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Modification of Classical Two Loop Autopilot Design using PI Controller and Reduced Order Observer (DGO)

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Abstract— A flight path rate demand modified two-loop lateral missile autopilot design methodology for a class of rear controlled guided missile, based on state feedback, output feedback, reduced order Das & Ghosal observer (DGO) and PI controller is proposed. The open loop undamped model of two-loop autopilot has been stabilized by using pole placement and state feedback. The non-minimum phase feature of rear controlled missile airframes is analyzed. An optimal pole placement has been done through an iterative method such that the adverse effect of non-minimum phase property on the response of missile autopilot can be reduced considerably and also the system stability can be made better. The overall response of the modified two-loop autopilot has been significantly improved over the previous ones - specially the flight path rate and body rate. The flight path rate (as well as the lateral acceleration) has reached the commanded unit step input without compromising on its settling time thus it has been able to eliminate the steady state error. The system can now also cop up with the body rate demand and even the Maximum body rate demand is decreased to a great extent as compared to the classical two-loop design. Reduced order Das & Ghosal observer (DGO), using Generalized Matrix Inverse) is implemented successfully in this design. A program has been developed to find out the optimal pole positions and feedback gains according to the desired time domain performance specifications. Finally a numerical example has been considered and the simulated results are discussed in details.

Keywords—Two Loop Pitch Missile Autopilot, Non-minimum phase system, Flight path rate, Body Rate, Rate Gyro, Accelerometers, Luenberger Observer, Das & Ghosal Observer, Generalized Matrix Inverse, PI controller.

I. INTRODUCTION

This paper is in continuation with the work reported in reference **[1]** by the present authors. It deals with the modified two-loop lateral missile autopilot design methodology in pitch plane based on time domain design approach. In literature **[3] & [4],** detailed design of classical two loop flight path rate demand autopilot (fig. 2.1) is given. Here the accelerometer provides the main output (flight path rate) feedback and the rate gyro enables the body rate feedback (inner loop) thus resulting in two loops. The authors presented three different design situations of two loop lateral autopilot for a class of guided missile. Frequency domain approach had been taken in those papers. In conventional two loop autopilot system there is no provision for direct control over the missile body rate. However, Tactical homing missiles require explicit control on body rate. Three such specific requirements are a) body rate generation not to exceed predetermined maximum; b) body acceleration limit; and c) it could produce moderate actuator rates without rate saturation for sudden pitch rate demands. In literature **[5]**, authors modified the design of two loop lateral autopilot and proposed an additional rate gyro feedback to be applied at the input of an integrating amplifier block (fig. 2.2) which integrates the body rate (i.e. pitch rate here) error to obtain direct control over the missile body rate. This enhanced model is referred to as Three Loop Lateral Autopilot. The three loop autopilot has a larger dc gain and a relatively small high frequency gain compared to a two-loop autopilot. This feature effectively improves the steady state performance and loop stiffness as well as reduces the initial negative peak of the time response. The three-loop autopilot attempts to reduce the adverse effect of non-minimum phase zeros. In reference **[6],** Prof. G. Das and T. K. Ghosal have derived a new method of reduced order observer construction based on Generalized Matrix Inverse theory **[14]** which possesses some certain advantages over the well known and well-established Luenberger observer **[11] & [12].** In paper **[7] & [8],** the present authors discussed about the intricate details of Luenberger observer and Das & Ghosal (DGO) observer. They have also carried out an exhaustive comparison (structure-wise and performance-wise) between the two observers. In literature **[10]**, Lin Defu, Fan Junfang, Qi Zaikang and Mou Yu have proposed a modification of classical three-loop lateral autopilot design using frequency domain approach. Also they have done performance and robustness comparisons between the two-loop and classical three-loop topologies. In order to eliminate the static error, Hui WANG, De-fu LIN, Jiang WANG and Zhen-xuan CHENG **[9]** introduced PI compensator to the classical two-loop autopilot and the contribution of PI was analyzed while the equivalent time constant estimates of two-loop autopilot with PI was discussed. Four different objective functions were given to the design procedures. It is concluded that it is the large gain at low frequency of PI solved the problem of static error. He has also discussed the “Lever Effect” feature of autopilot dynamics in details.

