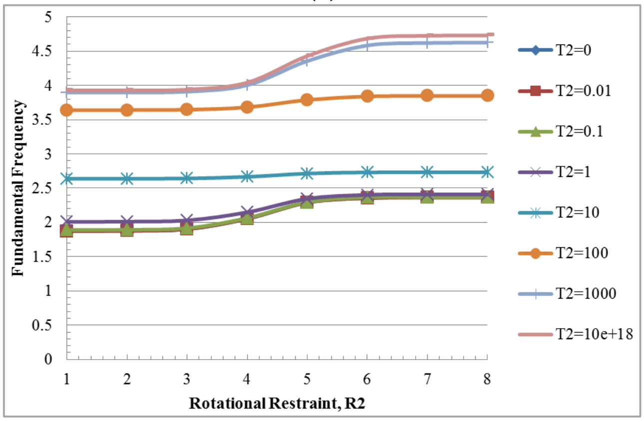
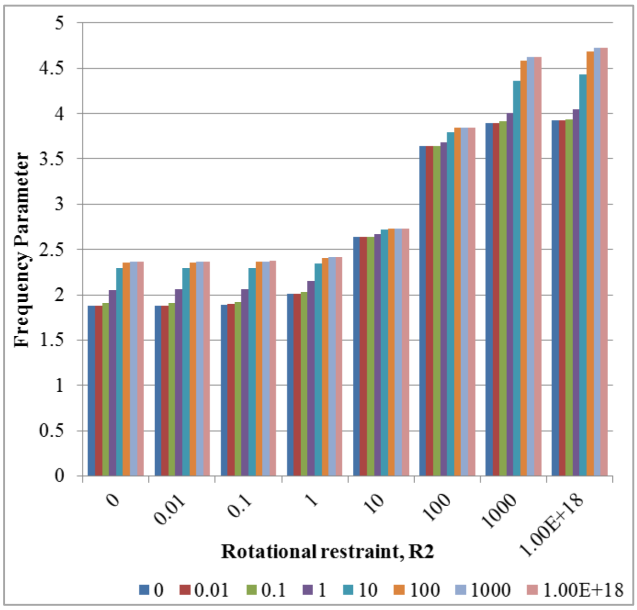


(c)

Figure 3. (a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=0.01).

Table3.First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter $K = 0.1$.

$R2$	$T2 = 0$	$T2 = 0.01$	$T2 = 0.1$	$T2 = 1$	$T2 = 10$	$T2 = 100$	$T2 = 1000$	$T2 = 10^{18}$
0	1.8769	1.8784	1.8918	2.0114	2.6394	3.6408	3.8983	3.9271
0.01	1.8797	1.8812	1.8945	2.0135	2.6398	3.6413	3.8995	3.9284
0.1	1.9038	1.9052	1.9179	2.0317	2.6431	3.6458	3.9105	3.9403
1	2.0551	2.0561	2.0653	2.1501	2.6667	3.6821	4.0046	4.0423
10	2.2918	2.2924	2.2976	2.3477	2.7151	3.7890	4.3565	4.4306
100	2.3570	2.3575	2.3619	2.4042	2.7313	3.8405	4.5848	4.6856
1000	2.3647	2.3652	2.3695	2.4110	2.7333	3.8476	4.6207	4.7256
10^{18}	2.3656	2.3661	2.3703	2.4117	2.7336	3.8485	4.6250	4.7303



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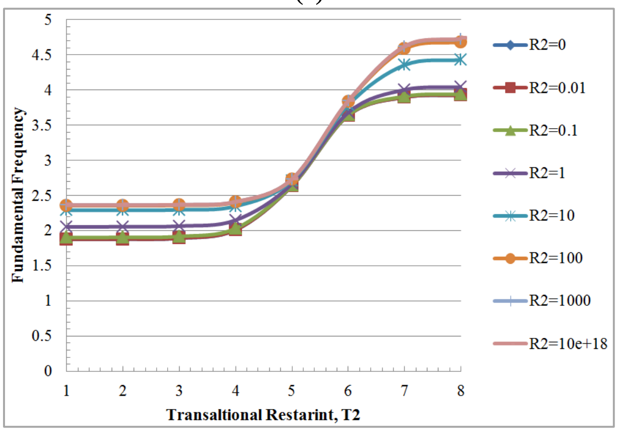
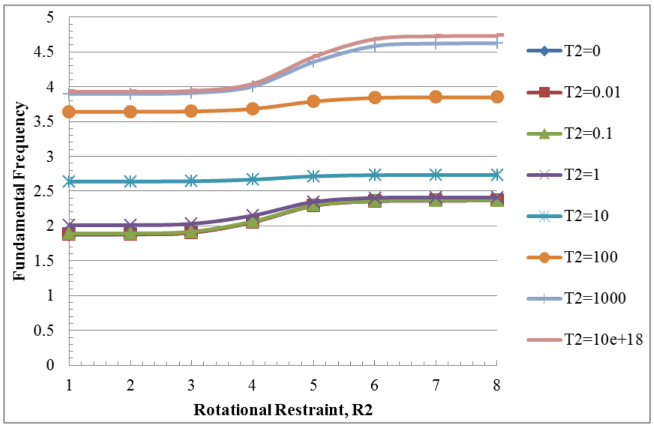




