International Journal of Engineering Research and Development
e-ISSN : 2278-067X, p-ISSN : 2278-800X, www.ijerd.com
Volume 2, Issue 6 (August 2012), PP. 01-07

Modified Design of Three Loop Lateral Missile Autopilot based on LQR and Reduced Order Observer (DGO)

Parijat Bhowmick[1], Prof. Gourhari Das[2]
[1,2]Dept. of Electrical Engg. (Control Systems), Jadavpur University, Kolkata, India

Abstract— A flight path rate demand modified three-loop lateral missile autopilot design methodology for a class of guided missile, based on state feedback, output feedback, reduced order Das & Ghosal observer (DGO) and Linear Quadratic Regulator (LQR), is proposed. The open loop undamped model of three-loop autopilot has been stabilized by using pole placement and state feedback. The non-minimum phase feature of rear controlled missile airframes is analyzed. The LQR technique is adopted to ensure optimal pole placement such that the adverse effect of non-minimum phase property on the response of missile autopilot can be reduced considerably and also the system stability can be made better. The overall response of the three-loop autopilot has been significantly improved over the previous ones. The flight path rate (as well as the lateral acceleration) has reached the commanded unit step input without compromising on its settling time thus it has been able to eliminate the steady state error. Performance (both transient and steady state) comparisons among the classical two loop design, three-loop design using frequency domain approach and the present one, are made. Reduced order Das & Ghosal observer is implemented successfully in this design. A program has been developed to find out the optimal pole positions and feedback gains according to the desired time domain performance specifications. Finally a numerical example has been considered and the simulated results are discussed in details.

Keywords— Three Loop Pitch Missile Autopilot , Non-minimum phase system, Angle of attack, Latex demand, Flight path rate demand loop, Rate Gyro, Accelerometers, Aerodynamic control, Luenberger Observer, Das & Ghosal Observer, Generalized Matrix Inverse, LQR, Ackermann.

I. INTRODUCTION

This paper is in continuation with the work reported in literature **[2]**. It deals with the modified three-loop lateral missile autopilot design methodology in pitch plane based on its state space model. In literature **[3] & [4],** detailed design of classical two loop flight path rate demand autopilot (fig. 2.1) is given. Here the accelerometer provides the main output (flight path rate) feedback and the rate gyro enables the body rate feedback (inner loop) thus resulting in two loops. The authors presented three different design situations of two loop lateral autopilot for a class of guided missile. Frequency domain approach had been taken in those papers. In conventional two loop autopilot system there is no provision for direct control over the missile body rate. However, Tactical homing missiles require explicit control on body rate. Three such specific requirements are a) body rate generation not to exceed predetermined maximum; b) body acceleration limit; and c) it could produce moderate actuator rates without rate saturation for sudden pitch rate demands. In literature **[5],** authors modified the design of two loop lateral autopilot and proposed an additional rate gyro feedback to be applied at the input of an integrating amplifier block (fig. 2.2) which integrates the body rate (i.e. pitch rate here) error to obtain direct control over the missile body rate. This enhanced model is referred to as Three Loop Lateral Autopilot. The three loop autopilot has a larger dc gain and a relatively small high frequency gain compared to a two-loop autopilot. This feature effectively improves the steady state performance and loop stiffness as well as reduces the initial negative peak of the time response. The three-loop autopilot attempts to reduce the adverse effect of non-minimum phase zeros. In reference **[6],** Prof. G. Das and T. K. Ghosal have derived a new method of reduced order observer construction based on Generalized Matrix Inverse theory **[15]** which possesses some certain advantages over the well known and well-established Luenberger observer **[12] & [13]**. In paper **[7] & [8],** the present authors discussed about the intricate details of Luenberger observer and Das & Ghosal (DGO) observer. They have also carried out an exhaustive comparison (structure-wise and performance-wise) between the two observers. In literature **[10]**, Lin Defu, Fan Junfang, Qi Zaikang and Mou Yu have proposed a modification of classical three-loop lateral autopilot design using frequency domain approach. Also they have done performance and robustness comparisons between the two-loop and classical three-loop topologies. In order to eliminate the static error, Hui WANG, De-fu LIN, Jiang WANG and Zhen-xuan CHENG **[9]** introduced PI compensator to the classical two-loop autopilot and the contribution of PI was analyzed while the equivalent time constant estimates of two-loop autopilot with PI was discussed. Four different objective functions were given to the design procedures. It is concluded that it is the large gain at low frequency of PI solved the problem of static error. He has also discussed the "Lever Effect" feature of autopilot dynamics in details. Tayfun Cimen has discussed in paper **[11]** the concept of *extended linearization* (also known as *state-dependent coefficient parameterization*) for *state-dependent nonlinear formulation* of the vehicle dynamics in a very general form for the development of a generic and practical autopilot design approach for missile flight control systems. Any *extended linearization control methods,* such as State-Dependent Riccati Equation (SDRE) methods, can then be applied to this state-dependent formulation for missile flight control system design. He has also discussed about different aspects of LQR based state feedback design since it generally gives good performance characteristics and stability margins, with the availability of the states required for implementation.

