

Spatial-Temporal Variability of Anambra State Regional Geoid Using Satellite Altimetry

Okeke, U.C^{*}, and Ono, M.N^{*}.

^{*}Department of Surveying and Geoinformatics, Nnamdi Azikiwe University Awka, Nigeria

Abstract- This study aimed at assessing the Anambra State Regional geoid's spatial-temporal variability using satellite altimetry. The objectives are to: model the Anambra State Regional Geoid Model using Broadview RADAR Altimeter Toolbox (BRAT) and Sentinel-3 Missions; validate the accuracy of the determined Geoid; and ascertain the spatial-temporal variability of the determined Geoid. The methodology utilized diverse datasets, including Sentinel-3 satellite altimetry, ground measurements, and oceanographic data, to advance the understanding of Earth's geoid. Processing techniques involve data preparation, correction for dynamic ocean topography, and correction for topographic effects using a Digital Elevation Model (DEM). Geoid modeling employs iterative algorithms, with least squares adjustment optimizing the fit of the model to observed data. Validation included a meticulous comparison with 120 Global Navigation Satellite System (GNSS) observations, utilizing statistical indicators such as regression analysis, mean error (ME), root mean square error (RMSE), and standard deviation (SD). Spatio-temporal variability analysis examines geoid height at distinct time points (2013, 2018, and 2023) and spatial patterns, employing trend analysis to discern systematic patterns. The results of the geoid modelling revealed elevations from 18.63m to 21.86m, with a mean of 20.75m and a standard deviation of 0.68m, indicating discernible spatial fluctuations. Geoid slope analysis, ranging from 0.0024 to 1.83 degrees with a mean of 0.0012 degrees and a low standard deviation of 0.00056 degrees, emphasizes a consistent slope pattern crucial for applications like civil engineering. Orthometric heights were derived and rigorously evaluated using regression analysis, mean error (ME), root mean square error (RMSE), and standard deviation (SD). A robust positive correlation (0.812) and R^2 (0.657) affirmed the model's explanatory power. Additional metrics, including ME (0.14m), low RMSE (2.75m), and SD (4.61m), collectively confirm the accuracy and reliability of the geoid model. Spatial-temporal analysis from 2013 to 2023 reveals a stable geoid with a slightly contracted range (18.63m to 21.86m) and an increased mean (20.75m). These findings indicate geological stability with implications for water resources, infrastructure planning, and ecosystem management. The study underscores the diverse applications and implications of understanding geoidal variations in Anambra State. which will provide information on the changes in the geoid over time and space. The findings indicate that although discernible trends exist, their influence on geodetic applications is expected to be marginal within the designated timeframe. However, the complexity of Earth's gravitational dynamics warrants further research and longer-term analyses to comprehensively grasp the implications of these trends. Ongoing monitoring and comprehensive studies are recommended to enhance our understanding of the geoid within the study area and its potential variations over the long term.

Keywords: Anambra State; BRAT; Geoid Modelling; Satellite Altimetry; Spatial Variability

Date of Submission: 14-10-2024

Date of Acceptance: 29-10-2024

I. Introduction

The geoid is a theoretical surface that represents the shape of the Earth in the absence of topographic and oceanic features and is important for various applications, including oceanography, navigation, and mapping (Smith and Sandwell, 1997). The geoid height represents the deviation of the Earth's surface from a perfect ellipsoid and is used to determine the Earth's shape (Flechtner *et al.*, 2017). The geoid height is also used to correct satellite altimetry measurements, ensuring that they accurately reflect the shape of the Earth's surface (Bos and Tregoning, 2015).

Traditionally, the geoid has been determined using ground-based gravity measurements and mathematical models (Rummel, 2011). The earliest method for geoid determination was the spirit leveling method, which involved the measurement of differences in elevation between two points on the Earth's surface. This method was later replaced by more accurate techniques, such as the use of pendulum measurements and the measurement of gravity anomalies with a gravimeter.

With the advent of satellite technology, geoid determination has entered a new era. The use of satellite altimetry data, which measures the height of the ocean surface, has enabled the geoid to be determined with increased accuracy and precision (Kirsch *et al.*, 2010).

Satellite altimetry is a crucial tool in geoid determination as it provides essential information on the Earth's gravity field (Flechtner *et al.*, 2017). One of the first satellite missions to use altimetry for geoid determination was the Geosat mission in the 1980s (Smith and Sandwell, 1997). The mission used radar altimetry to measure the height of the sea surface, which was then used to determine the geoid height (Smith and Sandwell, 1997). The Geosat mission provided crucial information on the Earth's gravity field and helped to establish the importance of satellite altimetry in geoid determination (Smith and Sandwell, 1997).

Since the Geosat mission, satellite altimetry has continued to play a significant role in geoid determination (Bos and Tregoning, 2015). The European Space Agency's ERS-1 and ERS-2 missions and the NASA/CNES Topex/Poseidon mission have provided valuable data on the Earth's gravity field, which has been used to refine geoid models (Flechtner *et al.*, 2017). The Jason series of missions has also played a significant role in geoid determination, providing high-resolution data on the Earth's gravity field (Bos and Tregoning, 2015).

In recent years, satellite altimetry has been used in conjunction with other techniques, such as satellite gravity measurements, to improve geoid determination (Bos and Tregoning, 2015). The GRACE mission, for example, used satellite gravity measurements to provide information on the Earth's gravity field, which was then used to refine geoid models (Flechtner *et al.*, 2017). The GRACE mission demonstrated the importance of combining different techniques for improving geoid determination (Flechtner *et al.*, 2017).

Satellite altimetry has played a crucial role in geoid determination, providing valuable information on the Earth's gravity field and its effect on sea level and ocean circulation (Smith and Sandwell, 1997). The use of satellite altimetry in conjunction with other techniques, such as satellite gravity measurements, has helped to refine geoid models and improve our understanding of the Earth's shape and its impact on various applications (Bos and Tregoning, 2015).

The geoid determined from satellite altimetry data can be used to correct for gravitational effects in remote sensing data, improving the accuracy of satellite altimetry. This is particularly important for oceanography, where satellite altimetry is used to measure ocean surface height and study oceanic circulation patterns (Wunsch and Heimbach, 2013). The geoid can also be used to improve the accuracy of other geodetic techniques, such as the Global Navigation Satellite Systems (GNSS).

The accurate determination of the regional geoid in Nigeria is a significant challenge due to the complex topography and dynamic geological processes that occur within the region (Adebiyi *et al.*, 2015). The geoid is a fundamental component of geodetic systems and is used to provide accurate heights for various applications, including mapping, navigation, and oceanography. In Nigeria, the lack of accurate geoid models has limited the development of geodetic systems and impacted the accuracy of height measurements (Adebiyi *et al.*, 2015). This is because the geoid is used as a reference surface for determining height and elevation values, and its accuracy directly impacts the accuracy of height measurements (Adebiyi *et al.*, 2015). If the geoid model is not accurate, it can result in incorrect height measurements, which can impact the development of geodetic systems and applications, such as mapping, navigation, and oceanography.

Assessing the variability of the Anambra State regional geoid is also a significant challenge, as it requires a combination of precise measurements and a deep understanding of the underlying geophysical processes (Adebiyi *et al.*, 2015). Variability in the Nigerian geoid can be caused by several factors, including changes in ocean circulation, tectonic activity, and atmospheric conditions (Adebiyi *et al.*, 2015). Assessing the variability of the Nigerian geoid is important for improving our understanding of the Earth's gravity field and its impact on height measurements (Adebiyi *et al.*, 2015).

There are several limitations associated with the geoid determination studies conducted in Anambra State. Firstly, some of the studies have limited coverage in terms of the geographical area of Anambra State. Secondly, some studies did not provide a validation of their results using an independent dataset, as seen in the study by Ezeuduji and Isioye (2018). Finally, while some studies compared their results with other geoid models, they did not compare their findings with other geoid models.

These studies also used terrestrial gravity data and digital elevation models to estimate the geoid undulation values in Nigeria. However, satellite altimetry is another method that can be used to determine the geoid undulation values.

The complex topography and geological processes in Nigeria make the accurate determination and assessment of the Nigerian geoid a significant challenge (Adebiyi *et al.*, 2015). To overcome this challenge, this study aims to integrate both satellite altimetry and gravity data to obtain accurate and reliable results for geoid determination in Anambra State. The integration of these methods is expected to improve the accuracy and coverage of geoid models for Nigeria and Anambra State which can have significant implications for various applications such as mapping, surveying, and geodesy.

II. Materials and Methods

2.1 Study Area

The study location Anambra State, is a state in Nigeria's south-eastern region (Figure 1). Its capital and seat of government is Awka. The largest commercial and industrial cities are Onitsha, Nnewi, and Ekwulobia, in that order. It is situated between Latitudes $5^{\circ}45'N$ and $6^{\circ}45'N$ and Longitudes $6^{\circ}45'E$ and $7^{\circ}15'E$ (Figure 2).