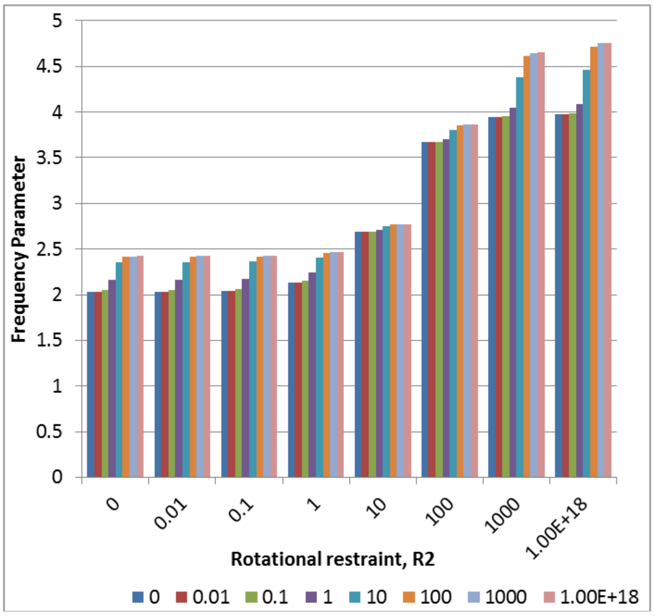
(b)(c)

Figure 4. (a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=0.1).

Table4.First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter $K = 1.0$.

$R2$	$T2 = 0$	$T2 = 0.01$	$T2 = 0.1$	$T2 = 1$	$T2 = 10$	$T2 = 100$	$T2 = 1000$	$T2 = 10^{18}$
0	2.0274	2.0285	2.0389	2.1337	2.6854	3.6683	3.9420	3.9733
0.01	2.0294	2.0305	2.0408	2.1353	2.6858	3.6683	3.9432	3.9746
0.1	2.0463	2.0474	2.0574	2.1487	2.6885	3.6725	3.9537	3.9860
1	2.1590	2.1599	2.1676	2.2401	2.7086	3.7059	4.0441	4.0842
10	2.3551	2.3556	2.3604	2.4064	2.7517	3.8061	4.3857	4.4626

100	2.4132	2.4137	2.4177	2.4571	2.7668	3.8553	4.6104	4.7144
1000	2.4202	2.4206	2.4246	2.4633	2.7687	3.8622	4.6460	4.7541
10^{18}	2.4210	2.4214	2.4254	2.4640	2.7689	3.8630	4.6502	4.7588



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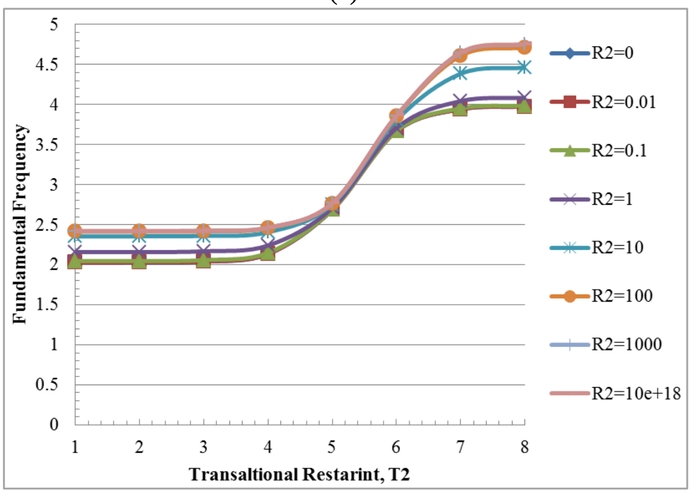

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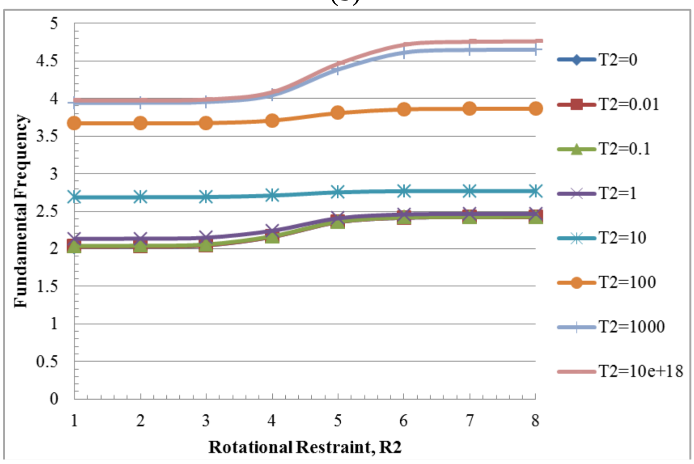

