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Roll Position Demand Autopilot Design using State Feedback and Reduced Order Observer (DGO)

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Abstract— A Roll Position demand (as well as Roll Position Control) missile autopilot design methodology for a class of guided missile, based on state feedback, Ackermann pole-placement and reduced order Das & Ghosal observer (DGO), is proposed. The open loop unstable model of the roll control (or roll position demand) autopilot has been stabilized by using pole placement and state feedback. The non-minimum phase feature of rear controlled missile airframes is analyzed. Actuator dynamics has also been included in this design to make the overall system practically suitable for use. The overall responses of the Roll Control autopilot has been significantly improved over the frequency domain design approach where phase lag & phase lead compensator were used. Closed loop system poles are selected on the basis of desired time domain performance specifications. This set of roots not only to ensure the system damping, but also to make sure that the system can be fast. Reduced order Das & Ghosal (DGO) observer is implemented successfully in this design to estimate the Aileron angle and its rate. Finally a numerical example is considered and the simulated results are discussed in details. The date set has been chosen here for which largest rolling moment would occur (at Mach No = 4) due to unequal incidence in pitch and yaw.

Keywords— Roll Position & Roll Rate Control Autopilot, Ailerons-Rollerons, Roll Gyro, Fin Servo Actuator, Aerodynamic control, Luenberger Observer, Das & Ghosal Observer, Generalized Matrix Inverse and Ackermann.

I. INTRODUCTION

The fundamental objective of autopilot design for missile systems is to provide stability with satisfactory performance and robustness over the whole range of flight conditions throughout the entire flight envelope that missiles are required to operate in, during all probable engagements. **Roll autopilot** is used to eliminate roll Angle error caused by disturbances, and maintain stable relationship of the body coordinate system and other relative coordinates, and avoid chaos of pitch and yaw signal..

Wang Lei, Meng Xiu-yun, Xia Qun-li and Guo-Tao **[1]** have proposed a Roll autopilot designing method with Pole placement Method and State-Observer Feedback. State feedback gain via pole placement is designed by selecting a suitable root of closed-loop transfer function. With pole-placement method full-order state observer with one or two measurable outputs are designed respectively. In this literature, taking the integral initial values disturbance of roll angle as an example the authors have illustrated that in order to stabilize the missile's roll angle position, not only roll angle error should be eliminated, but also the transition process of good quality is required. The quality of control system mainly depends on the distributive position of the system poles in the root plane. Pole-placement is to select state-feedback gain, and make the system poles located in expected position in root plane, so as to achieve an appropriate damping coefficient and undamped natural frequency. Only when the state parameters for feedback are observable, the pole placement method can be used. When the state parameters used for feedback are not observable or requirement of a certain filter properties, a state observer should be designed. They focused on the application of the pole placement used to design autopilot feedback gain and to design the state observer. In the design of the feedback gain, the focus is to select a set of good performance roots on the root locus. This set of roots not only to ensure the system damping, but also to make sure that the system can be fast. Through the state observer's responses it can be seen that, when all the intermediate variables of the system are immeasurable, the observer can be designed with only one measurable output γ (Roll angle). Although the observer is the estimated means taken only when the intermediate state cannot be measured, we still can construct the intermediate state of the original system. It is concluded to the available rule: When γ and $\dot{\gamma}$ of the system can be measured, the system outputs of the observer with two measurable outputs can be much more accurate than that with one measurable output.

In literature **[2] & [3],** detailed design of classical two loop flight path rate demand autopilot is given. Here the accelerometer provides the main output (flight path rate) feedback and the rate gyro enables the body rate feedback (inner loop) thus resulting in two loops. The authors presented three different design situations of two loop lateral autopilot for a class of guided missile. Frequency domain approach had been taken in those papers. In conventional two loop autopilot system there is no provision for direct control over the missile body rate. However, Tactical homing missiles require explicit control on body rate. Three such specific requirements are a) body rate generation not to exceed predetermined maximum; b) body acceleration limit; and c) it could produce moderate actuator rates without rate saturation for sudden pitch rate demands. In literature **[4]**, authors modified the design of two loop lateral autopilot and proposed an additional rate gyro feedback to be applied at the input of an integrating amplifier block which integrates the body rate (i.e. pitch rate here) error to obtain direct control over the missile body rate. This enhanced model is referred to as Three Loop Lateral Autopilot. The three loop autopilot has a larger dc gain and a relatively small high frequency gain compared to a two-loop autopilot. This feature effectively improves the steady state performance and loop stiffness as well as reduces the initial negative peak of the time response. The three-loop autopilot attempts to reduce the adverse effect of non-minimum phase zeros. In reference **[5],** Prof. G. Das and T. K. Ghosal have derived a new method of reduced order observer construction based on Generalized Matrix Inverse theory **[16]** which possesses some certain advantages over the well known and well-established Luenberger observer **[12] & [13].** In paper **[6] & [7],** the present authors discussed about the intricate details of Luenberger observer and Das & Ghosal (DGO) observer. They have also carried out an exhaustive comparison (structure-wise and performance-wise) between the two observers. In literature **[11]**, Lin Defu, Fan Junfang, Qi Zaikang and Mou Yu have proposed a modification of classical three-loop lateral autopilot design using frequency domain approach. Also they have done performance and robustness comparisons between the two-loop and classical three-loop topologies.