The unique contribution of this paper is the modified design technique of conventional two-loop lateral missile autopilot in state space domain by reduced order Das & Ghosal observer (DGO) based state feedback and a PI controller in the forward path. It has been established through this paper that the initial negative peak occurring in the time response due to the non-minimum phase zeros, the reduction of which posed a major challenge so far in autopilot design, is reduced drastically without compromising much on its setting time and steady state values. More over the steady state performance is also improved much) as compared to the classical two-loop design **[1]**. This configuration has also been able to lower the maximum body rate and the fin deflection rates which are the major issues of structural failure of a missile. This modified design (fig 2.3) of two-loop autopilot is even more advantageous than the classical three-loop **[2]** design (fig. 2.2) since in case of the latter there is always some steady state error in flight path rate response. Reduced order Das & Ghosal observer has been applied effectively to estimate fin deflection and its rate.

II. DEVELOPMENT OF MODIFIED TWO-LOOP AUTOPILOT FROM THE CONVENTIONAL ONE

The following block diagrams (fig. 2.1 & 2.2) represents the transfer function model of flight path rate demand two loop and three loop autopilot respectively in pitch plane **[3], [4] & [5].**

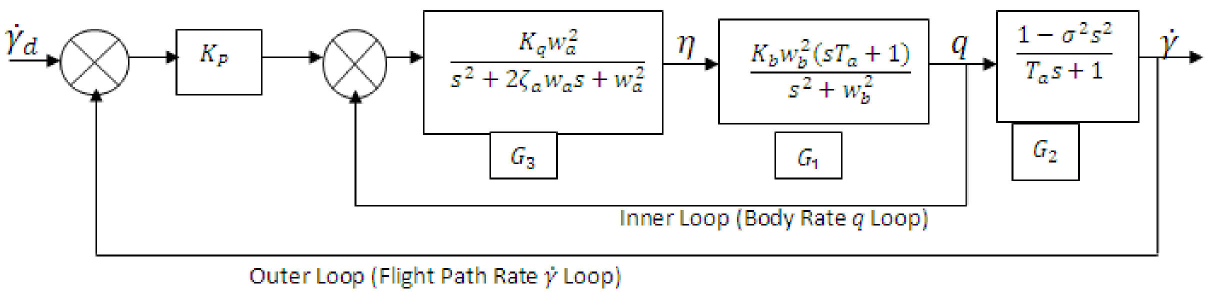


Fig – 2.1: Conventional Flight Path Rate Demand Two Loop Autopilot Configuration

Where G_1 and G_2 are the Aerodynamic transfer functions; G_3 is the Actuator transfer function;

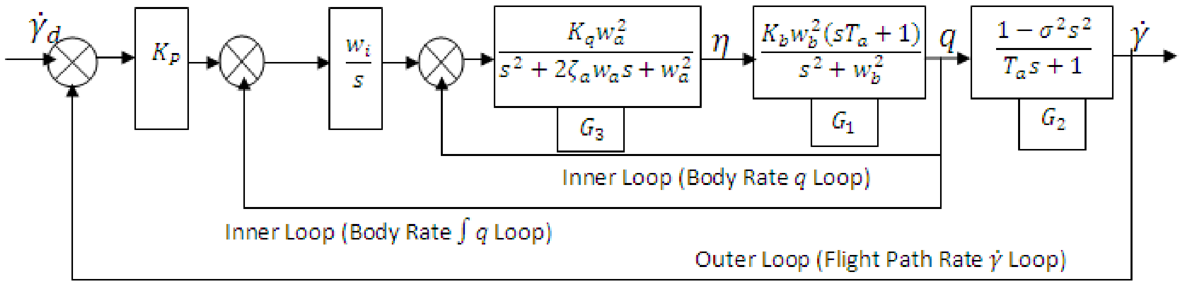


Fig – 2.2: Conventional Three Loop Autopilot Configuration (Transfer Function Model)

The open loop model i.e. the cascaded combination- $G_1 G_2 G_3$ of fig. 2.1 can be converted to the corresponding state space model given by $\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}$ & $\boldsymbol{y} = \boldsymbol{Cx}$ (discussed in the next section) and the conventional two-loop configuration can be converted to an equivalent state space model.