The unique contribution of this paper is the new design technique of classical three-loop lateral missile autopilot in state space domain by incorporating output feedback and reduced order Das & Ghosal observer (DGO) based LQR state feedback with it. It has been established through this paper that the initial negative peak occurring in the time response due to the non-minimum phase zeros, the reduction of which posed a major challenge so far in autopilot design, is reduced drastically with a minor change on its setting time. Another worth noting modification is achieved that the steady state value of flight path rate response has reached unity (in case of commanded unit step input), which was much below unity in case of classical three-loop design. This is very much important in case of intercepting missiles. More over the steady state performance is also improved much as compared to the classical three-loop design (described in table 6.1). A generalized program has been successfully developed and implemented to choose the proper weightage matrices required for LQR algorithm to calculate the optimal pole locations as per desired time domain performance specifications.

II. DEVELOPMENT OF MODIFIED THREE-LOOP AUTOPILOT FROM THE CONVENTIONAL ONE

The following block diagrams (fig. 2.1 & 2.2) represents the transfer function model of flight path rate demand two loop and three loop autopilot respectively in pitch plane **[3], [4] & [5].**

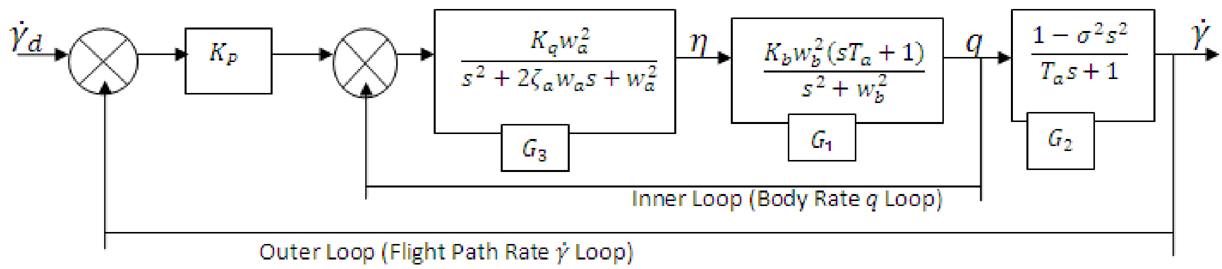


Fig-2.1: Conventional Flight Path Rate Demand Two Loop Autopilot Configuration (Transfer Function Model)

Where G_1 and G_2 are the Aerodynamic transfer functions; ***G_3 is the Actuator transfer function***;