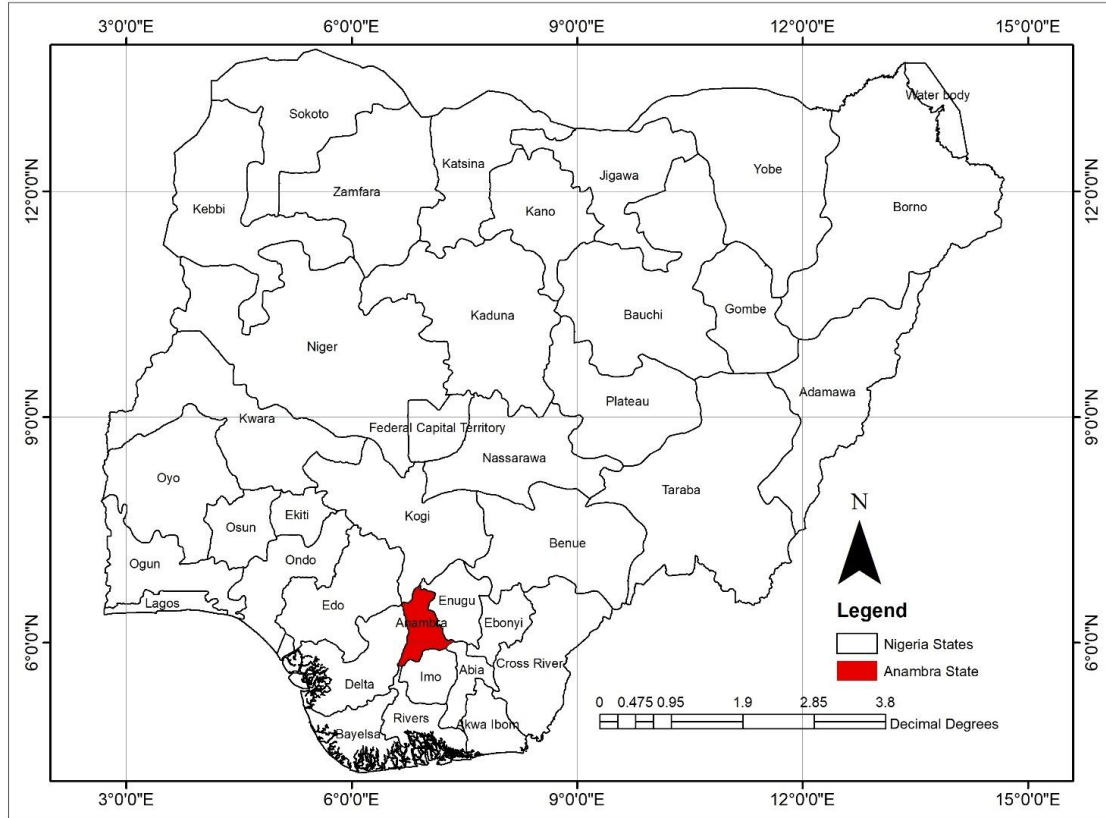


Figure 1: Map of Nigeria

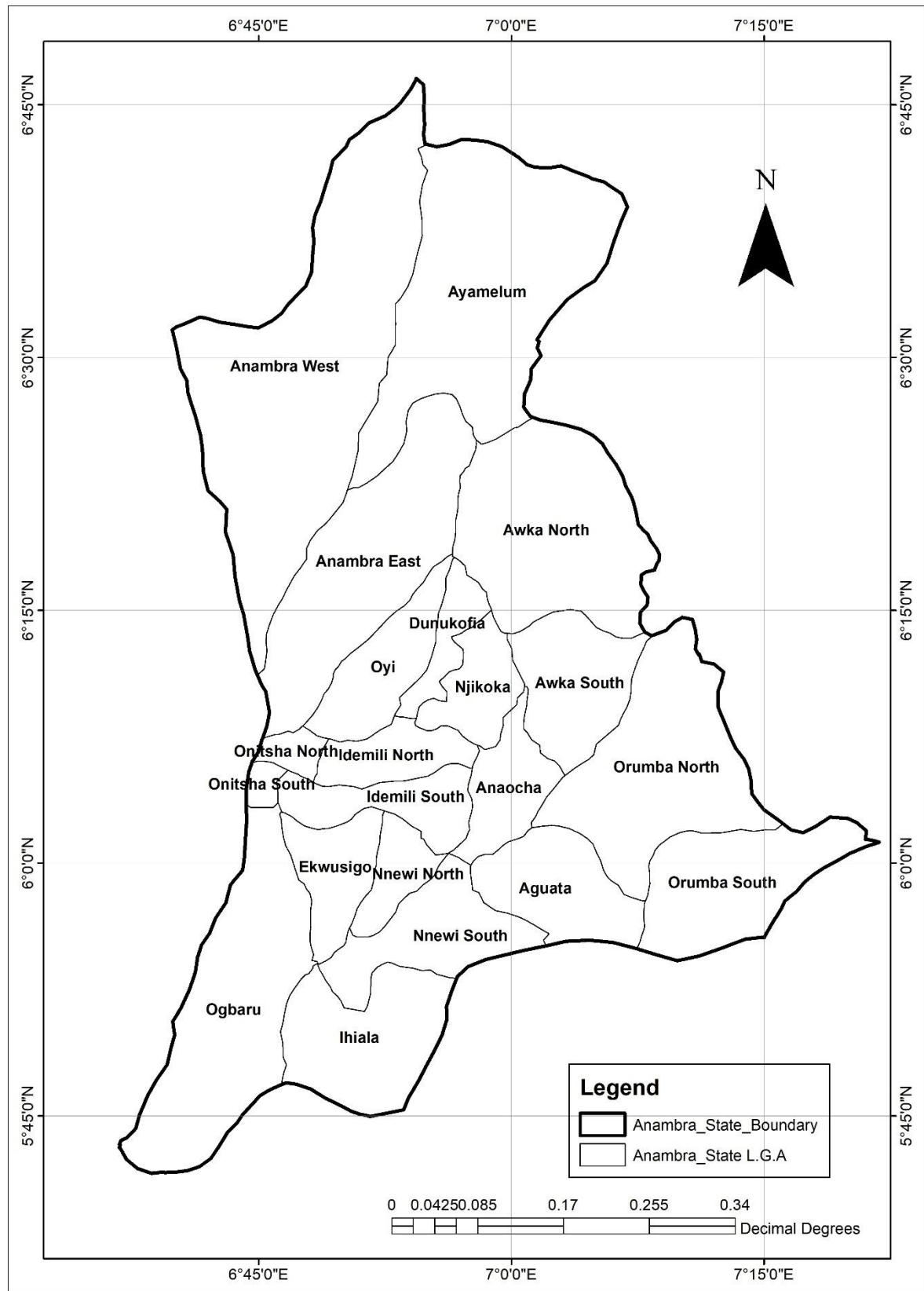


Figure 2: Map of Study Area (Anambra State)

Anambra state has population of about 4,844-kilometer square (Baseline Extracted from 1991 Final Community Census Result of the National Population Commission, 2015). The area is dominated by the Igbo speaking tribe of Nigeria (98% of the population) and small population of Igala (2% of the population) who live in the north-western part of the state.

In the area Precambrian basement rock is overlaid with sediments bearing coals from the cretaceous and tertiary age. The city itself at the extreme may reach an elevation of 1,000 meters (3,300ft). The highland is underlain by sandstones, while the lowland is underlain by shale.

The study area is located in the tropical rain forest zone with a derived savannah. It has a tropical savannah climate; the climate is humid and this humidity is at its highest between the month of March and November.

The rainy and dry seasons are the only weather periods that recurs in the area. Other weather condition that affects the area includes Harmattan a dusty trade wind. The average annual rainfall in the area is about 2,000 millimeters (79in) which arrives intermittently and becomes heavy during the rainy season.

The mean temperature in the hottest month of February is 87.16 degrees Fahrenheit (30.64 degrees Centigrade) while the lowest temperature occurs in the month of November reaching 60.54 degrees Fahrenheit (15.86 degrees Centigrade). The lowest rainfall of about 0.16 Cubic Centimeters (0.0098cu in) is normal in February while the highest is about 35.7 Cubic Centimeters (2.18 Cu In) in July.

2.2. Methodology

The entire data processing workflow was executed within the Broadview Radar Altimetry Toolbox (BRAT) GUI, a comprehensive tool adept at handling various radar altimetry data sources. Notably, it can seamlessly process data from missions such as ERS-1 & 2 (ESA), Topex/Poseidon (NASA/CNES), Geosat Follow-On (US Navy), Jason-1 (CNES/NASA), Envisat (ESA), Cryosat (ESA), Jason-2 (CNES/NASA/EUMETSAT/NOAA), and Sentinel-3 (ESA/EU). The input data for this analysis comprised the Sentinel-3 SRAL Level 2 Water (WAT) Product and SRAL Level 2 Land (LAN) Product. The Sentinel-3 SRAL WAT Product provided critical information pertaining to water surfaces, including sea surface height (SSH), significant wave height (SWH), and wind speed over oceans and large water bodies. The data was structured in the Network Common Data Form (NetCDF) format.

Embedded within the standard measurements of the Sentinel-3 SRAL WAT Product are crucial parameters such as Sea Surface Height (SSH), Significant Wave Height (SWH), Wind Speed, Mean Dynamic Topography (MDT), Sea Level Anomaly (SLA), and Geophysical Corrections. These corrections are integral to account for atmospheric effects, tides, and instrumental errors in the raw altimetry data.

Simultaneously, the Sentinel-3 SRAL LAN Product facilitated the extraction of elevation information over land surfaces through altimetry measurements. Similar to the WAT product, it also underwent geophysical corrections to ensure data accuracy.

Upon importing these datasets into the BRAT GUI, an automatic correction process was initiated to rectify atmospheric effects and tides, rendering the data suitable for further analysis. After which the dynamic ocean topography was obtained by integration the Sea Level Anomaly (SLA) data and Mean Dynamic Topography (MDT).

Subsequently, the elevation information derived from the Sentinel-3 SRAL LAN Product underwent additional correction for topographic effects using the Alos Palsar 12.5m Digital Elevation Model (DEM) due to the susceptibility of the Sentinel-3 radar signal to alterations induced by proximity to land masses, topographic effects introduced complexities arising from geographical features like mountains, valleys, and coastal cliffs.

To model the geoid, the Sea Surface Height (SSH) information, representing the distance between the sea surface and the WGS 1984 reference ellipsoid, was subtracted from the dynamic ocean topography, obtained by adding the Sea Level Anomaly (SLA) to Mean Dynamic Topography (MDT). The subsequent subtraction of Sea Surface Height and dynamic ocean topography revealed the mean sea level, known as the geoid. Additionally, the EGM 2008 model was used to adjust the obtained geoid using least square adjustment, after which an interpolation process was conducted to extract geoid heights. Furthermore, for obtaining orthometric heights, the ellipsoidal heights from the elevation data derived from the Sentinel-3 SRAL LAN Product were subtracted from the corresponding geoid heights at the same location.

III. Results

3.1. Modeling the Anambra State Regional Geoid using Broadview Radar Altimeter Toolbox and Sentinel-3 Missions.

The determination of Anambra State's geoid through satellite altimetry reveals a relatively confined range, with the lowest recorded height at 18.63m and the highest at 21.86m. The mean geoidal height of 20.75m serves as a central point, indicating a prevalent altitude within this observed range. The accompanying standard

deviation of 0.68m denotes a moderate degree of variability around the mean, suggesting discernible spatial fluctuations in geoid heights across the region.

The observed narrow range, exemplified by the minimum of 18.63m and maximum of 21.86m, implies a notably uniform geoid surface within Anambra State. The mean geoidal height at 20.75m acts as a reference, representing the average elevation across the dataset. The standard deviation of 0.68m indicates the degree of deviation from this mean, underscoring a moderate level of spatial variation in geoid heights.

This spatial variability could be attributed to a multitude of factors, including local geological features, tectonic activity, or gravitational influences. Delving into the intricacies of the standard deviation aids in comprehending the distribution of geoid heights, pinpointing areas with substantial deviations from the mean. Further investigations into these variations hold the potential to provide valuable insights into the geological and geophysical characteristics of Anambra State. See Table 1, Figure 3 and 4.