<u>Notations & Symbols used:</u>

$\dot{\boldsymbol{\gamma}}$ *is flight path rate*; q *is pitch rate*;
$\boldsymbol{w_a}$ *is natural frequency of Actuator*; $\boldsymbol{\zeta_a}$ *is damping ratio of actuator*; $\boldsymbol{K_P}, \boldsymbol{K_q}, \boldsymbol{K_b}$ *are the control gains*;
$\boldsymbol{w_b}$ *is weather cock frequency*; $\boldsymbol{T_a}$ *is the incidence lag of the air frame*;
$\boldsymbol{\eta}$ *is Elevator deflection*; $\boldsymbol{\sigma}$ *is aquantity whoose invese determines the locations ofnon – minimum phase zeros*; $\boldsymbol{w_i}$ *integrator gain*;

Now the modified configuration of two-loop lateral autopilot using Das & Ghosal observer, output feedback and PI controller is presented below in fig. 2.3. The gain matrix $\boldsymbol{K} = [\boldsymbol{K_1} \quad \boldsymbol{K_2} \quad \boldsymbol{K_3} \quad \boldsymbol{K_4}]$ is obtained by an iterative method on the basis of the optimal pole locations.

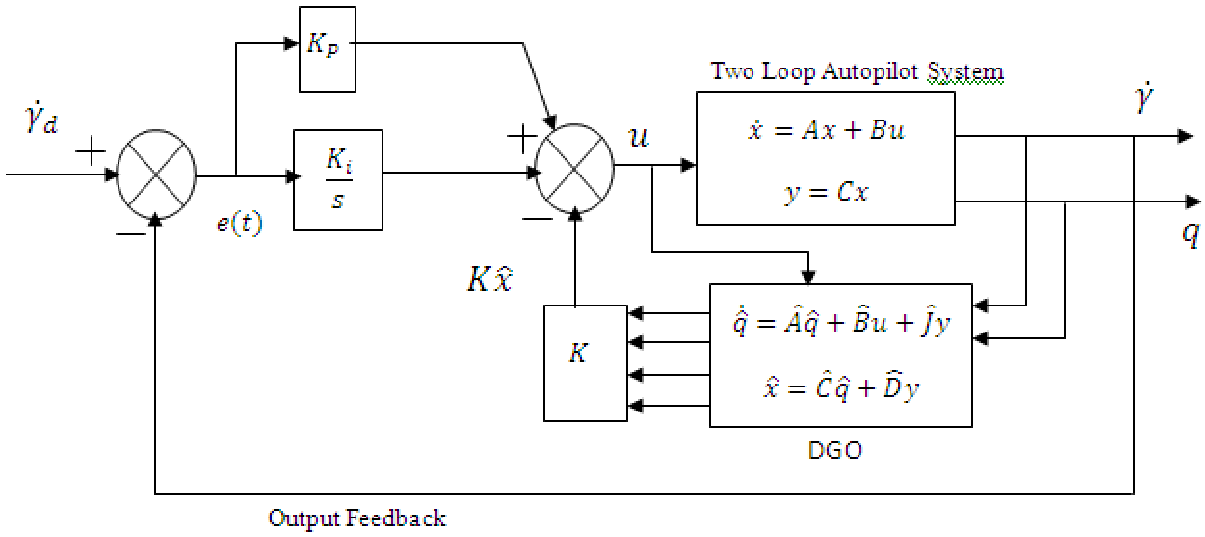
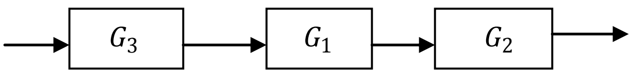


Fig – 2.3: Modified Two-Loop Autopilot Configuration using PI Controller, Output Feedback, Das & Ghosal Observer (DGO)

III. STATE VARIABLE MODELING OF TWO LOOP AUTOPILOT

The open loop model of two loop autopilot (fig. 2.1)



can be converted to state variable form based on the following four state variables:

$x_1 = \dot{\gamma}\ (Flight\ path\ rate\ demand);$
$x_2 = q\ (pitch\ rate);$
$x_3 = \eta\ (elevator\ deflection);$
$x_4 = \dot{\eta}\ (rate\ of\ chance\ of\ elevator\ deflection$