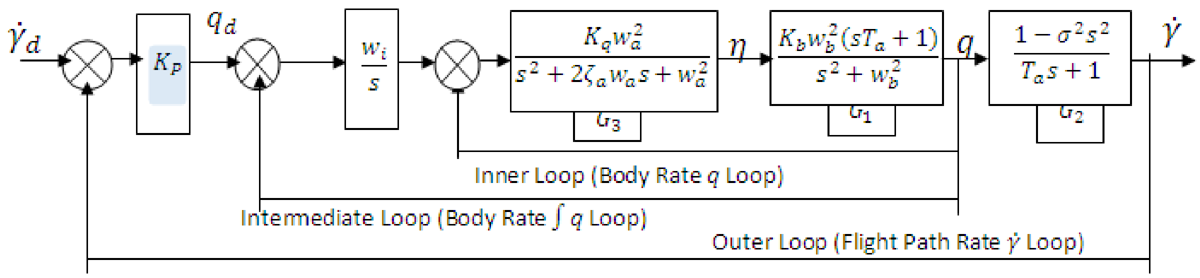


Fig – 2.2: Conventional Three Loop Autopilot Configuration (Transfer Function Model)

The open loop model i.e. the cascaded combination $-\frac{w_i}{s}\boldsymbol{G_1 G_2 G_3}$ of fig. 2.2 can be converted to the corresponding state space model given by $\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}$ & $\boldsymbol{y} = \boldsymbol{Cx}$ (discussed in the next section) and the conventional three-loop configuration can be converted to an equivalent state space model.

Now the modified configuration of three-loop lateral autopilot using Das & Ghosal observer (DGO) and LQR is presented below in fig. 2.3. The gain matrix $\boldsymbol{K_{opt}} = [\boldsymbol{K_1} \quad \boldsymbol{K_2} \quad \boldsymbol{K_3} \quad \boldsymbol{K_4} \quad \boldsymbol{K_5}]$ is obtained by using **LQR** command in Matlab (7.12, 2012a version) environment on the basis of optimal pole locations. A numerical program has been developed to choose the proper weightage of Q & R matrices required for the LQR algorithm to work on the basis of given performance specifications (e.g. settling time, steady state values, initial negative peak, overshoot, rise time, delay time etc.). Using the best weightage (Q & R matrices) combinations (got from the program just discussed) LQR decides the optimal pole locations and gain values ($\boldsymbol{K_{opt}}$ matrix).