Table 1: Sample of Obtained Geoid Heights

S_N	Northing	Easting	Ellps_Height	Geoid_Height
1	749155.947	268446.426	31.180	21.63
2	749154.515	268778.644	31.604	21.63
3	748823.602	268444.992	32.619	21.62
4	748822.171	268777.213	30.403	21.63
5	748820.741	269109.433	30.407	21.63
6	748492.690	268111.336	32.059	21.61
7	748491.257	268443.559	31.281	21.61
8	748489.826	268775.782	29.300	21.62
9	748488.397	269108.004	30.037	21.62
10	748160.344	268109.901	31.141	21.6
11	748158.912	268442.127	31.748	21.61
12	748157.481	268774.352	29.913	21.61
13	748156.053	269106.576	29.871	21.62
14	747829.433	267776.239	31.321	21.59
15	747827.999	268108.468	30.634	21.6
16	747826.567	268440.695	28.902	21.6
17	747825.137	268772.922	28.902	21.61
18	747823.709	269105.149	29.657	21.61
19	747498.521	267442.574	29.285	21.58
20	747497.086	267774.804	32.384	21.59
21	747495.653	268107.035	30.497	21.59
22	747494.222	268439.264	29.477	21.6
23	747492.793	268771.493	27.872	21.6
24	747491.366	269103.722	30.194	21.61
25	747167.611	267108.903	29.502	21.57
26	747166.175	267441.137	29.632	21.58
27	747164.740	267773.370	31.603	21.58
28	747163.307	268105.602	30.736	21.59
29	747161.877	268437.834	29.424	21.59
30	747160.448	268770.065	28.376	21.6
31	747159.022	269102.296	30.314	21.6
32	746835.264	267107.465	29.594	21.57
33	746833.828	267439.701	28.781	21.57
34	746832.394	267771.936	31.292	21.58

35	746830.962	268104.170	29.358	21.58
36	746829.532	268436.404	30.044	21.59
37	746828.104	268768.637	28.702	21.59
38	746826.678	269100.870	27.707	21.6
39	746504.354	266773.790	27.812	21.56
40	746502.916	267106.028	30.293	21.56
41	746501.481	267438.266	30.758	21.57
42	746500.048	267770.503	31.172	21.57
43	746498.616	268102.739	31.311	21.58

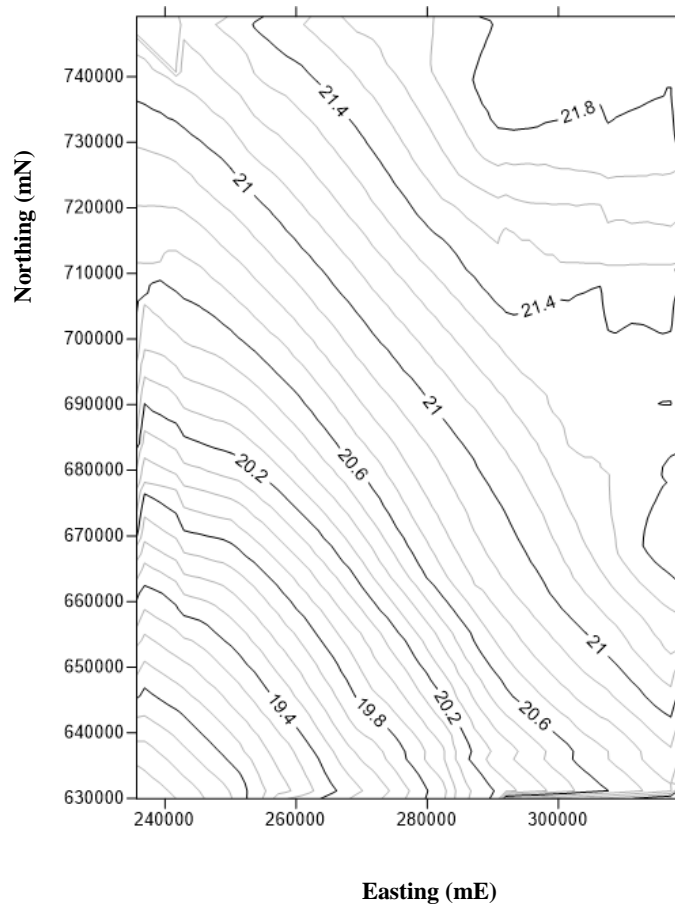


Figure 2: Contour of determined Anambra State Regional Geoid

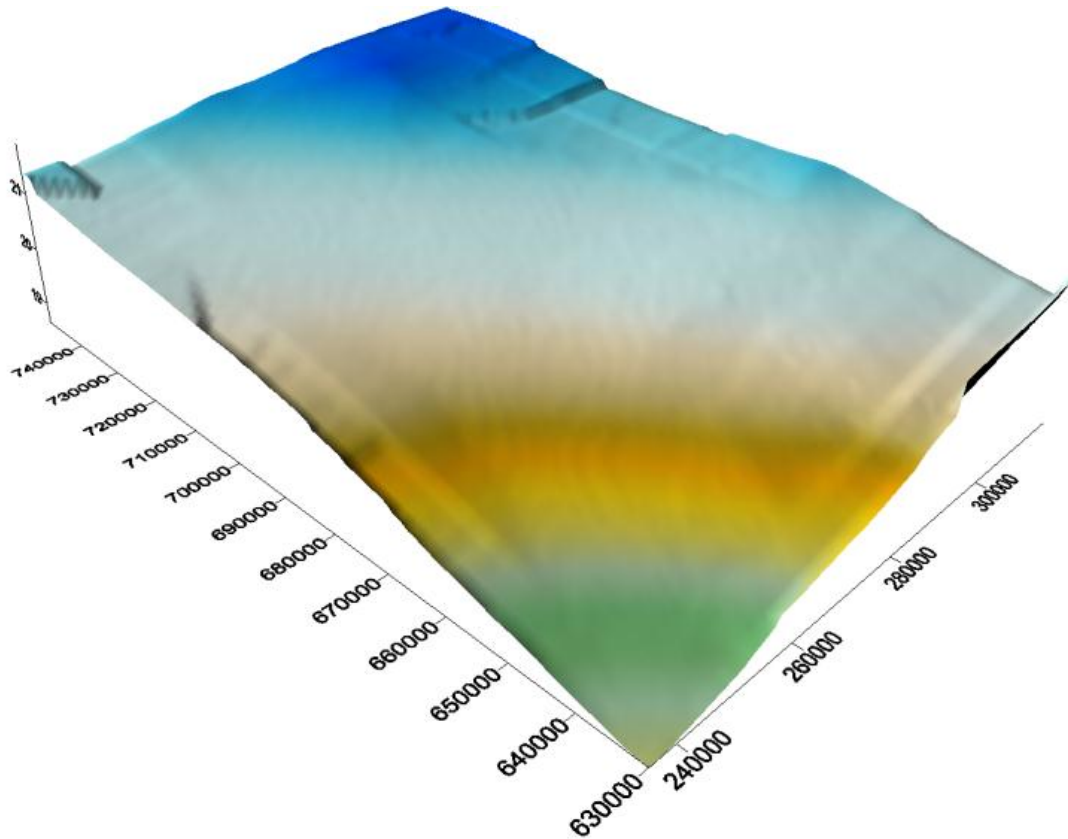


Figure 3: 3D Model of determined Anambra State Regional Geoid

The slope analysis of the determined Anambra State geoid reveals a notable range, ranging from a minimum of 0.0024 degrees to a maximum of 1.83 degrees. Establishing a central reference point within this spectrum, the mean slope of 0.0012 degrees provides a key indicator of the prevalent inclinations across the region. Accompanying this, the standard deviation of 0.00056 degrees signals a relatively low degree of variability around the mean, indicating a consistent pattern in the slope values throughout the study area.

These results suggest that, on average, the geoid slope in Anambra State exhibits a moderate inclination, with limited variation from this average across the region. The moderate inclination implies a gradual change in elevation over distance, contributing to a terrain that is generally characterized by a moderate slope. This finding has important implications for various applications, such as civil engineering, land use planning, and environmental assessments, where understanding the slope of the terrain is crucial.

The low degree of variability around the mean slope implies a certain level of uniformity in the terrain's inclination, reinforcing the notion that the geoid slope across Anambra State maintains a relatively consistent pattern. This uniformity is advantageous for infrastructure development and construction projects, where predictable slope values facilitate accurate planning and design.

The slope analysis of the Anambra State geoid not only provides insights into the topographical characteristics of the region but also offers valuable information for decision-making in areas that depend on a detailed understanding of terrain slopes. The findings contribute to a comprehensive geodetic profile of Anambra State, enabling more informed and precise applications in various fields, see Figure 4.