The unique contribution of this paper is the modified design technique of Roll Position Demand (as well as Roll Position Control) autopilot in state space domain by reduced order Das & Ghosal observer based Ackermann state feedback. Full state feedback has been used to make the system stability better and to regulate all the states. Actuator dynamics which is neglected in **[1]** has also been taken in to account in this design. A very fast servo actuator is considered here so that the demanded roll position can be achieved quickly or it can be made zero in case of Roll Position Control autopilot. The system is designed in a very effective manner such that there is no steady state error in the output thus it can follow the commanded input exactly. More over the overshoots occurring in the Roll position response in case of frequency domain **[14]** design approach has been fully eliminated along with the maximum Roll rate has also come down significantly. It has been established through this paper that the aileron deflections and their rates are also reduced to a great extent which poses major issues of concern regarding structural failure of missiles.

II. STATE OBSERVER

To implement state feedback control [control law is given **by** $\boldsymbol{u} = \boldsymbol{r} - \boldsymbol{K\hat{x}}$ ... ... $(\mathbf{2.1})$] by pole placement, all the state variables are required to be feedback. However, in many practical situations, all the states are not accessible for direct measurement and control purposes; only inputs and outputs can be used to drive a device whose outputs will estimate the state vector. This device (or computer program) is called State Observer. Intuitively the observer should have the similar state equations as the original system (i.e. plant) and design criterion should be to minimize the difference between the system output $\boldsymbol{y} = \boldsymbol{Cx}$ and the output $\hat{\boldsymbol{y}} = \boldsymbol{C\hat{x}}$ as constructed by the observed state vector $\hat{\boldsymbol{x}}$. This is equivalent to minimization of $\boldsymbol{x} - \hat{\boldsymbol{x}}$. Since $\boldsymbol{x}$ is inaccessible, $\boldsymbol{y} - \hat{\boldsymbol{y}}$ is tried to be minimized. The difference $(\boldsymbol{y} - \hat{\boldsymbol{y}})$ is multiplied by a gain matrix (denoted by **M**) of proper dimension and feedback to the input of the observer. There are two well-known observers namely – **Luenberger Observer [12] & [13]** and **Das & Ghosal Observer [5]** while the second one has some genuine advantages over the first one. Das & Ghosal Observer construction procedure is essentially based on the Generalized Matrix Inverse Theory and Linear space mathematics.

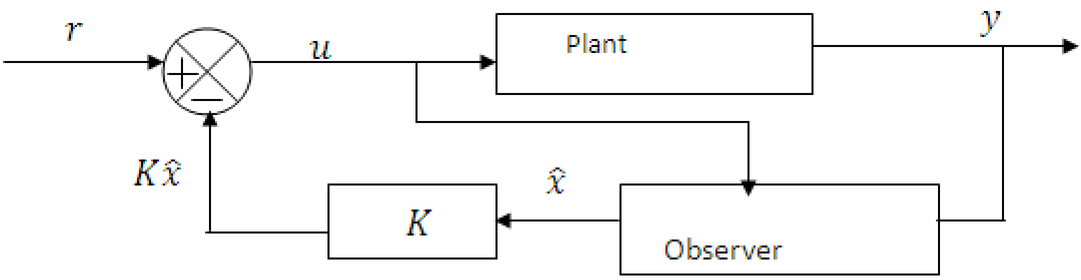


Fig - 2.1: General Block Diagram of an Observer based State Feedback Control System

III. DEVELOPMENT OF MODIFIED ROLL POSITION DEMAND AUTOPILOT FROM THE CONVENTIONAL ONE

The following block diagram **(fig. 3.1)** represents the transfer function model of a Roll Position Control Autopilot. The model has been taken from Garnell & East **[14].**

The transfer function model shown in **fig. 3.1** has been converted into state variable form. Das & Ghosal observer is applied to estimate the Aileron angle and its rate where as the Roll angle and the Roll rate are measured externally Position gyro and Roll gyro, respectively. Full state feedback is used instead of partial state feedback (as in case of transfer function model) to ensure effective pole-placement such that desired time domain performance can be achieved.