Out of them x_1 *and* x_2 have been considered to be as outputs. Thus two-loop autopilot model is a SIMO (single input – multiple outputs) system. Such that the A, B & C matrices become (taken from **[1] & [2]**):

$$\mathbf{A} = \begin{bmatrix} -\frac{1}{T_a} & \frac{(1+\sigma^2 w_b^2)}{T_a} & -\frac{K_b \sigma^2 w_b^2}{T_a} & -K_b \sigma^2 w_b^2 \\ -\frac{(1+w_b^2 T_a^2)}{T_a(1+\sigma^2 w_b^2)} & \frac{1}{T_a} & \frac{K_b w_b^2 T_a (1-\sigma^2/T_a^2)}{(1+\sigma^2 w_b^2)} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -w_a^2 & -2\zeta_a w_a \end{bmatrix};\ \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ K_q w_a^2 \end{bmatrix} \dots\dots (3.1a)$$

$$\text{and } \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \dots\dots (3.1b)$$

Now the state space equivalent model of conventional two-loop autopilot (fig. 2.1) can be represented by the state space equation given by (3.1a & 3.1b).

Ultimately the modified two loop flight path rate demand autopilot (fig. 2.3) has been introduced by incorporating Das & Ghosal observer (DGO), output feedback and PI controller along with the state space equivalent model and an iterative method has been adopted to find out the optimal pole positions based on the desired time domain performance criteria (e.g. steady state value, settling time, peak overshoot, undershoot, rising time etc.). The combined system (fig. 2.3) is governed by the following set of equations:

$\dot{x} = Ax + Bu \dots\dots (3.2)$

$y = Cx \dots\dots (3.3)$

$\dot{x_i} = r - x_1 = r - C_1 x = e(t) \dots\dots (3.4)$

Where $C_1 = [1 \quad 0 \quad 0 \quad 0]\ and\ e(\boldsymbol{t}) is\ error\ signal;$

$u = K_i X_i + K_p e(t) - K\hat{x} \dots\dots (3.5)$

The closed loop poles, calculated from the characteristics equation of the overall transfer function of the conventional two-loop autopilot configuration (fig. 2.1), have also been considered for performance comparison with those obtained from the present modified two-loop configuration with DGO and PI controller (fig. 2.3).

IV. REDUCED ORDER DAS & GHOSAL OBSERVER – GOVERNING EQUATIONS

Reduced order Das and Ghosal observer **[6]** which is constructed by using Generalized Matrix Inverse **[14],** is governed by the following equations and conditions.

$x = C^g y + L\,h \dots\dots\dots (4.1\,)$ (eqn. 13 of [6])

$\dot{h(t)} = L^g AL\, h(t) + L^g AC^g\, y(t) + L^g\, B\, u(t) \dots\dots\dots (4.2)$ (eqn. 15 of [6])

$\dot{y} = CALh + CAC_g\, y + CB\, u \dots\dots (4.3)$ (eqn. 18 of [6])

$\dot{\hat{h}} = (L^g AL - MCAL)\hat{h} + (L^g AC^g - MCAC^g)y + (L^g - MCB)u + M\dot{y} \dots\dots (4.4)$ (eqn. 19 of [6])

$\dot{\hat{q}} = (L^g AL - MCAL)\hat{q} + \{(L^g AC^g - MCAC^g) + (L^g AL - MCAL)M\}y + (L^g - MCB)u \dots\dots (4.5)$ (eqn. 20 of [6])

$where\ \hat{q} = \hat{h} - My \dots\dots (4.6)$ (Page-374 of [6])

Equation (4.5) can be expressed in short form: $\dot{\hat{q}} = \hat{A}\hat{q} + \hat{J}y + \hat{B}u \dots\dots (4.7)$

$And\ \ \hat{x} = L\hat{q} + (C^g + LM)y \dots\dots (4.8)$ (eqn. 21 of [6])

Equation (4.8) can also be expressed in short form: $\hat{x} = \hat{C}\hat{q} + \hat{D}y \dots\dots (4.9)$

Finally equation (3.2) can be rewritten by using eqns. (3.3), (3.4), (3.5) & (4.8) -

$\dot{x} = \left(A - BK_p C_1 - BK\hat{D}C\right) x + BK_i X_i - BK\hat{C}\hat{q} + BK_p r \dots\dots (4.10)$

Finally equation (4.7) can be rewritten by using eqns. (3.3), (3.4), (3.5) & (4.8) -