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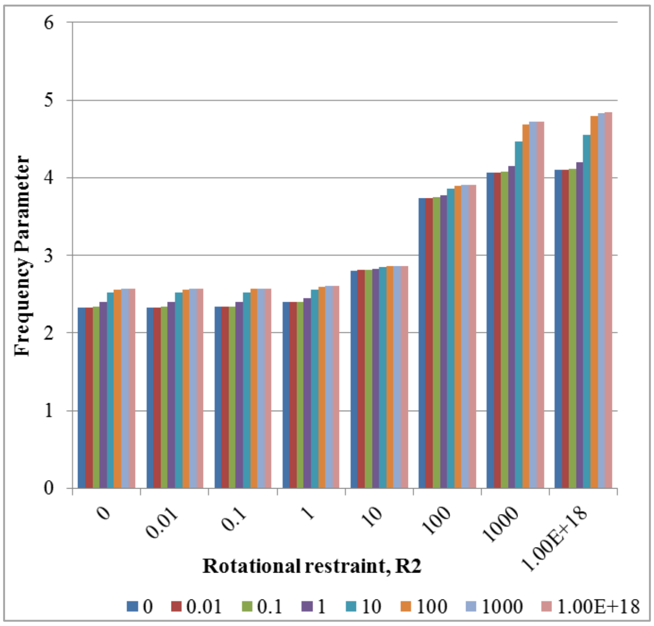
Figure 5. (a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=1).

Table5.First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter K= 2.0.

$R2$	$T2 = 0$	$T2 = 0.01$	$T2 = 0.1$	$T2 = 1$	$T2 = 10$	$T2 = 100$	$T2 = 1000$	$T2 = 10^{18}$
0	2.3283	2.3290	2.3353	2.3953	2.8073	3.7422	4.0649	4.1038
0.01	2.3292	2.3299	2.3362	2.3961	2.8075	3.7425	4.0660	4.1049
0.1	2.3372	2.3379	2.3441	2.4029	2.8092	3.7458	4.0753	4.1152
1	2.3952	2.3958	2.4012	2.4525	2.8224	3.7723	4.1558	4.2037
10	2.5187	2.5191	2.5230	2.5601	2.8539	3.8555	4.4700	4.5554
100	2.5618	2.5622	2.5655	2.5983	2.8662	3.8987	4.6848	4.7983
1000	2.5672	2.5676	2.5709	2.6031	2.8678	3.9049	4.7195	4.8374
10^{18}	2.5678	2.5682	2.5715	2.6036	2.8680	3.9056	4.7235	4.8420



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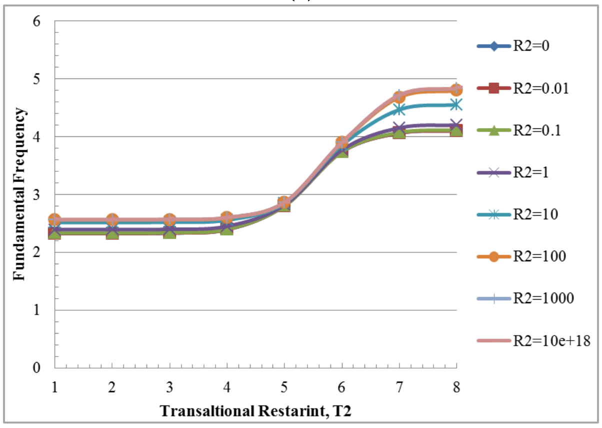
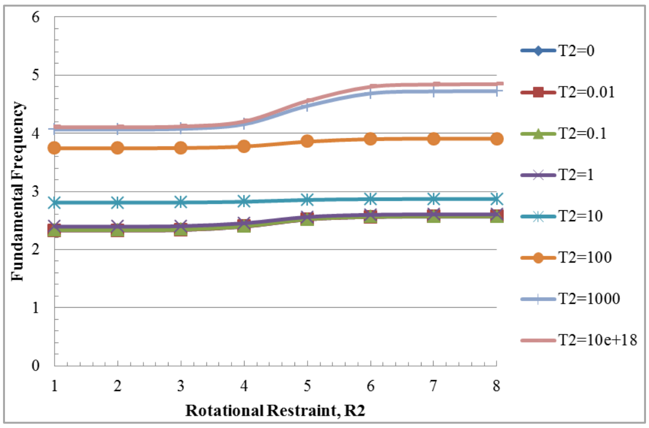