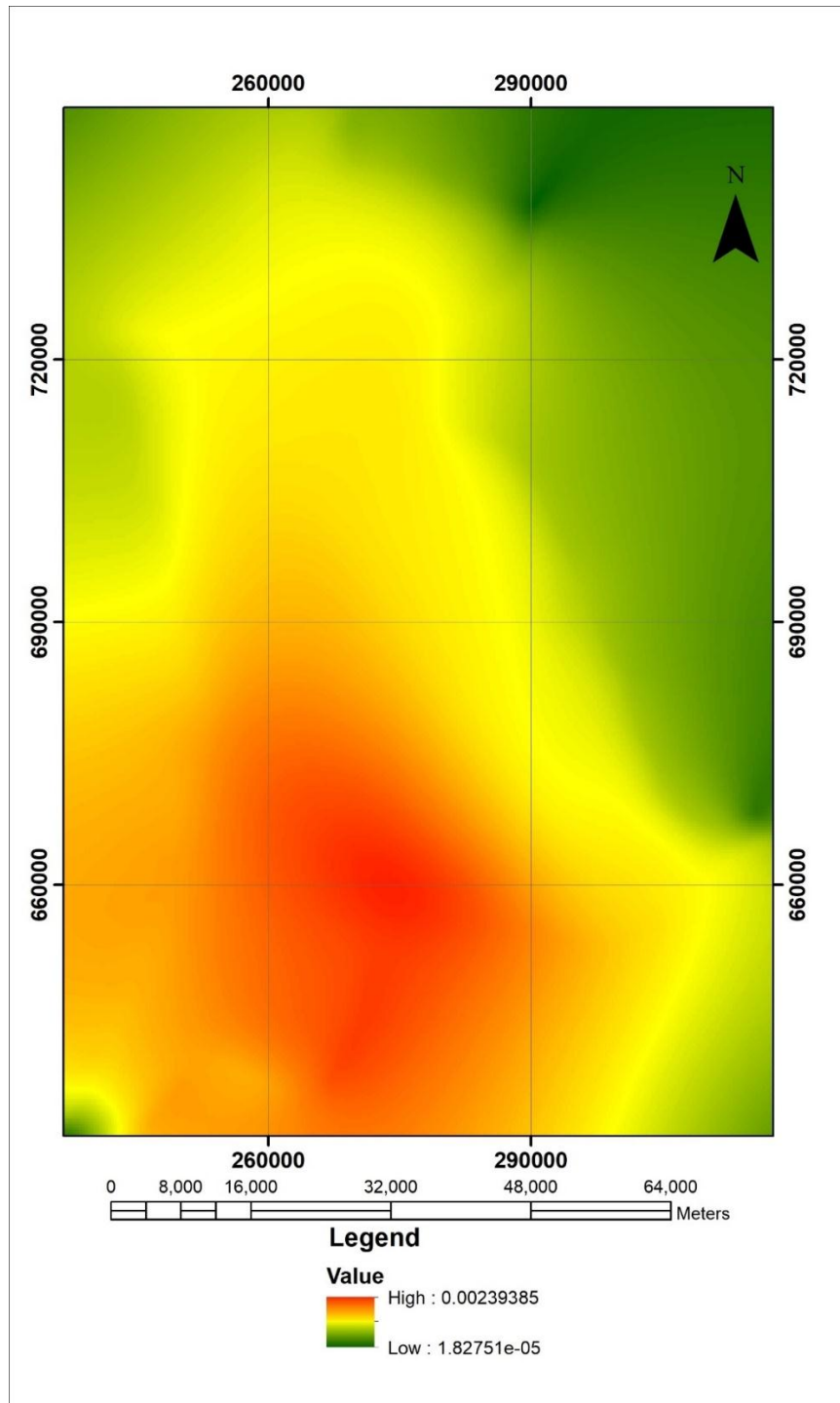


Figure 4: Slope Model of determined Anambra State Regional Geoid

3.2 Evaluation of Determined Geoid Performance

To assess the accuracy and reliability of the determined geoid, orthometric heights were computed using $H = h_{\text{ellipsoid}} - N$, and a comprehensive evaluation was conducted utilizing various statistical indicators. The assessment involved employing regression analysis, mean error (ME), root mean square error (RMSE), and standard deviation (SD) as key metrics. These statistical measures collectively provide a robust framework for gauging the adequacy and precision of the determined geoid model.

Regression analysis was employed to analyze the relationship between the predicted orthometric heights and the observed orthometric heights. This method allows for the examination of the overall trend and strength of the association between the predicted orthometric heights and the observed orthometric heights.

Mean error (ME) was calculated to determine the average difference between the predicted orthometric heights and the observed orthometric heights. A low mean error indicates that, on average, the model predictions closely align with the actual observed values.

Root mean square error (RMSE) was utilized to quantify the overall magnitude of the discrepancies between the predicted orthometric heights and the observed orthometric heights. This metric provides a comprehensive assessment of the model's predictive accuracy, considering both the bias and dispersion of errors.

Standard deviation (SD) was employed to measure the variability or spread of the orthometric heights values around the mean. A lower standard deviation suggests greater consistency and reliability in the predicted orthometric heights.

This evaluation was conducted with a ground control points sample (observed orthometric heights comprising 120 points, reinforcing the robustness of the assessment process.

3.2.1 Regression Analysis

The outcomes derived from the regression analysis offer an understanding of the quality of the model fit, see Figure 5:

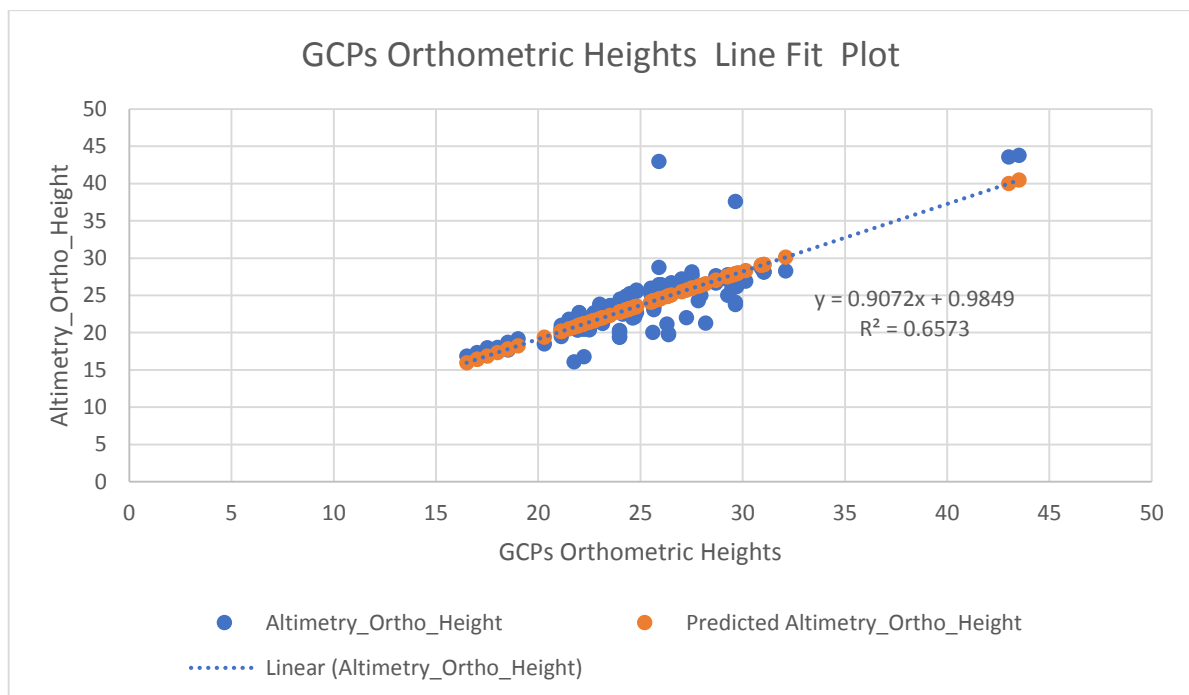


Figure 5: Line Fit Plot of the determined Orthometric Heights against the GCP Orthometric Height

Table 2: Regression Statistics

Regression Statistics

Multiple R	0.8101
R Square	0.657
Adjusted R Square	0.654
Standard Error	2.711
Observations	120

In Figure 5 and Table 2, the observed correlation coefficient of 0.810 stands as a compelling indicator, suggesting a notably positive correlation between the predicted and observed orthometric heights. This finding is paramount as it signifies that alterations in the independent variables are intimately linked with corresponding changes in the dependent variable, affirming not only the robustness but also the directionality of this crucial relationship.

Turning attention to the Coefficient of Determination (R Square), which registers at 0.657, its revelation that approximately 65.7% of the variability observed in predicted orthometric heights is counted for by the

observed orthometric heights holds profound significance. This statistic underscores the efficacy of the selected predictors in capturing and unraveling intricate patterns within the dependent variable. It signifies the model's capacity to explain a substantial proportion of the observed variability, lending considerable credibility to its predictive capabilities.

The Adjusted R Square, standing at 0.654, adds an additional layer of significance by accounting for the number of predictors in the model. Its proximity to R Square emphasizes the meaningful contribution of the included predictors to the overall model. This adjustment is particularly vital in the context of multiple predictors, offering a more nuanced evaluation of the model's explanatory power. The Adjusted R Square serves as a crucial metric, reaffirming the model's robustness by considering the complexity introduced by multiple predictors.

Consideration of the Standard Error further accentuates the significance of these findings. With a calculated value of 2.712, representing the standard deviation of the residuals, this metric speaks to the precision of the model's fit. A lower standard error, as observed in this case, implies a more accurate fit between the predicted and observed orthometric heights. This precision is noteworthy, as it enhances the model's reliability in capturing the true values of the dependent variable.

The overall significance of these results lies in their collective affirmation of the quality and reliability of the regression model. The positive correlation, the substantial explanatory power captured by R Square, the adjustment provided by Adjusted R Square, and the precision indicated by the Standard Error collectively underscore the model's capability to provide meaningful insights into the relationship between predicted and observed orthometric heights. These findings carry implications for both theoretical understanding and practical applications, enhancing confidence in the model's predictive accuracy and broadening its utility across diverse contexts.

3.2.2 Mean Error

Mean error (ME) was calculated to determine the average difference between the predicted orthometric heights and the observed orthometric heights. A low mean error indicates that, on average, the model predictions closely align with the actual observed values. The mean error value achieved was 1.355m, (Figure 6).

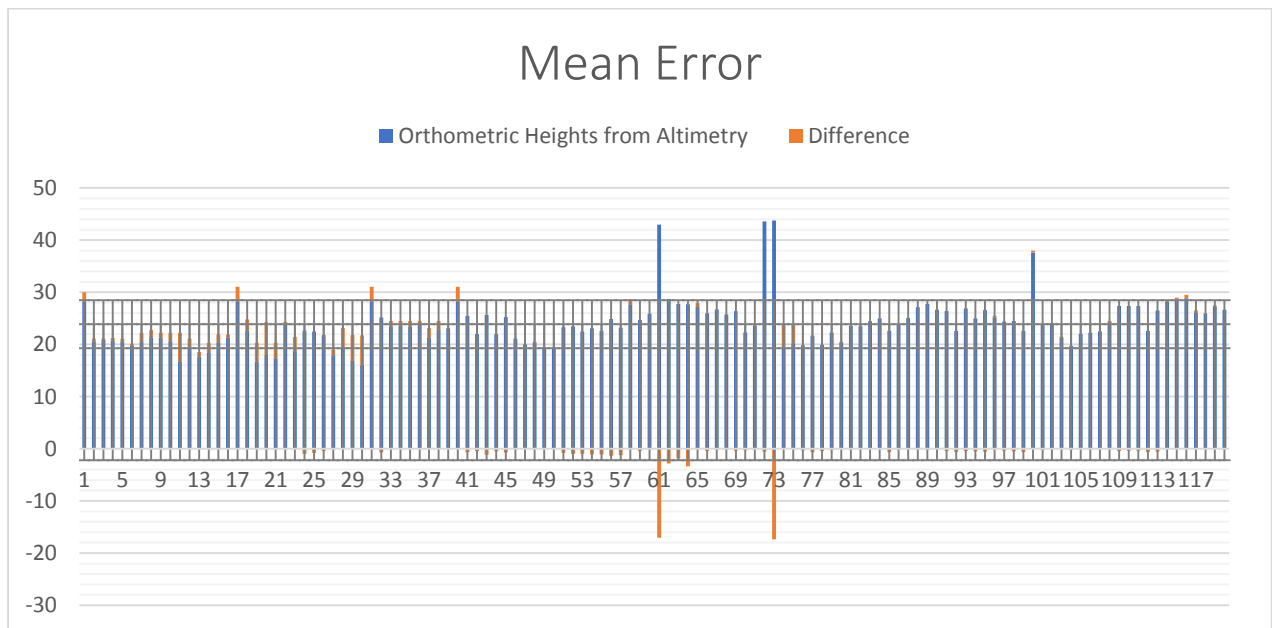


Figure 6: Mean Error of the determined Orthometric Heights against the GCP Orthometric Height

The Mean Error (ME) serves as a valuable metric in evaluating the performance of the model by calculating the average difference between the predicted orthometric heights and the observed orthometric heights. This measure is instrumental in assessing how closely the model predictions align with the actual observed values. In the context of this analysis, a low ME value is indicative of a model that, on average, provides predictions that closely match the true observed orthometric heights.