$\dot{\hat{q}} = \left(\hat{J}C - \hat{B}K\hat{D}C - \hat{B}K_p C_1\right)x + \hat{B}K_i X_i + \left(\hat{A} - \hat{B}K\hat{C}\right)\hat{q} + \hat{B}K_p r \dots\dots (4.11)$

Combining eqns. (4.10), (3.4) and (4.11) into a matrix, we get

$$\begin{bmatrix} \dot{x} \\ \dot{X}_i \\ \dot{\hat{q}} \end{bmatrix} = \begin{bmatrix} \left(A - BK_p C_1 - BK\hat{D}C\right) & BK_i & -BK\hat{C} \\ -C_1 & 0_{(1\times1)} & 0_{(1\times2)} \\ \left(\hat{J}C - \hat{B}K\hat{D}C - \hat{B}K_p C_1\right) & \hat{B}K_i & \left(\hat{A} - \hat{B}K\hat{C}\right) \end{bmatrix} \begin{bmatrix} x \\ X_i \\ \hat{q} \end{bmatrix} + \begin{bmatrix} BK_{p\,(4\times1)} \\ I_{(1\times1)} \\ \hat{B}K_{p\,(2\times1)} \end{bmatrix} [r] \dots\dots (4.12\ a) \quad and$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ X_i \\ \hat{q} \end{bmatrix} \dots\dots (4.12\ b)$$

$and\ \hat{x} = \hat{C}\hat{q} + \hat{D}y = \hat{C}\hat{q} + \hat{D}Cx \dots\dots (4.13\ c).$

V. SIMULATION RESULTS

The following numerical data for a class of guided missile have been taken for Matlab simulation:

$T_a = 0.36\ sec;\ \sigma^2 = 0.00029\ \mathrm{sec}^2;\ w_b = 11.77\frac{rad}{sec};$

$\zeta_a = 0.6;\ K_b = -10.6272\ per\ sec;$

$v = 470\frac{m}{sec};\ K_p = 5.51;\ \ K_q = -0.07;$

$w_a = 180\frac{rad}{sec};\ \ w_i = 22.02;$

Using these values, the open loop state space model of two-loop autopilot [given by eqns. (3.1a & b)] becomes,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} -2.77 & 1.1860 & 2.8894 & 0.4269\\ 0 & 0 & 0 & 1\\ -50.6161 & -508.388 & 2.77 & 0\\ 0 & -32400 & 0 & -216\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ -3888\end{bmatrix} u \ldots\ldots (5a)$$

$$y = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \ldots\ldots (5b)$$

Finally unit step simulation is done on the combined model (i.e. two-loop autopilot + state feedback + DGO + PI controller), given by matrix eqns. (4.12a), (4.12b) & (4.12c), is simulated by using Matlab software (version 7.12, 2012a) and the results obtained, are presented below:

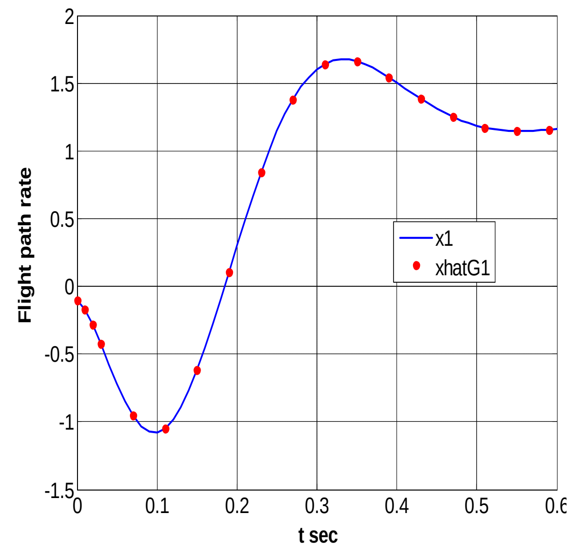


Figure 7.1: **Flight Path Rate**

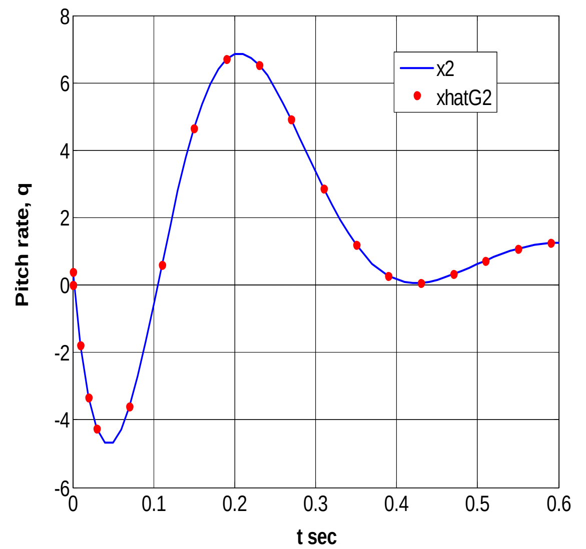


Figure 7.2: **Body rate (Pitch rate)**

N.B: In all of the above four graphs the **blue** lines indicate the modified two-loop autopilot states (as per fig. 2.3). While the **red dotted** lines indicate the estimated states obtained by using reduced order Das & Ghosal observer (DGO).

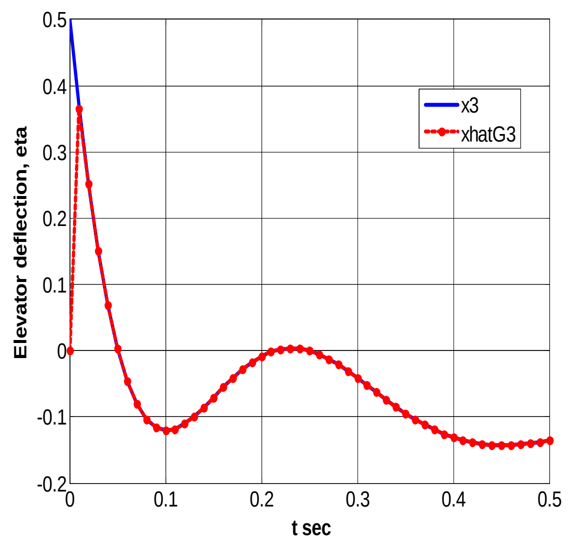


Figure 7.3: **Elevator Deflection**

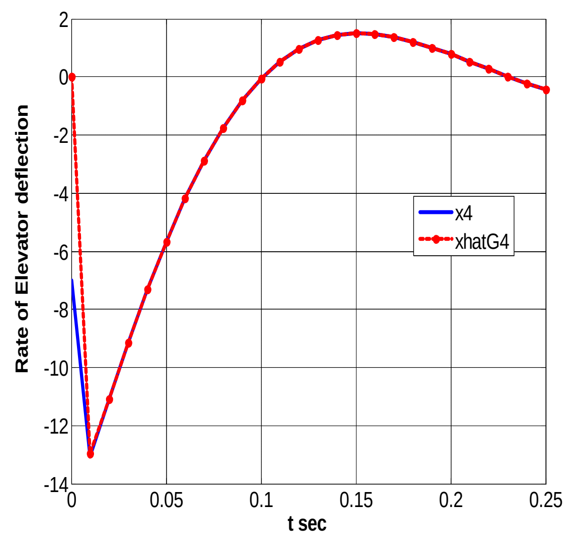


Figure 7.4: **Rate of Elevator Deflection**

VI. OBSERVATIONS AND DISCUSSIONS

From the simulation results, it can be inferred that the present method of modified two-loop autopilot design using **PI controller** and **Das & Ghosal Observer (DGO)** is a very effective and efficient one. It has improved the overall responses, specially the *initial negative peak* of $\dot{\gamma}, \eta, \dot{\eta}$ responses due to the presence of non-minimum phase zeros, without sacrificing the settling time period and final steady state values. Moreover it has reduced the peak body rate (q) demand as compared to the previous ones and also increased the steady state level of $\dot{\gamma}$ to a great extent. It has also been established through the simulation that Das & Ghosal observer has successfully caught the system states within less than 0.02 seconds and that also without any steady state error or oscillations. Further the experiments have also been carried out on the very well known and well used Luenberger observer **[11] & [12]** in place of Das & Ghosal **[6]** and it is seen that Luenberger observer will work only when the C matrix lies in the standard form $[I \;\; \vdots \;\; 0]$ otherwise not. Like here C matrix is in the standard form. Otherwise it will require an additional coordinate transformation to bring back the C matrix in the standard form and then only Luenberger observer will succeed. This is a one of the greatest advantages **[8], [9]** of Das & Ghosal observer over Luenberger.

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