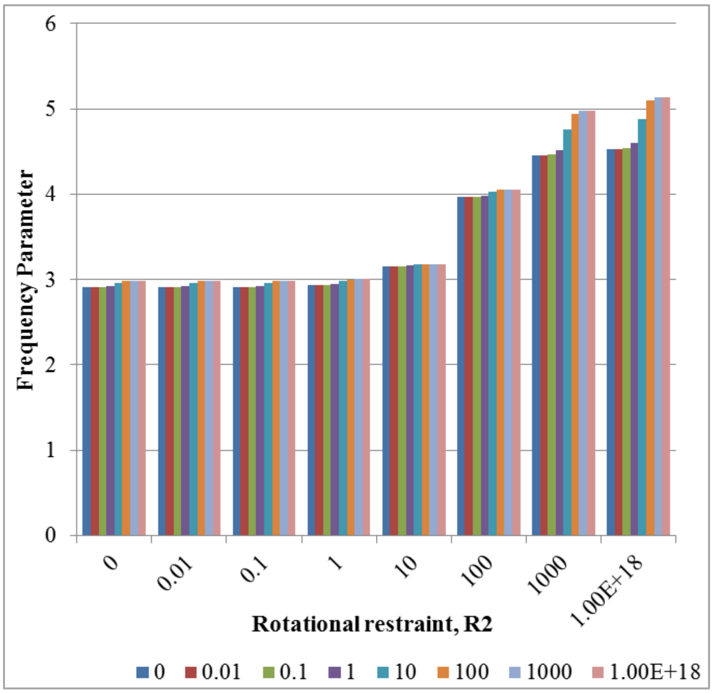


(b) (c)

Figure 6. (a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=2).

Table6.First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter K= 4.0.

$R2$	$T2 = 0$	$T2 = 0.01$	$T2 = 0.1$	$T2 = 1$	$T2 = 10$	$T2 = 100$	$T2 = 1000$	$T2 = 10^{18}$
0	2.9057	2.9060	2.9087	2.9352	3.1564	3.9670	4.4550	4.5263
0.01	2.9059	2.9062	2.9089	2.9354	3.1564	3.9671	4.4557	4.5271
0.1	2.9075	2.9078	2.9105	2.9368	3.1569	3.9686	4.4620	4.5344
1	2.9204	2.9207	2.9232	2.9485	3.1611	3.9809	4.5178	4.5986
10	2.9593	2.9596	2.9618	2.9837	3.1740	4.0253	4.7567	4.8774
100	2.9786	2.9789	2.9809	3.0012	3.1807	4.0526	4.9414	5.0943
1000	2.9813	2.9816	2.9836	3.0037	3.1816	4.0568	4.9731	5.1312
10^{18}	2.9817	2.9819	2.9839	3.0040	3.1817	4.0573	4.9768	5.1356



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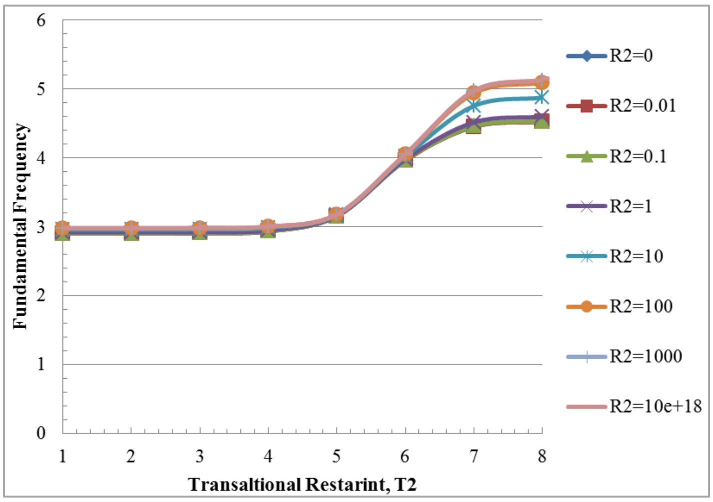

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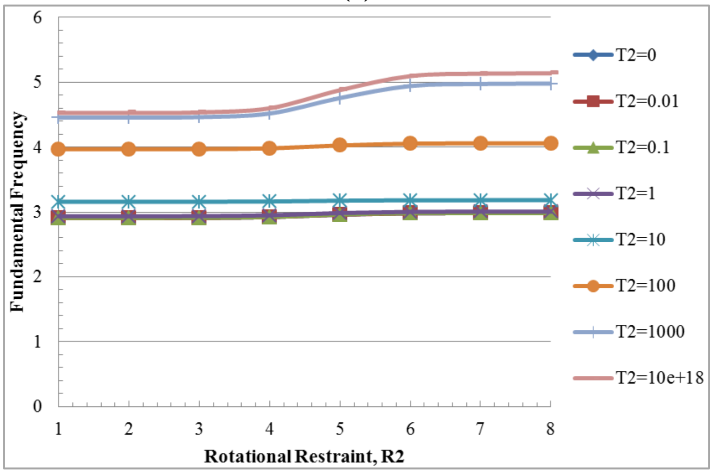


(c)

Figure 7. (a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=4).

Table7. (5)First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter $K = 10.0$.