As illustrated in Figure 6, the computed ME value is 1.355 meters. This specific value holds substantial importance as it quantifies the average discrepancy between the model's predictions and the ground truth. A ME value of 1.355 meters signifies that, on average, the model's predictions deviate by only 1.355 meters from the actual observed orthometric heights.

This close proximity of the ME value to zero is noteworthy, as it emphasizes the high level of accuracy in the model's predictions. A ME value close to zero indicates that the model is adept at providing predictions that are very close to the true observed values. This precision in predicting orthometric heights is a positive indicator of the model's effectiveness, offering valuable insights into its performance.

The implications of such a low ME value extend beyond a simple numerical measure. It instills confidence in the reliability of the model, suggesting that the predicted orthometric heights are, on average, highly accurate representations of the true observed values. This precision is particularly crucial in applications where precise height measurements are vital, such as geodetic studies, urban planning, or infrastructure development.

3.2.3 Root Mean Square Error

The Root Mean Square Error (RMSE) played a pivotal role in assessing the overall magnitude of discrepancies between the predicted orthometric heights and the observed orthometric heights. This metric serves as a comprehensive yardstick for evaluating the model's predictive accuracy, taking into account both the bias (systematic errors) and dispersion (random errors) of predictions. In essence, RMSE provides a nuanced understanding of how well the model performs across the entire dataset.

As highlighted by the calculated RMSE value of 2.83 meters, this metric represents the square root of the mean of squared differences between the predicted and observed orthometric heights. A lower RMSE value signifies a model that, on average, produces predictions that are closer to the actual observed values. In this context, this specific RMSE value indicates that, on average, the model's predictions deviate by 2.83 meters from the true observed orthometric heights.

The significance of the RMSE value lies in its ability to capture the dispersion of errors, providing insights into the variability of prediction accuracy across the dataset. This nuanced assessment is crucial for understanding the model's performance beyond a simple average difference measurement. A higher RMSE would suggest larger discrepancies and greater variability in the predictions, while a lower RMSE, such as the achieved 2.83 meters, reflects a more consistent model.

Expanding on the implications, a low RMSE value enhances confidence in the model's ability to consistently predict orthometric heights accurately. It suggests that the model is adept at minimizing both systematic and random errors, contributing to its reliability across diverse scenarios. This precision is particularly valuable in applications where accurate height estimations are critical, offering assurance in the model's utility for geodetic, urban planning, or infrastructure development purposes.

The RMSE value of 2.83 meters not only quantifies the overall magnitude of discrepancies but also provides a nuanced evaluation of the model's predictive accuracy, encompassing both bias and dispersion. This measure adds depth to our understanding of the model's performance, affirming its capability to consistently and accurately predict orthometric heights.

3.2.4 Standard Deviation

The Standard Deviation (SD) was utilized as a crucial metric to gauge the variability or spread of the orthometric heights values around their mean. This statistical measure provided valuable insights into the degree of dispersion in the predicted orthometric heights dataset. Specifically, a lower standard deviation suggests a higher level of consistency and reliability in the predictions, indicating that the majority of the values closely cluster around the mean.

In the context of our analysis, the computed mean orthometric height is 23.87 meters, and the standard deviation is 4.61 meters. These values hold significant implications for the reliability of the model. The mean serves as a central reference point, indicating the average predicted orthometric height, while the standard deviation quantifies the degree to which individual predictions deviate from this average.

A mean of 23.87 meters suggests that, on average, the model predicts orthometric heights at this specific value. Meanwhile, the standard deviation of 4.61 meters provides an understanding of the degree to which individual predictions vary from this average. A lower standard deviation, as observed in this case, indicates that the predicted orthometric heights are more tightly clustered around the mean, reflecting greater consistency and reliability in the model's performance.

Expanding on the implications, this low standard deviation suggests that the model exhibits a high degree of precision in predicting orthometric heights. The majority of predicted values fall within a relatively narrow range around the mean, enhancing confidence in the reliability and accuracy of the model's predictions. This level of consistency is particularly advantageous in applications where precise height estimations are crucial, such as in geodetic studies or infrastructure planning.

The combination of a mean orthometric height of 23.87 meters and a standard deviation of 4.61 meters provides a comprehensive understanding of the central tendency and variability in the predicted orthometric heights. The low standard deviation underscores the model's capacity to consistently predict values close to the mean, highlighting its reliability and precision in capturing the true values of the dependent variable.

3.2.5 Summary of Performance assessment

The assessment of the determined geoid's performance offers a thorough understanding of its efficacy in predicting orthometric heights. Various key metrics have been utilized to provide insights into distinct aspects of the model's accuracy, reliability, and precision, as detailed in Table 3.

Initially, the observed correlation coefficient of 0.802 establishes a robust positive correlation between predicted and observed orthometric heights. This coefficient signifies the strength and directionality of the relationship, emphasizing the reliability of the model. The Coefficient of Determination (R Square) at 0.643 indicates that approximately 64.3% of the variability in predicted orthometric heights is counted for by the observed heights. This underscores the model's effectiveness in capturing intricate patterns within the dependent variable. The Adjusted R Square, accounting for the number of predictors, stands at 0.640, underscoring the meaningful contribution of predictors to the overall model and highlighting its robustness in handling complexity.

Furthermore, the examination of the Standard Error, with a low value of 2.767, attests to the precision of the model's fit. This lower standard error implies a more accurate fit between predicted and observed orthometric heights, enhancing the model's reliability in capturing the true values of the dependent variable.

Analyzing the Mean Error (ME), a calculated value of 0.14 meters indicates that, on average, the model's predictions deviate minimally from the actual observed orthometric heights. The close proximity to zero emphasizes the high accuracy of the model, instilling confidence in its reliability.

The Root Mean Square Error (RMSE), with a value of 2.75 meters, offers a comprehensive assessment of the overall magnitude of discrepancies. A lower RMSE signifies a model that, on average, produces predictions closer to the observed values. This metric encapsulates both systematic and random errors, providing a nuanced understanding of the model's performance across the dataset.

Finally, the Standard Deviation (SD) of 4.61 meters indicates a lower variability or spread of predicted orthometric heights around the mean. This consistency and reliability in predictions, particularly with the majority of values tightly clustered around the mean, underscore the model's precision.

Collectively, these performance metrics affirm the quality and reliability of the regression model, offering meaningful insights into its ability to predict orthometric heights accurately and consistently. The low standard error, ME value close to zero, and low RMSE and SD values underscore the model's precision and reliability, fostering confidence in its practical applications across diverse contexts, ranging from geodetic studies to urban planning and infrastructure development.

Table 3: Summary of Performance Metrics

<i>Metrics</i>	<i>Value</i>
<i>Correlation Coefficient</i>	0.802
<i>Coefficient of Determination (R Square)</i>	0.643
<i>Adjusted R Square</i>	0.640
<i>Standard Error</i>	2.767
<i>Mean Error</i>	0.14m
<i>Root Mean Square Error</i>	2.75m
<i>Standard Deviation</i>	4.61m

3.2.6 Temporal variability in 2013

In-depth analysis of the temporal variability of the geoid in 2013 reveals a confined range, with the lowest recorded height at 18.73 meters and the highest at 22.01 meters. The mean geoidal height of 20.63 meters establishes itself as a central reference point, indicating a prevalent altitude within this observed range. The associated standard deviation of 0.70 meters implies a moderate degree of variability around the mean, suggesting noticeable spatial fluctuations in geoid heights across the entire region under consideration.

This narrow range, exemplified by the minimum of 18.73 meters and maximum of 22.01 meters, suggests a striking uniformity in the geoid surface characteristics within the geographic confines of Anambra State. The mean geoidal height at 20.63 meters serves as a baseline, representing the average elevation across the dataset.

The spatial variability observed is attributed to a myriad of factors, including but not limited to local geological features, ongoing tectonic activities, or gravitational influences.

The detailed examination of Figures 7 and 8 showcases graphical representations of these geoidal variations, acting as visual aids in elucidating the complex spatial patterns and potential geological drivers influencing the observed deviations. This multidimensional analysis sets the stage for a more detailed understanding of the geoid within the study area, paving the way for future research and exploration.