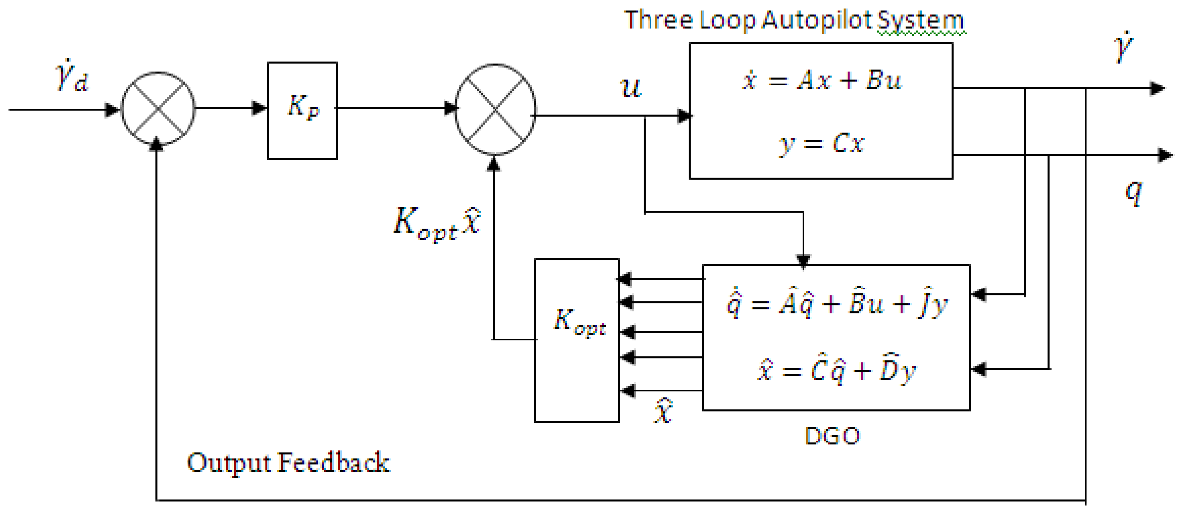


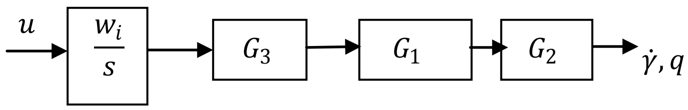
Fig – 2.3: Modified Three Loop Autopilot Configuration using Output Feedback, Das & Ghosal Observer (DGO) and LQR

Notations & Symbols used:

$\dot{\boldsymbol{\gamma}}$ *is flight path rate; q is pitch rate;*
$\boldsymbol{w_a}$ *is natural frequency of Actuator;* $\boldsymbol{\zeta_a}$ *is damping ratio of actuator;* $\boldsymbol{K_P, K_q, K_b}$ *are the control gains;*
$\boldsymbol{w_b}$ *is weather cock frequency;* $\boldsymbol{T_a}$ *is the incidence lag of the air frame;* $\boldsymbol{\eta}$ *is Elevator deflection;*
$\boldsymbol{\sigma}$ *is aquantity whoose invese determines the locations of non – minimum phase zeros;*
$\boldsymbol{w_i}$ *integrator gain;*

III. STATE VARIABLE MODELING OF THREE LOOP AUTOPILOT

The open loop model of three loop autopilot (fig. 2.2)



can be converted to state variable form based on the following five state variables:

$x_1 = \dot{\gamma}$ (*Flight path rate demand*);
$x_2 = \eta$ (*elevator deflection*);
$x_3 = q$ (*pitch rate*);
$x_4 = \dot{\eta}$ (*rate of chance of elevator deflection*;
$x_5 = X_i$ *Integrator State*;

Out of them x_1 *and* x_2 have been considered to be the outputs. Thus three loop autopilot model is a SIMO (single input – multiple output) system. Such that it's state variable model become:

This configuration is now governed by the standard state space equations:

$\dot{x} = Ax + Bu$ *and* $y = Cx$ …… (3.1)

and $u = r - K_{opt}\,\hat{x}$ …… (3.2) Where A, B & C matrices are:

$$\begin{bmatrix} -\frac{1}{T_a} & -\frac{K_b\sigma^2 w_b^2}{T_a} & \frac{(1+\sigma^2 w_b^2)}{T_a} & -K_b\sigma^2 w_b^2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\frac{(1+w_b^2 T_a^2)}{T_a(1+\sigma^2 w_b^2)} & \frac{K_b w_b^2 T_a(1-\sigma^2/T_a^2)}{(1+\sigma^2 w_b^2)} & \frac{1}{T_a} & 0 & 0 \\ 0 & -w_a^2 & 0 & -2\zeta_a w_a & K_q w_a^2 w_i \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \dots \dots (3.3a)$$

$$\text{and } y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \dots \dots (3.3b)$$

The closed loop poles, calculated from the characteristics equation of the overall transfer function of the conventional three-loop autopilot configuration (fig. 2.2), have also been considered for performance comparison with those obtained by applying LQR algorithm on the modified configuration (fig. 2.3).