$R2$	$T2 = 0$	$T2 = 0.01$	$T2 = 0.1$	$T2 = 1$	$T2 = 10$	$T2 = 100$	$T2 = 1000$	$T2 = 10^{18}$
0	4.1965	4.1966	4.1973	4.2043	4.2715	4.7462	5.7208	6.0483
0.01	4.1965	4.1966	4.1973	4.2043	4.2715	4.7462	5.7210	6.0486
0.1	4.1966	4.1966	4.1973	4.2043	4.2715	4.7463	5.7227	6.0512
1	4.1971	4.1972	4.1979	4.2049	4.2718	4.7470	5.7384	6.0754
10	4.1999	4.2000	4.2007	4.2075	4.2732	4.7508	5.8251	6.2091
100	4.2027	4.2028	4.2035	4.2101	4.2746	4.7548	5.9259	6.3648
1000	4.2033	4.2033	4.2040	4.2107	4.2749	4.7556	5.9479	6.3984
10^{18}	4.2033	4.2034	4.2041	4.2107	4.2750	4.7557	5.9506	6.4026

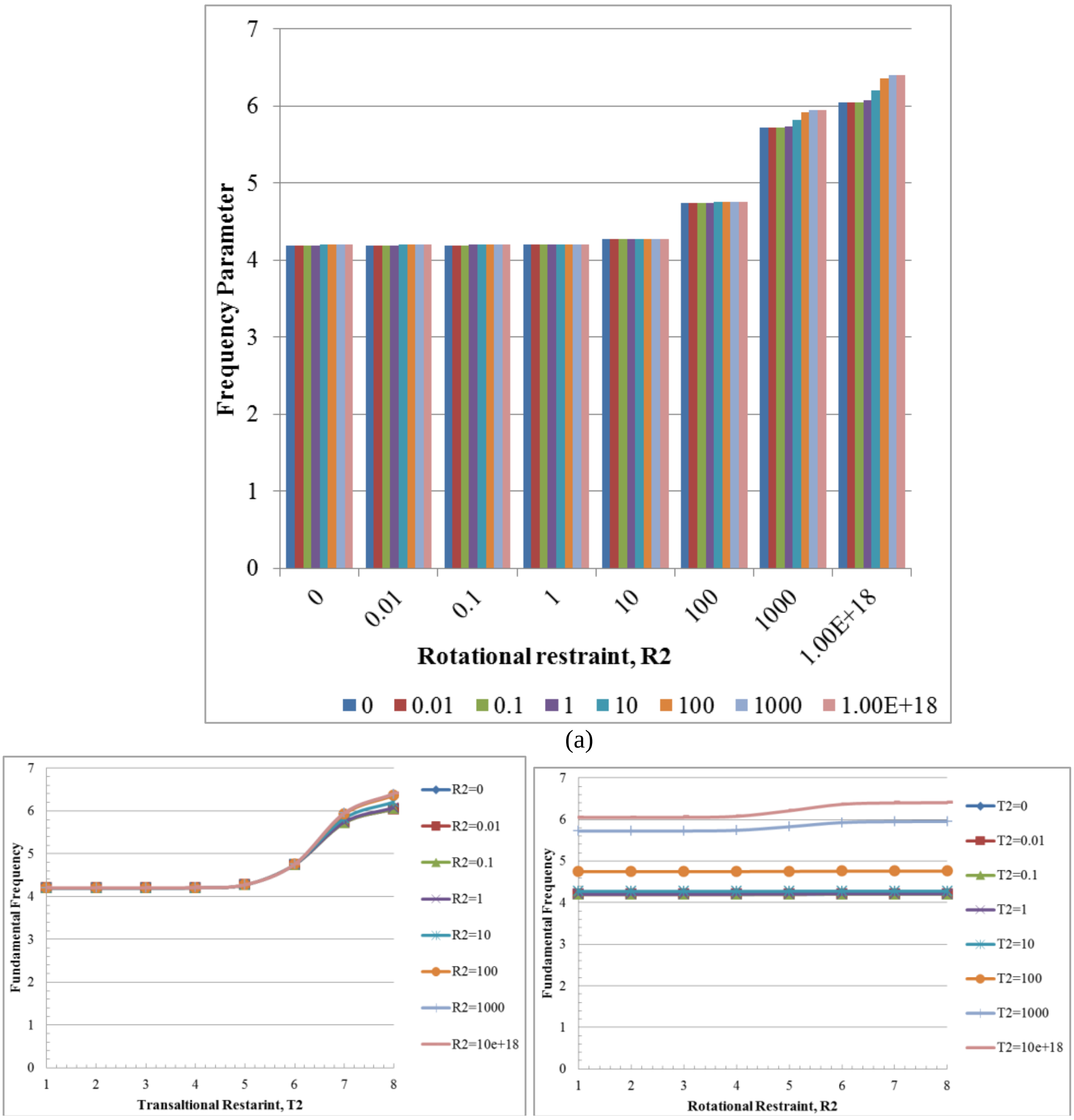


Figure 8. (a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=10).