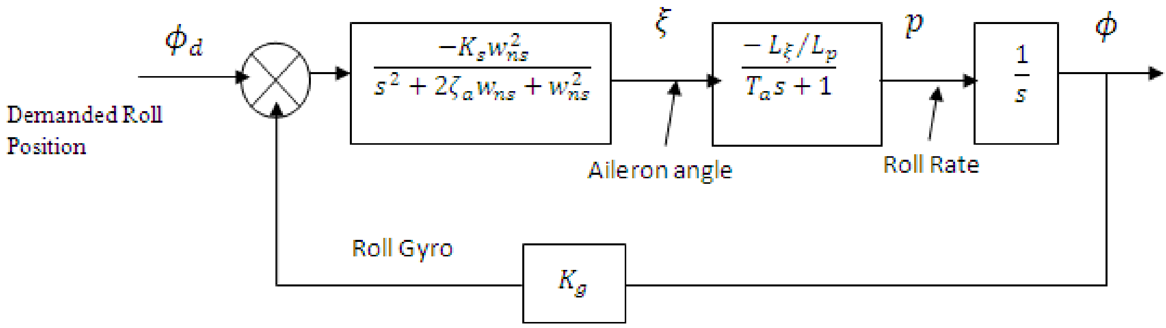


Fig - 3.1: Conventional Roll Position Control (or Roll Position Demand) Autopilot

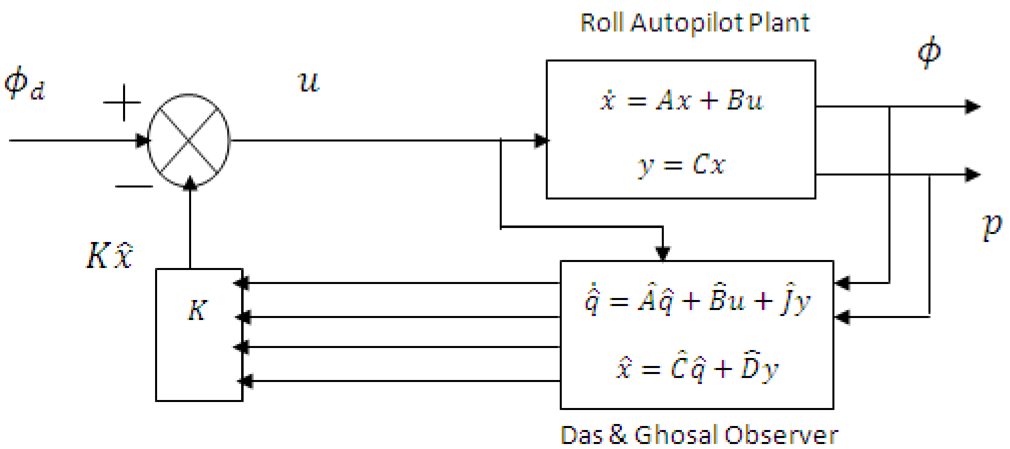


Fig – 3.2: State Feedback Design of Roll Control Autopilot using Das & Ghosal Observer (DGO)

<u>**Notations & Symbols used:**</u>

$\boldsymbol{\phi}$ *is Roll Angle*; $\boldsymbol{p}$ *is Roll rate*; $\boldsymbol{L_\xi}$ *is Roll Moment*;
$\boldsymbol{L_p}$ *is Damping Derivative in Roll* ;
$\boldsymbol{\xi}$ *is Aileron deflection angle*; $\dot{\boldsymbol{\xi}}$ *Rate of Aileron Deflection*; $\boldsymbol{T_a}$*is Airframe Time constant*;
$\boldsymbol{w_{ns}}$ *is natural frequency of Actuator*;
$\boldsymbol{\zeta_a}$ *is damping ratio of actuator*; $\boldsymbol{K_s}$ *is the Servo gain*;
$\boldsymbol{K_g}$*is Rate Gyro gain*;
$\boldsymbol{\phi_d}$*Demanded Roll Position*; $\boldsymbol{M}$ *is Mach Number*;
$\boldsymbol{A}$ *is Roll Moment of Inertia*;