IV. REDUCED ORDER DAS & GHOSAL OBSERVER (DGO) – GOVERNING EQUATIONS

Reduced order Das and Ghosal observer **[6]** is governed by the following equations and conditions -

$x = C^g y + L\, h \dots \dots \dots (4.1)$ (eqn. 13 of [6])

$\dot{h(t)} = L^g AL\, h(t) + L^g AC^g\, y(t) + L^g\, B\, u(t) \dots \dots \dots (4.2)$ (eqn. 15 of [6])

$\dot{y} = CALh + CAC_g\, y + CB\, u \dots \dots (4.3)$ (eqn. 18 of [6])

$\dot{\hat{h}} = (L^g AL - MCAL)\hat{h} + (L^g AC^g - MCAC^g)y + (L^g B - MCB)u + M\dot{y} \dots \dots (4.4)$ (eqn. 19 of [6])

$\dot{\hat{q}} = (L^g AL - MCAL)\hat{q} + \{(L^g AC^g - MCAC^g) + (L^g AL - MCAL)\}My + (L^g B - MCB)u \dots \dots (4.5)$ (eqn. 20 of [6])

$where\ \hat{q} = \hat{h} - My \dots \dots (4.6)$ (Page-374 of [6])

Equation (4.5) can be expressed in short form: $\dot{\hat{q}} = \hat{A}\hat{q} + \hat{J}y + \hat{B}u \dots \dots (4.7)$

$And\ \ \hat{x} = L\hat{q} + (C^g + LM)y \dots \dots (4.8)$ (eqn. 21 of [6])

Equation (4.8) can also be expressed in short form: $\hat{x} = \hat{C}\hat{q} + \hat{D}y \dots \dots (4.9)$

After applying the observer (DGO) the combined state space model of three-loop autopilot and DGO (fig. 2.3) becomes (using eqns. 3.1, 3.2, 4.7 & 4.9):

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{q}} \end{bmatrix} = \begin{bmatrix} A - BK\hat{D}C & -BK\hat{C} \\ (\hat{J} - \hat{B}K\hat{D})C & (\hat{A} - \hat{B}K\hat{C}) \end{bmatrix} \begin{bmatrix} x \\ \hat{q} \end{bmatrix} + \begin{bmatrix} B \\ \hat{B} \end{bmatrix} [r] \dots \dots (4.10a) and$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \hat{q} \end{bmatrix} \dots \dots (4.10b)$$

$$and\ \hat{x} = \hat{C}\hat{q} + \hat{D}y = \hat{C}\hat{q} + \hat{D}Cx \dots \dots (4.10c).$$

V. SIMULATION RESULTS

The following numerical data for a class of guided missile have been taken for Matlab simulation:

$T_a = 0.36\ sec;\ \sigma^2 = 0.00029\ \text{sec}^2;$
$w_b = 11.77 \frac{rad}{sec};\ \zeta_a = 0.6;$
$K_b = -10.6272\ per\ sec;$
$v = 470 \frac{m}{sec};\ K_p = 4.95;\ \ K_q = -0.12;$
$w_a = 180 \frac{rad}{sec};\ \ w_i = 22.02;$

Using these values, the open loop state space model of three-loop autopilot [given by eqns. (3.3a) & (3.3b)] becomes,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\end{bmatrix}=\begin{bmatrix}-2.77 & 1.1860 & 2.8894 & 0.4269 & 0\\ 0 & 0 & 0 & 1 & 0\\ -50.6161 & -508.388 & 2.77 & 0 & 0\\ 0 & -32400 & 0 & -216.0 & -85613.76\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 0\\ 0\\ 1\end{bmatrix}u \ldots\ldots(5.1a) and$$

$$y=\begin{bmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\end{bmatrix}\ldots\ldots(5.1b)$$

Finally unit step simulation is done on the combined model (i.e. three-loop autopilot + observer + state feedback + output feedback + LQR), given by matrix eqns. (4.10a), (4.10b) & (4.10c) by using Matlab software (**version 7.12, 2012a**) and the results obtained, are presented below:

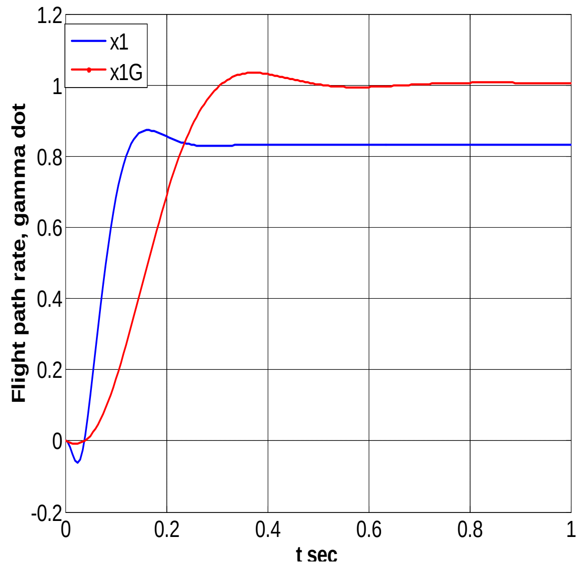


Figure 7.1: **Flight path rate**

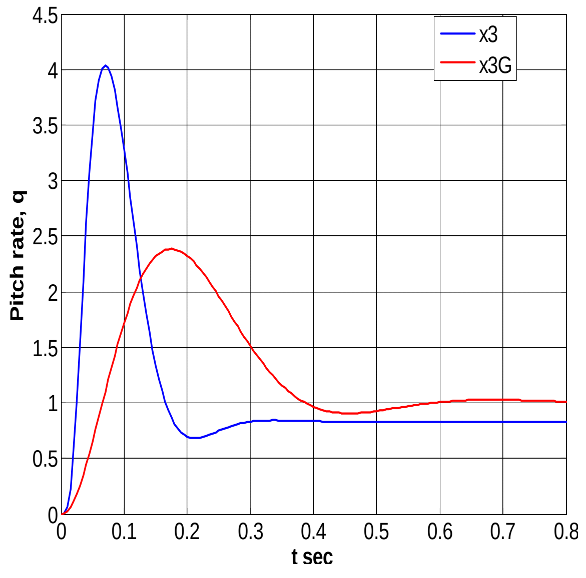


Figure 7.2: **Body rate (Pitch rate)**

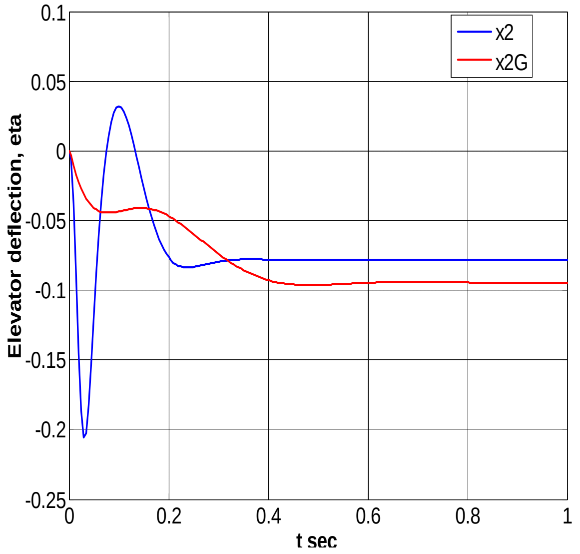


Figure 7.3: **Elevator Deflection**

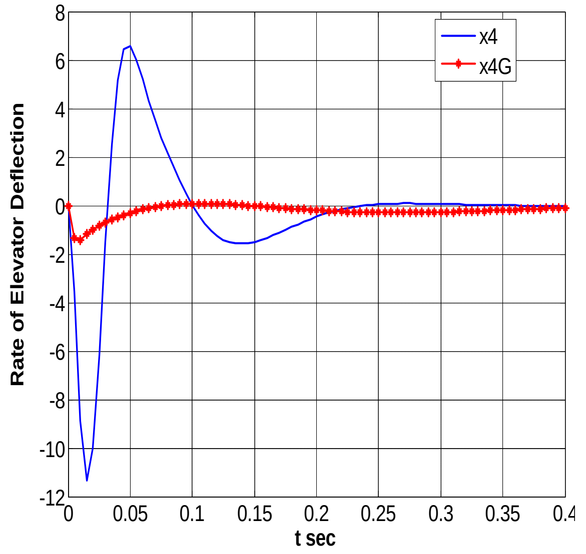


Figure 7.4: **Rate of change of Elevator Deflection**

N.B: In all of the above four graphs the **blue** lines indicate the classical three-loop autopilot response for frequency domain design approach (as per fig. 2.2) and corresponding state feedback design approach using Ackermann's pole-placement. While the **red** lines indicate the response for LQR based state feedback design methodology along with reduced order Das & Ghosal observer (fig. 2.3).

VI. OBSERVATIONS AND DISCUSSIONS

From the **table 6.1**, it can be inferred that the present method of three-loop autopilot design using output feedback, **LQR** and **DGO** is a very effective and efficient one. It has improved the overall responses, specially the *initial negative peak* of $\dot{\gamma}, \eta, \dot{\eta}$ responses due to the presence of non-minimum phase zeros, without sacrificing the settling time period and final steady state values. The steady state response of flight path rate has reached to unity which is one of the greatest features of this modified design methodology. Moreover it has reduced the body rate (q) demand as compared to the previous ones **[2], [5]**. It is seen from the simulation graphs that the responses (blue continuous line) obtained from the transfer function model and the corresponding state space model for the same set of closed loop poles; overlap each other indicating that both the modeling schemes are compatible. It has also been established through the simulation that Das & Ghosal observer has successfully caught the system states within less than 0.02 seconds and that also without any steady state error or oscillations. Further the experiments have also been carried out on the very well known and well used Luenberger observer **[12] & [13]** in place of Das & Ghosal **[6]** and