Table8. (6)First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter K= 20.0.

$R2$	$T2=0$	$T2=0.01$	$T2=0.1$	$T2=1$	$T2=10$	$T2=100$	$T2=1000$	$T2=10^{18}$
0	5.7585	5.7585	5.7588	5.7613	5.7861	6.0080	7.0717	8.1827
0.01	5.7585	5.7585	5.7588	5.7613	5.7861	6.0080	7.0717	8.1829
0.1	5.7585	5.7585	5.7588	5.7613	5.7861	6.0080	7.0720	8.1838
1	5.7586	5.7586	5.7588	5.7613	5.7861	6.0080	7.0745	8.1927
10	5.7589	5.7589	5.7591	5.7616	5.7863	6.0081	7.0915	8.2541
100	5.7594	5.7594	5.7596	5.7621	5.7867	6.0083	7.1226	8.3673
1000	5.7595	5.7595	5.7598	5.7623	5.7868	6.0083	7.1321	8.4020
10^{18}	5.7595	5.7596	5.7598	5.7623	5.7868	6.0083	7.1334	8.4067

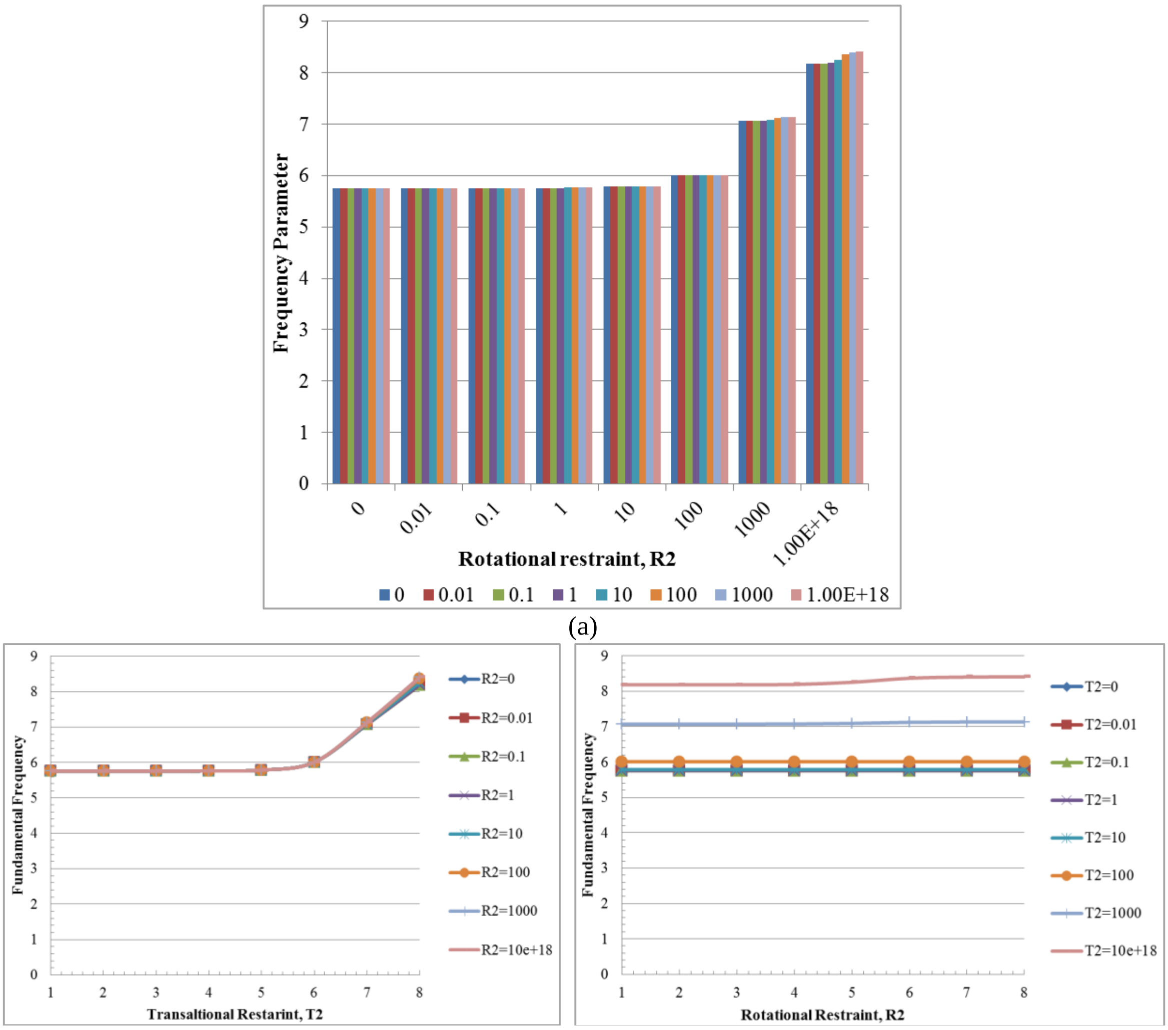
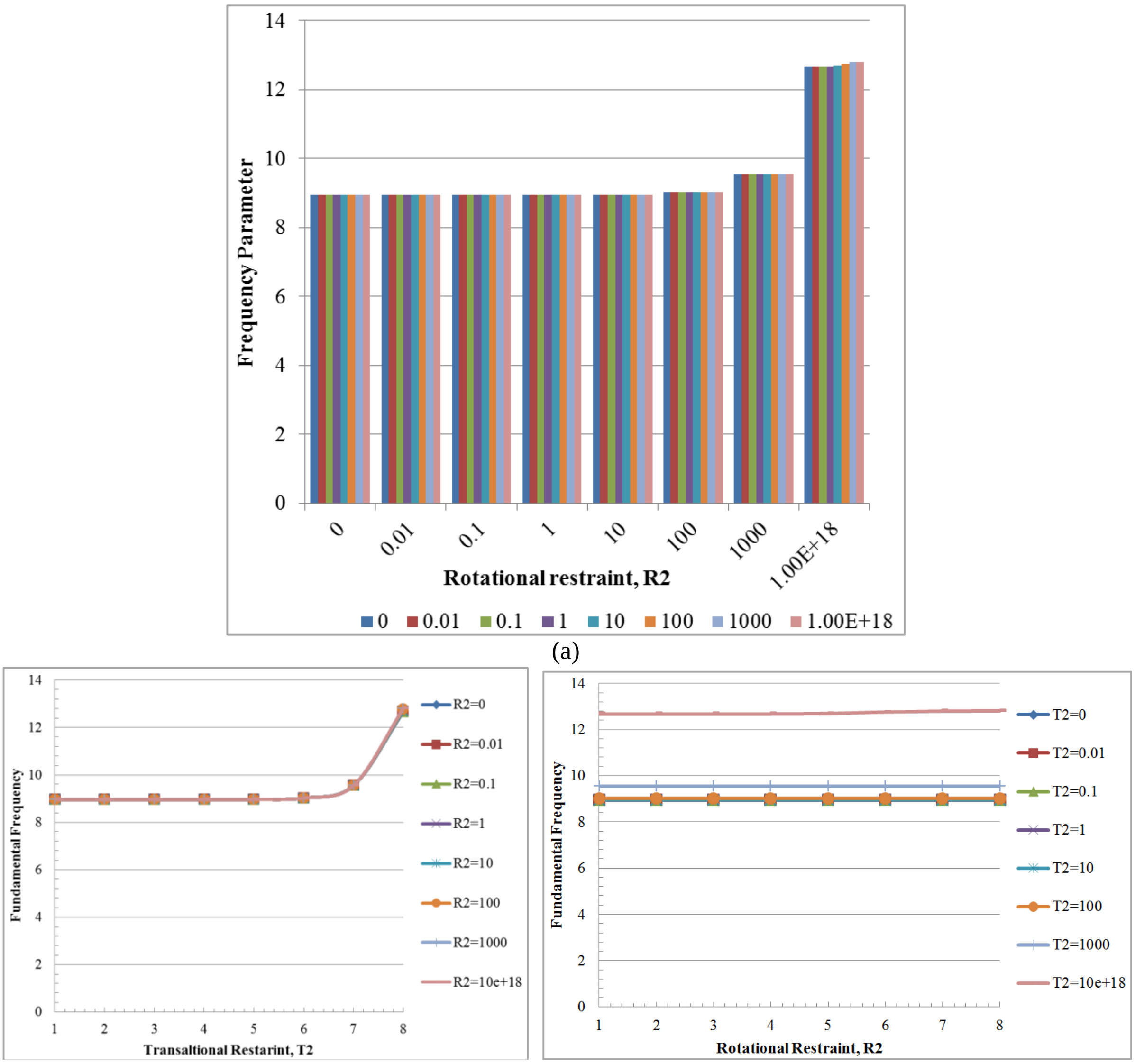


Figure 9. (a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=20).

Table9. (7)First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter K= 50.0.