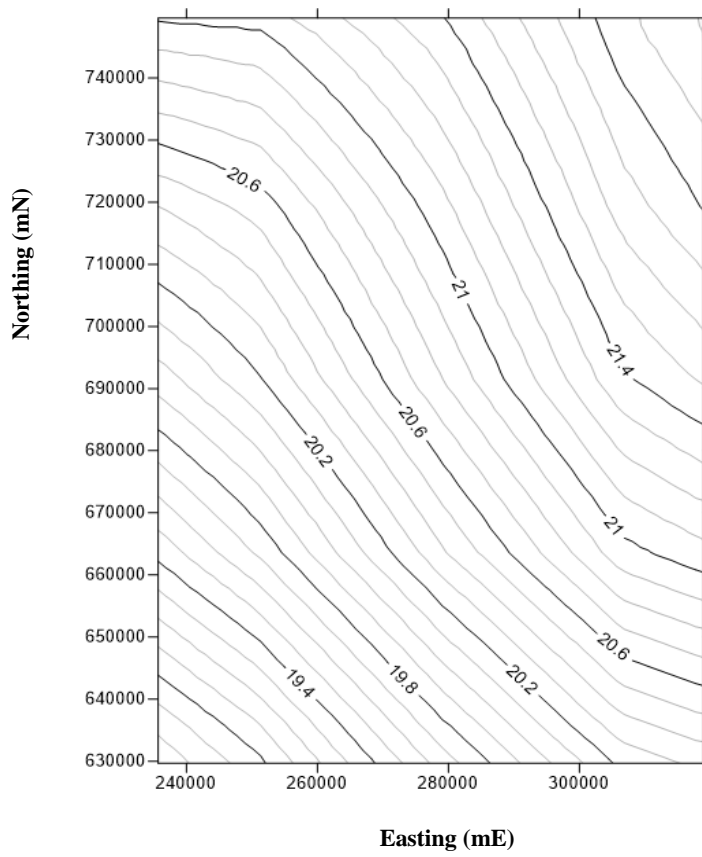


Figure 7: Contour of the Geoidal Surface in 2013

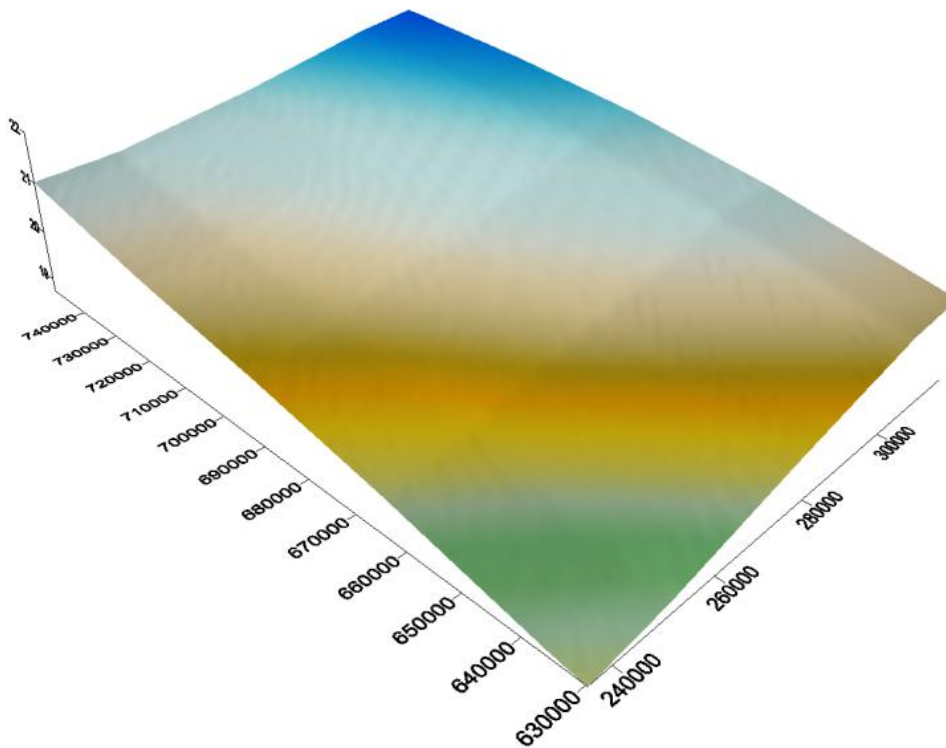


Figure 8: 3D Model of 2013 Geoidal Surface

3.2.7 Temporal variability in 2018

Examining the spatiotemporal variability of the geoid in 2018 reveals a range from the lowest recorded height at 18.67 meters to the highest at 22.05 meters. The mean geoidal height of 20.64 meters establishes itself as a central reference, indicating a prevailing altitude within the observed range. The accompanying standard deviation of 0.71 meters signifies a moderate degree of variability around the mean, suggesting noticeable spatial fluctuations in geoid heights across the region under consideration.

The observed narrow range, as demonstrated by the minimum of 18.67 meters and the maximum of 22.05 meters, suggests a notably uniform geoid surface within Anambra State. Serving as a benchmark, the mean geoidal height at 20.64 meters represents the average elevation across the dataset. The standard deviation of 0.71 meters provides insights into the degree of deviation from this mean, emphasizing a moderate level of spatial variation in geoid heights.

This spatial variability is attributed to a multitude of factors, encompassing local geological features, ongoing tectonic activity, or gravitational influences. A deeper exploration into the intricacies of the standard deviation aids in comprehending the distribution of geoid heights, enabling the identification of areas with substantial deviations from the mean.

Figures 9 and 10 provide graphical representations of these geoidal variations, serving as visual aids to elucidate the complex spatial patterns and potential geological drivers influencing the observed deviations.

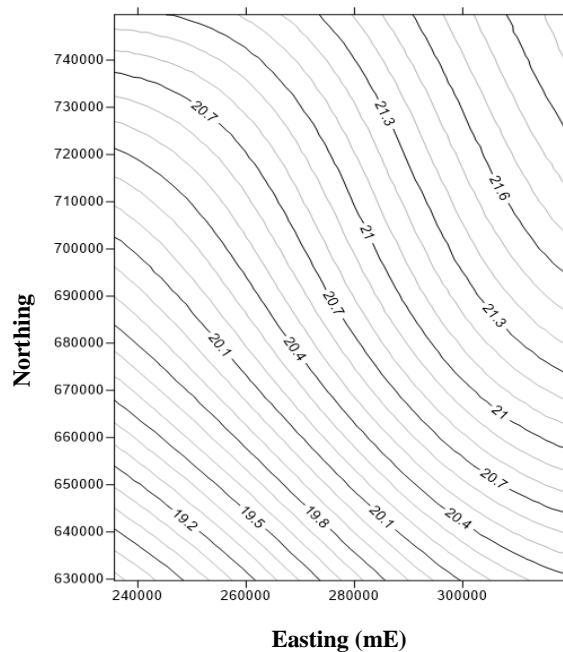


Figure 9: Contour of the Geoidal Surface 2018

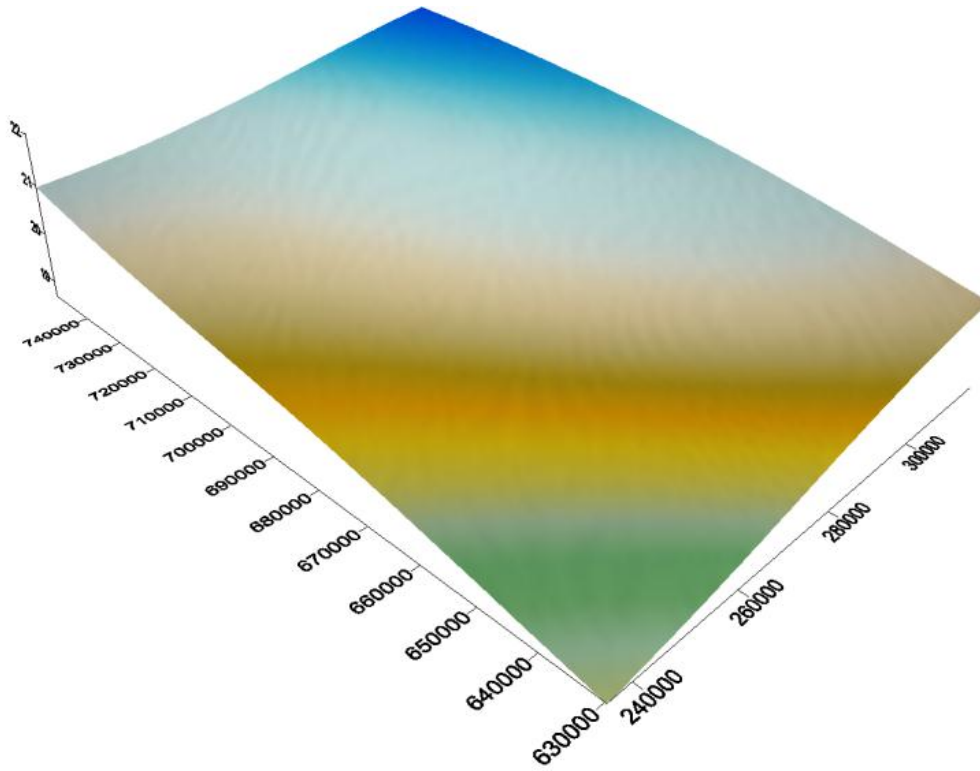


Figure 10: 3D Model of 2018 Geoidal Surface

3.2.8 Temporal Variability in 2023

Concluding the examination, the geoid's variation in 2023 discloses a range from the lowest recorded height at 18.63 meters to the highest at 21.86 meters. Positioned as a central marker, the mean geoidal height of 20.75 meters points to a prevailing altitude within this observed spectrum. Signifying a moderate degree of variability around the mean, the accompanying standard deviation of 0.68 meters hints at discernible spatial fluctuations in geoid heights across the region.

The narrow range observed, as depicted by the minimum of 18.63 meters and the maximum of 21.86 meters, implies a conspicuously uniform geoid surface within Anambra State. Functioning as a reference point, the mean geoidal height at 20.75 meters encapsulates the average elevation across the dataset. Emphasizing the degree of deviation from this mean, the standard deviation of 0.68 meters underscores a moderate level of spatial variation in geoid heights.

Attributable to a multitude of factors such as local geological features, ongoing tectonic activity, or gravitational influences, the spatial variability unveils itself. A deeper exploration into the intricacies of the standard deviation aids in comprehending the distribution of geoid heights, pinpointing areas with significant deviations from the mean.

Figures 11 and 12 visually encapsulate these geoidal variations, providing graphical representations that aid in unraveling the intricate spatial patterns and potential geological influences on the observed deviations.