it is seen that Luenberger observer will work only when the C matrix lies in the standard form $[I \;\; \vdots \;\; 0]$ otherwise not. But here C matrix is not in the standard form. So it will require an additional coordinate transformation to bring back the C matrix in the standard form and then only Luenberger observer will succeed. This is a one of the greatest advantages **[7] & [8]** of Das & Ghosal observer (DGO) over Luenberger.

Table - 6.1: Comparisons among Different Three-Loop Autopilot Design Methods

Comparison Parameters	3 Loop Autopilot Frequency Domain Design (I)	3 Loop Autopilot State Feedback Design by Ackermann (II)	3 Loop Autopilot State Feedback Design by LQR and Das & Ghosal Observer (III)	Percentage Change $\frac{(I)-(III)}{(I)} \times 100\%$
Initial negative peak of Flight path rate response due to non-minimum phase zeros	-0.0544	-0.0544	-0.0076	**86.03% improved**
Steady state value of Flight path rate response	0.8319	0.8319	1.0	**20.20% improved**
Settling time of Flight path rate response (5% tolerance band)	0.24 sec	0.24 sec	0.28 sec	**16% increased**
Maximum value of Body rate (q)	4.04	4.04	2.386	**41% decreased**
Settling time of Body rate (q)	0.5 sec	0.5 sec	0.5 sec	**No Change**
Steady state value of Body rate (q)	0.8319	0.8319	1.00	**20.20% increased**
Initial negative peak of Elevator deflection	-0.2056	-0.2056	-0.04435	**78.43% decreased**
Positive maximum of Elevator deflection	0.0322	0.0322	Nil	**Improved much**
Steady state value of Elevator deflection	-0.07828	-0.07828	-0.09462	**20% increased**
Initial negative peak in rate of Elevator deflection	-10.03	-10.03	-1.406	**86% decreased**
Positive maximum of rate of Elevator deflection	6.58	6.58	Nil	**Improved much**
Settling time of rate of Elevator deflection	0.35 sec	0.35 sec	0.40 sec	**14% increased**

ACKNOWLEDGEMENT

I would like to thank my friend **Sanjay Bhadra and Avijit Banerjee,** Control System section**,** Electrical Engineering Department, Jadavpur University, for his constant support and motivation.

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