$R2$	$T2 = 0$	$T2 = 0.01$	$T2 = 0.1$	$T2 = 1$	$T2 = 10$	$T2 = 100$	$T2 = 1000$	$T2 = 10^{18}$
0	8.9544	8.9544	8.9544	8.9551	8.9615	9.0242	9.5508	12.6729
0.01	8.9544	8.9544	8.9544	8.9551	8.9615	9.0242	9.5508	12.6730
0.1	8.9544	8.9544	8.9544	8.9551	8.9615	9.0242	9.5508	12.6732
1	8.9544	8.9544	8.9544	8.9551	8.9615	9.0242	9.5509	12.6755
10	8.9544	8.9544	8.9545	8.9551	8.9615	9.0242	9.5513	12.6945
100	8.9544	8.9544	8.9545	8.9551	8.9615	9.0242	9.5527	12.7600
1000	8.9545	8.9545	8.9545	8.9552	8.9616	9.0242	9.5535	12.7979
10^{18}	8.9545	8.9545	8.9545	8.9552	8.9616	9.0242	9.5536	12.8043



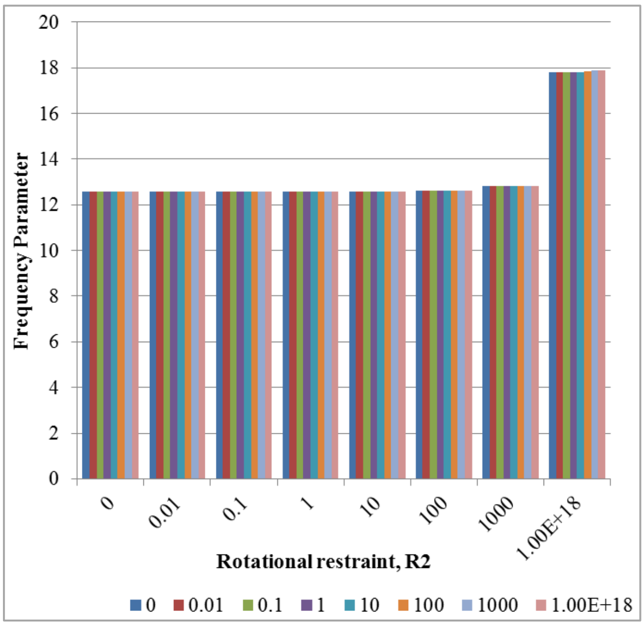
(a)

(b) (c)

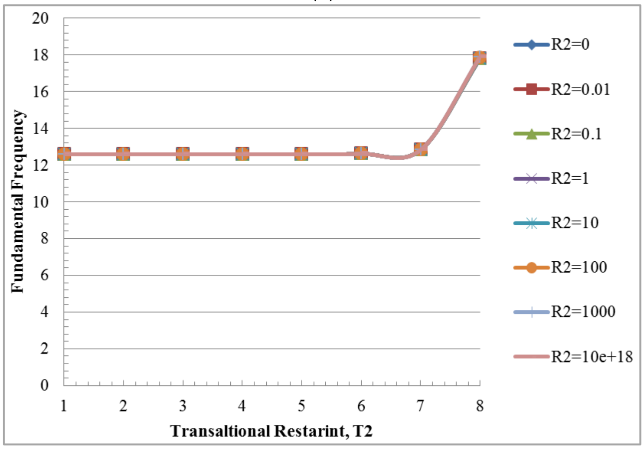
Figure 10. (a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=50).

Table10. (8)First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter K= 100.0.

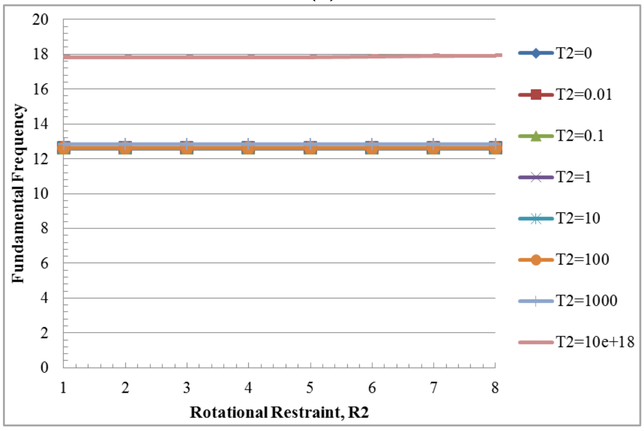
$R2$	$T2 = 0$	$T2 = 0.01$	$T2 = 0.1$	$T2 = 1$	$T2 = 10$	$T2 = 100$	$T2 = 1000$	$T2 = 10^{18}$
0	12.5970	12.5970	12.5971	12.5973	12.5996	12.6222	12.8378	17.8182
0.01	12.5970	12.5970	12.5971	12.5973	12.5996	12.6222	12.8378	17.8182
0.1	12.5970	12.5970	12.5971	12.5973	12.5996	12.6222	12.8378	17.8183
1	12.5970	12.5970	12.5971	12.5973	12.5996	12.6222	12.8378	17.8191
10	12.5970	12.5970	12.5971	12.5973	12.5996	12.6222	12.8378	17.8264
100	12.5971	12.5971	12.5971	12.5973	12.5996	12.6222	12.8379	17.8634
1000	12.5971	12.5971	12.5971	12.5973	12.5996	12.6222	12.8379	17.9006
10^{18}	12.5971	12.5971	12.5971	12.5973	12.5996	12.6222	12.8380	17.9089



(a)



(b)



(c)

Figure 11. (a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=100).

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