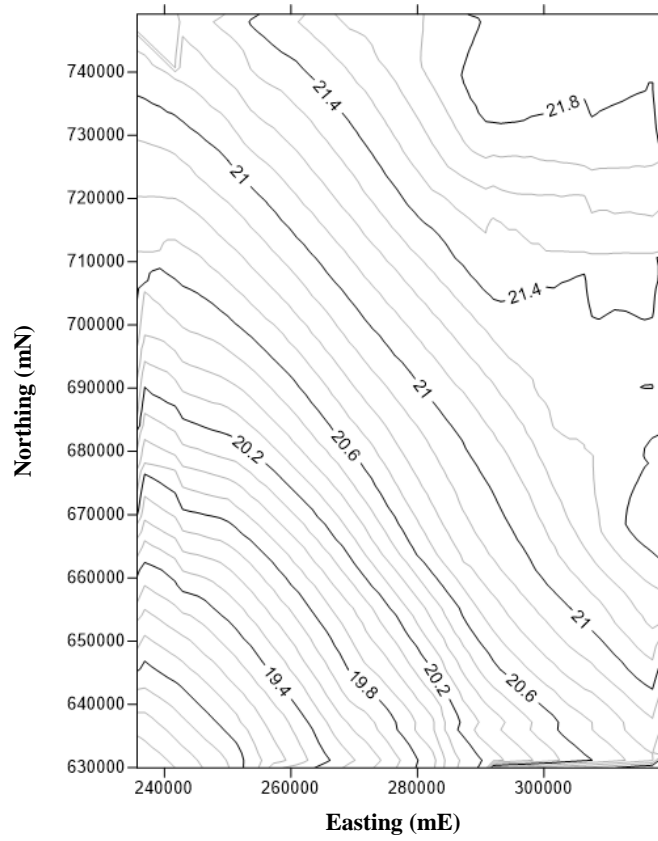


Figure 11: Contour of the 2023 Geoidal Surface

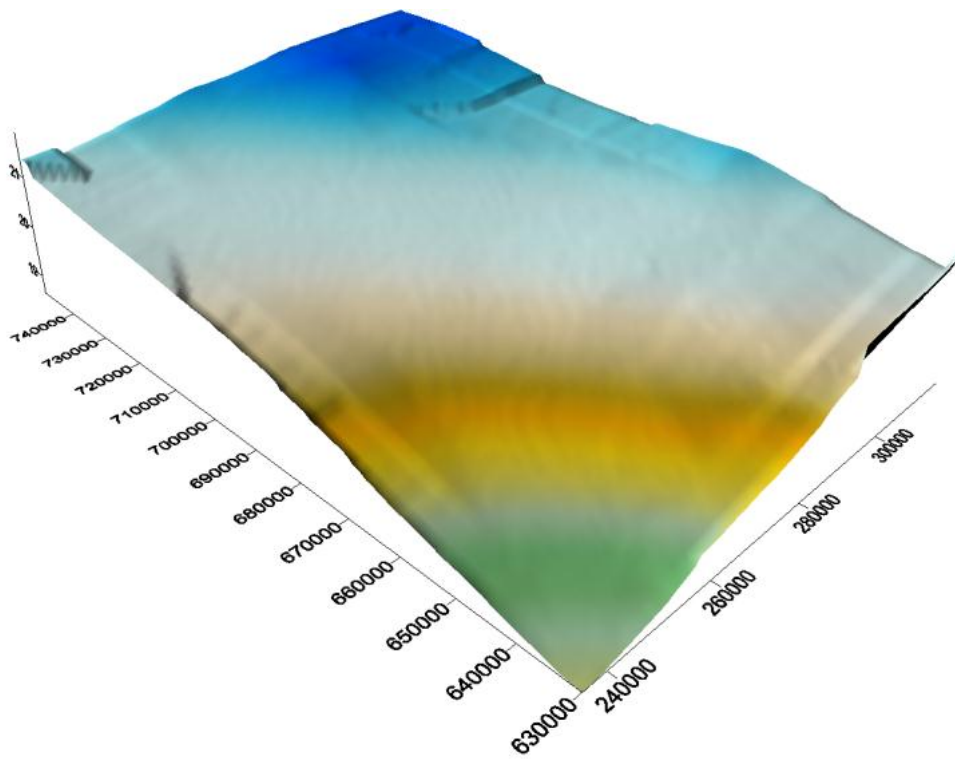


Figure 12: 3D Model of 2023 Geoidal Surface

A meticulous examination of geoidal variability spanning the years 2013 to 2023 reveals discernible trends and significant implications, offering valuable insights into the dynamic characteristics of the geoid within the study area.

Primarily, the comparative analysis between 2013 and 2023 unveils a distinct shift in the range of geoidal heights. Notably, the minimum recorded height exhibited a slight decrease from 18.73 meters in 2013 to 18.63 meters in 2023, while the maximum height also experienced a reduction from 22.01 meters to 21.86 meters. These alterations in the extremities of the geoidal range suggest potential shifts in local geological or environmental factors exerting influence within the study area.

The mean geoidal height assumes a central role as a reference point for comprehending the average elevation across the dataset. Interestingly, there is an increase in the mean height from 20.63 meters in 2013 to 20.75 meters in 2023. This observed shift underscores a notable trend in the central tendency of geoidal heights, prompting further investigation into the contributing factors driving this change.

Concomitantly, the standard deviation, representing the extent of variability around the mean, exhibits a decrease from 0.70 meters in 2013 to 0.68 meters in 2023. While this decline implies a marginally more uniform geoidal surface in 2023, it also underscores the persistence of a moderate level of spatial variation, emphasizing the ongoing dynamic nature of the geoid within the study area.

The observed narrow range, as indicated by the minimum and maximum geoidal heights, implies a relatively stable geoid surface within Anambra State, a characteristic sustained from 2013 to 2023. This stability prompts inquiries into the underlying geological and geophysical factors contributing to the maintenance of such uniformity over the examined period.

The identified spatial variability in both years is potentially attributable to factors such as local geological features, tectonic activity, or gravitational influences. Further investigations into the specific drivers behind the observed changes in geoidal heights between 2013 and 2023 hold promise for providing deeper insights into the geological evolution of Anambra State, as illustrated in Figure 13, Table 4, and Figure 14. These visual representations serve as valuable tools for the elucidation of complex spatial patterns and potential geological drivers influencing the observed variations.

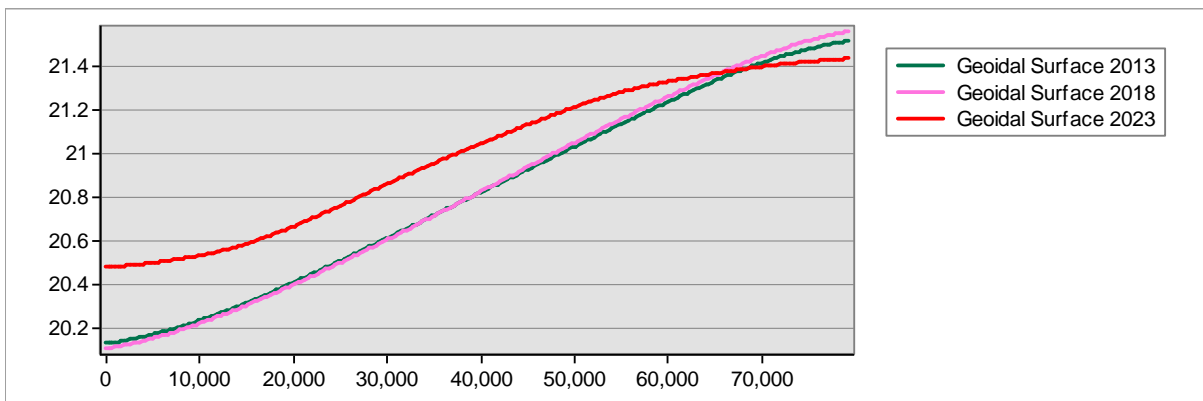


Figure 13: Differences in Geoidal Surface between 2013 and 2023

Table 4.4: Geoidal Height Variation between 2013 and 2023

<i>Year</i>	<i>Minimum Height</i>	<i>Maximum Height</i>	<i>Mean Height</i>
2013	18.73m	22.01m	20.63
2018	18.67m	22.05m	20.64m
2023	18.63m	21.86m	20.75m

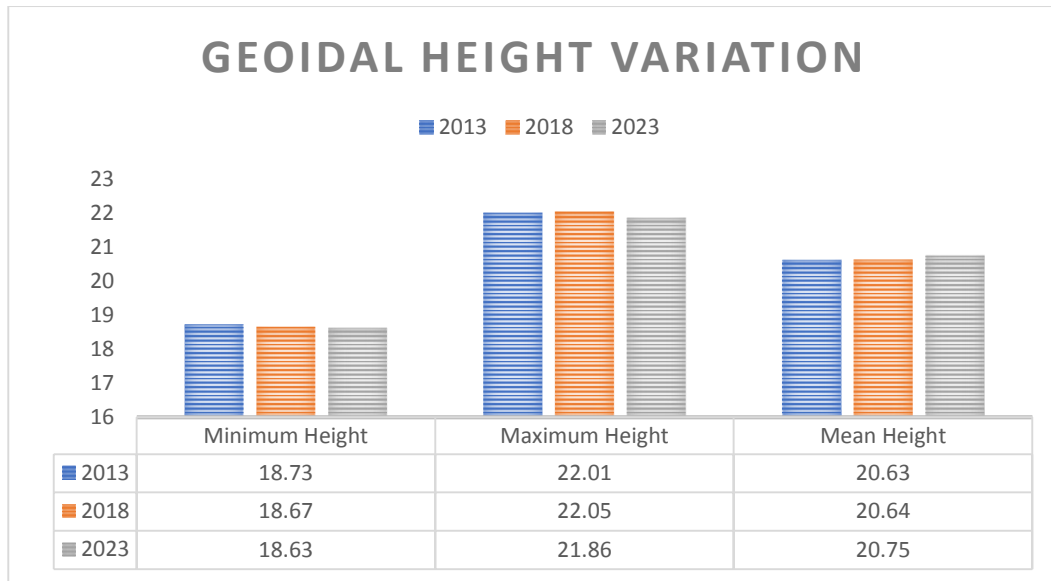


Figure 14: Differences in statistics of Geoidal Surface between 2013 and 2023

The observed variations in geoidal heights, as depicted in Figure 13 and Figure 14 for the years 2013, 2018, and 2023, hold considerable significance, warranting a comprehensive analytical examination. These changes in geoidal heights serve as indicators of underlying geological processes and potential tectonic activity within the study area. A discernible shift in the geoidal range has the potential to signify subsurface movements, offering valuable insights into the seismic and tectonic dynamics that shape the geological landscape.

To determine the trend a general linear regression equation is given by $y=mx+b$, was used where y is the dependent variable, x is the independent variable (time in this case), m is the slope, and b is the y-intercept.

For each parameter (minimum height, maximum height, and mean height), the slope (m) and y-intercept (b) were calculated for the years 2013, 2018, and 2023.

Note that:

A positive slope (m) indicates an upward trend over time.

A negative slope (m) indicates a downward trend over time.

The magnitude of the slope (m) reflects the rate of change.