IV. STATE VARIABLE MODELING OF ROLL POSITION DEMAND AUTOPILOT

The open loop model of Roll Position Autopilot is shown in fig. 4.1

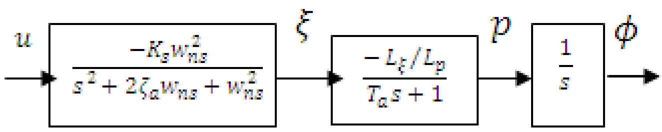


Fig – 4.1: Open Loop Model of Roll Position Control Autopilot

The above open loop model can be converted to state variable form based on the following four state variables:

$\boldsymbol{x_1} = \boldsymbol{\phi}$ (***Roll Angle***)
$\boldsymbol{x_2} = \boldsymbol{p}$ (***Roll Rate***)
$\boldsymbol{x_3} = \boldsymbol{\xi}$ (***Aileron Deflection***)
$\boldsymbol{x_4} = \dot{\boldsymbol{\xi}}$ (***Rate of Aileron Deflection***)

Out of these four state variables, $\boldsymbol{x_1}$ ***and*** $\boldsymbol{x_2}$ **have** been considered to be as outputs. Thus Roll Control Autopilot model is a SIMO (single input – multiple outputs) system. The state equations obtained are as follows:

$\dot{x}_1 = x_2$ ... ... (4.1)

$\dot{x}_2 = -\frac{1}{T_a} x_2 - \frac{L_\xi}{T_a L_p} x_3$ ... ... (4.2)

$\dot{x}_3 = x_4$ ... ... (4.3)

$\dot{x}_4 = -2\zeta_a w_{ns} x_4 - w_{ns}^2 x_3 - K_s w_{ns}^2 u$ ... ... (4.4)

So the A, B, C matrices take the forms as:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{T_a} & -\frac{L_\xi}{T_a L_p} & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -w_a^2 & -2\zeta_a w_a \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -K_s w_a^2 \end{bmatrix};$$

and $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ ... ... (4.5)

The combined system (fig. 3.2) is governed by the following equations:

$\dot{x} = Ax + Bu; \quad y = Cx$ ... ... (4.6)

$u = r - K\hat{x}$ ... ... (4.7)

V. REDUCED ORDER DAS & GHOSAL OBSERVER (DGO) – GOVERNING EQUATIONS

Reduced order Das and Ghosal observer **[5]** is governed by the following equations and conditions.

$x = C^g y + L h$ ... ... (5.1) (eqn. 13 of [5])

$\dot{h}(t) = L^g AL\, h(t) + L^g AC^g\, y(t) + L^g\, B\, u(t)$ ... ... (5.2) (eqn. 15 of [5])

$\dot{y} = CALh + CAC_g\, y + CB\, u$ ... ... (5.3) (eqn. 18 of [5])

$\dot{\hat{h}} = (L^g AL - MCAL)\hat{h} + (L^g AC^g - MCAC^g)y + (L^g - MCB)u + M\dot{y}$ ... ... (5.4) (eqn. 19 of [5])

$\dot{\hat{q}} = (L^g AL - MCAL)\hat{q} + \{(L^g AC^g - MCAC^g) + (L^g AL - MCAL)M\}y + (L^g - MCB)u$ ... ... (5.5) (eqn. 20 of [5])

$where\ \hat{q} = \hat{h} - My$ ... ... (5.6) (Page-374 of [5])

Equation (5.5) can be expressed in short form: $\dot{\hat{q}} = \hat{A}\hat{q} + \hat{J}y + \hat{B}u$ ... ... (5.7)

$And\ \ \hat{x} = L\hat{q} + (C^g + LM)y$ ... ... (5.8) (eqn. 21 of [5])
Equation (5.8) can also be expressed in short form: $\hat{x} = \hat{C}\hat{q} + \hat{D}y$ ... ... (5.9)

Then equation (4.6) can be rewritten by using eqns. (4.7) & (5.9) as:

$\dot{x} = \left(A - BK\hat{D}C\right)x + Br - BK\hat{C}\hat{q}$ ... ... (5.10)

Equation (5.7) can be rewritten by using eqns. (4.7) & (5.9) as:

$\dot{\hat{q}} = \left(\hat{J} - \hat{B}K\hat{D}\right).Cx + \hat{B}r + \left(\hat{A} - \hat{B}K\hat{C}\right)\hat{q}$ ... ... (5.11)

Combining eqns. (5.10) and (5.11) into a matrix, we get

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\hat{\boldsymbol{q}}} \end{bmatrix} = \begin{bmatrix} (\boldsymbol{A} - \boldsymbol{BK\hat{D}C}) & -\boldsymbol{BK\hat{C}} \\ (\hat{\boldsymbol{J}} - \hat{\boldsymbol{B}}\boldsymbol{K\hat{D}})\boldsymbol{C} & (\hat{\boldsymbol{A}} - \hat{\boldsymbol{B}}\boldsymbol{K\hat{C}}) \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \hat{\boldsymbol{q}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{B} \\ \hat{\boldsymbol{B}} \end{bmatrix} [\boldsymbol{r}] \ldots \ldots (\mathbf{5.12\ a})\, and$$

$$\boldsymbol{Y} = [\mathbf{1} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0}] \begin{bmatrix} \boldsymbol{x} \\ \hat{\boldsymbol{q}} \end{bmatrix} \ldots \ldots (\mathbf{5.12b})$$

$\boldsymbol{and\ \hat{x} = \hat{C}\hat{q} + \hat{D}y = \hat{C}\hat{q} + \hat{D}Cx} \dots\dots (\mathbf{5.12\ c})$.

VI. SIMULATION RESULTS

The following numerical data for a class of guided missile have been taken for Matlab simulation:

$T_a = 0.0257\ sec;\ L_\xi = -13500;\ L_p = -37.3$

$K_s = -1.48;\ K_g = 1;\ w_{ns} = 180\frac{rad}{sec};\ \zeta_a = 0.6;$

$M = 2.8;\ A = 0.96;$

Using these values, the open loop state space model of three-loop autopilot [given by eqns. (1) & (2)] becomes,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -38.9105 & -14083 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -32400 & -216\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ -47952\end{bmatrix}u \dots\dots (6.1\ a) and$$

$$y = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \dots\dots (6.1\ b)$$

Open loop poles of the system: $\boldsymbol{P} = [0 \quad -38.91 \quad -108 + j144 \quad -108 - j144]$

Desired closed loop system poles taken: $\boldsymbol{P_{desired}} = [-300 + j250 \quad -300 - j250 \quad -72 \quad -61]$

Desired observer poles taken: $\boldsymbol{Ob} = [-1740 \quad -1800]$

State Feedback Gain matrix is given by using **Ackermann's formula:**

$\boldsymbol{K} = [\,0.9918 \quad 0.0219 \quad -3.6971 \quad -0.0100]$

Observer Gain matrix is also obtained by using Ackermann's formula: $\boldsymbol{M} = \begin{bmatrix}0 & -0.2360\\ 0 & -169.1141\end{bmatrix}$

In the design of Das & Ghosal observer L matrix has been chosen from the linearly independent columns of $(I_{4\times4} - C^g C)$ matrix.

$$[I_{4\times4} - C^g C] = \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\ so\ the\ L\ matrix\ can\ be\ formed\ as\ L = \begin{bmatrix}0 & 0\\ 0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix};$$

Finally unit step simulation is done on the combined model (i.e. Roll Position Demand Autopilot + DGO + state feedback), given by matrix eqns. (5.12a), (5.12b) & (5.12c), by using Matlab software (version 7.12, 2012a) and the results obtained, are presented below:

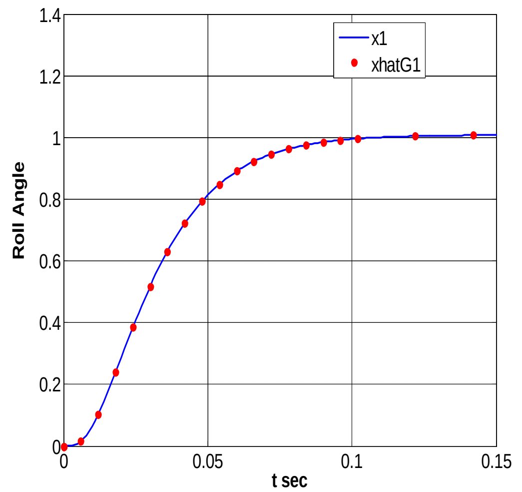


Figure 7.1: Roll Angle

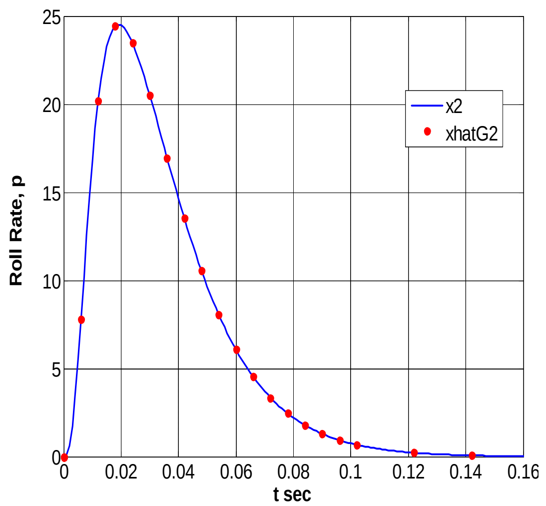
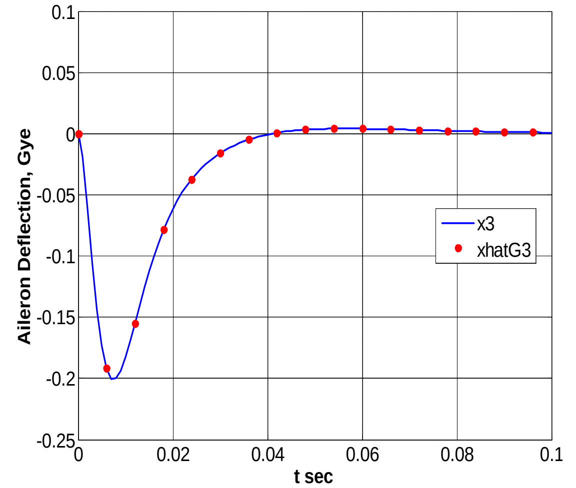


Figure 7.2: Roll Rate



Fiaure 7.3: Aileron Deflection

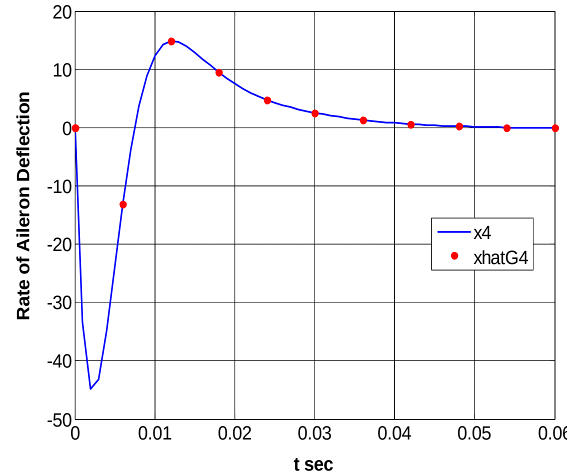


Figure 7.4: Rate of Aileron Deflection

N.B: In all of the above four graphs the **blue lines** indicate the original states of the Roll Position Demand Autopilot and the **red starred lines** indicate the estimated states by reduced order Das & Ghosal observer (DGO)

VII. OBSERVATIONS AND DISCUSSIONS

It can be readily inferred that the overall responses are improved and very much satisfactory. The Roll position demand is met exactly without any steady state error. No oscillations or peak is observed. Moreover within 0.1 second it has reached the steady level which is a very graceful performance. Aileron deflections are seen to be very low. It has also come down to zero within only 0.04 seconds. It is established through the simulation that Das & Ghosal observer has successfully caught the system states within less than 0.02 seconds and that also without any steady state error or oscillations. Further the experiments have also been carried out on the very well known and well used Luenberger observer **[12] & [13]** in place of Das & Ghosal **[5]** and it is seen that Luenberger observer will work only when the C matrix lies in the standard form $[I \ \vdots \ 0]$ otherwise not. Like here C matrix is in the standard form. Otherwise it will require an additional coordinate transformation to bring back the C matrix in the standard form and then only Luenberger observer will succeed. This is a one of the greatest advantages **[6], [7]** of Das & Ghosal observer over Luenberger. In the simulation and actual practice in the system it should be considered to combine with the difficulty of measuring the variables and the measurement accuracy requirements, in order to choose the proper root to meet the engineering requirements.

VIII. ACKNOWLEDGEMENT

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