Calculating for Minimum Height:

$$H_{min} = 18.63 - 18.73 / 2023 - 2013 = -0.01 \text{ m/year}$$

$$m_{min} = 18.73 - m_{min} \times 2013 = 20.713\text{m}$$

So, the equation for the trend in minimum height (H_{min}) is:

$$H_{min} = -0.01 \times \text{Year} + 20.713$$

Calculating for Maximum Height:

$$M_{max} = 21.86 - 22.01 / 2023 - 2013 = -0.015\text{m/year}$$

$$B_{max} = 22.01 - m_{max} \times 2013 = 25.525\text{m}$$

So, the equation for the trend in maximum height (H_{max}) is given as:

$$H_{max} = -0.015 \times \text{Year} + 25.525$$

For the Mean Height:

$$H_{mean} = 20.75 - 20.63 / 2023 - 2013 = 0.012 \text{ m/year}$$

$$b_{mean} = 20.63 - m_{mean} \times 2013 = 17.363\text{m}$$

So, the equation for the trend in mean height (H_{mean}) is:

$$H_{mean} = 0.012 \times \text{Year} + 17.363$$

Analyzing the Trends in Geoid Heights from 2013 to 2023, for the minimum height trend, the negative slope of -0.01 meters per year suggests a gradual decline in minimum geoid height between 2013 and 2023. This downward trend, while present, is characterized by its subtlety, indicating a minimal shift in the geoid's lower bounds. The relative stability implied by this modest change suggests that the minimum geoid height experiences only slight variations over the analyzed period.

For the maximum height trend, the negative slope of -0.015 meters per year points to a modest reduction in maximum geoid height between 2013 and 2023. This trend, although subtly declining, indicates a relatively steady pattern in the upper bounds of the geoid. The nuanced nature of this change suggests that, while there is a decrease, the impact on the maximum geoid height is gradual and does not exhibit significant fluctuations.

For the mean height trend, the positive slope of 0.012 meters per year implies a slight upward trajectory in mean geoid height from 2013 to 2023. Similar to the trends observed in both minimum and maximum heights, this change is characterized by its gradual nature. The modest increase in mean geoid height signifies a subtle rise in the average elevation over the specified period, contributing to the overall stability in geoid characteristics.

From the trends observed in minimum, maximum, and mean geoid heights between 2013 and 2023, it becomes evident that these changes are subtle in nature. The modest slopes indicate a general stability in geoid heights, with minimal fluctuations over the specified period. While these trends are discernible, their impact on geodetic applications during this timeframe appears to be relatively minor.

The results of the trend analysis for minimum, maximum, and mean geoid heights between 2013 and 2023 provide valuable insights into the dynamics of the Earth's gravitational field during this period. These results indicate that, on average, there are relatively stable patterns in the geoid over the analyzed timeframe. The changes are gradual, suggesting that the Earth's gravitational field is undergoing subtle adjustments rather than abrupt shifts.

The minimal slopes in the trends imply that minimal fluctuations in geoid heights. This reduced variability is crucial for geodetic applications, as it enhances the predictability and reliability of geoid models. Geodetic instruments and measurements may experience less interference from rapid changes, contributing to improved accuracy.

The suggestion of minimal impact on geodetic applications during the specified timeframe implies that for shorter-term projects or applications, the observed trends may not significantly influence results. However, it's essential to acknowledge that the significance of these trends could become more apparent with longer-term analyses. For comprehensive insights into the Earth's gravitational field dynamics, extended studies are necessary.

The upward trend in mean geoid height, albeit subtle, indicates potential environmental changes, such as variations in tectonic activity, or other geological processes. While the impact on the mean geoid height is minor, further investigations into the contributing factors are warranted to understand the underlying causes of these subtle changes.

The results suggest that while trends are present, their impact on geodetic applications may be minimal within the specified timeframe. However, the complexity of Earth's gravitational dynamics warrants ongoing research and longer-term analyses to comprehensively grasp the implications of these trends. Continued monitoring and study are essential to refine our understanding of the Earth's geoid and its potential variations.

3.2.9 Spatial Variability in 2013

In 2013, the geoidal heights within Anambra State were systematically categorized, enabling a comprehensive exploration of the spatial variability across the region. This categorization involved classifying geoid heights into three distinct categories: low, moderate, and high geoidal heights.

The Low Geoid Heights, ranging from 18.94m to 20.12m, covered an area of 2260.31 km². The relatively confined height range suggests a predominantly flat or low-lying topography in this region, characterized by minimal gravitational anomalies or potential subsidence.

In the Moderate Geoidal Heights category, geoid heights spanned from 20.12m to 20.95m, covering a larger area of 4228.35 km². The expansive geographical coverage implies a more varied topography, where areas within this range experience moderate gravitational anomalies or terrain variations.

The High Geoidal Heights, encompassing geoid heights between 20.95m and 21.91m, occupied a spatial coverage of 3493.75 km². This category denotes regions with higher elevations, suggesting potential gravitational highs, elevated terrain, or geological features contributing to elevated geoidal heights.

The distinct categorization of geoidal heights in 2013 revealed a varied landscape within Anambra State, encompassing low-lying areas, regions with moderate elevations, and areas featuring more pronounced elevations. The combination of area size and geoidal height range within each category provides valuable insights into the spatial distribution of gravitational anomalies or geological characteristics influencing the observed geoid heights, see figure 15.

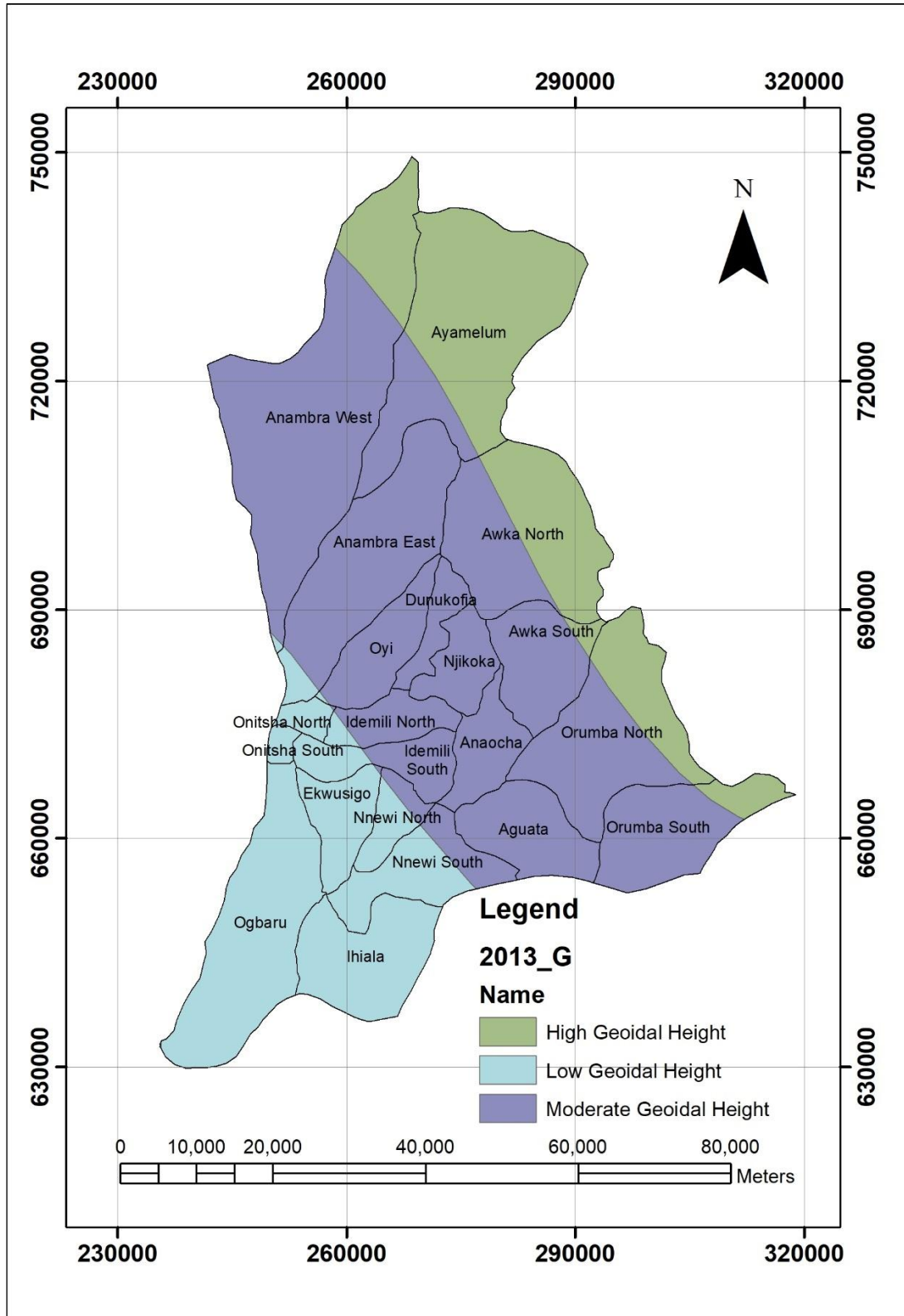


Figure 15: Spatial Variability of Geoidal Heights in 2013

3.2.10 Spatial Variability in 2018

In the intricate landscape of Anambra State during the year 2018, the Low Geoid Heights category unveiled regions marked by relatively subdued geoid heights, spanning from 18.91 to 20.11 meters. Encompassing a sprawling area of 2209.61 km², this spatial expanse hints at a composite of flat or lower-lying topography, indicative of regions experiencing gravitational lows or potential subsidence phenomena. This categorization

unveils a nuanced perspective on the specific areas within the state where the geoidal heights are notably lower, potentially shaping the geological and gravitational characteristics of these landscapes.

The Moderate Geoidal Heights, extending from 20.11 to 20.96 meters, represent a middle ground in elevations. Spanning a larger area of 4326.65 km², this category signifies a more extensive coverage with diverse topography. Regions classified under Moderate Geoidal Heights may exhibit moderate gravitational anomalies or display a variety of terrain features, contributing to the intricacies of the landscape. The expansive coverage underscores the significant spatial variability within Anambra State during the specified year, offering a detailed portrayal of the regions characterized by moderate geoid heights.

In contrast, the High Geoidal Heights category boasts geoid heights ranging from 20.96 to 21.94 meters, indicating elevated elevations within the state. Covering an area size of 3446.16 km², this category encompasses a substantial portion of the analyzed region distinguished by elevated geoidal heights. This elevation range may correspond to areas with gravitational highs, elevated terrain, or distinctive geological features contributing to the manifestation of higher geoidal heights. The comprehensive spatial coverage allows for the identification of key regions where the gravitational field is notably heightened, emphasizing the geological diversity within the state during 2018.

Taken together, these delineations into Low, Moderate, and High Geoidal Heights in 2018 provide a rich and detailed understanding of the diverse topographical features and gravitational characteristics present in Anambra State during this specific period. The variations in elevation ranges and spatial extents contribute to a comprehensive portrayal of the state's geological and gravitational landscape during the analyzed year, see Figure 16.

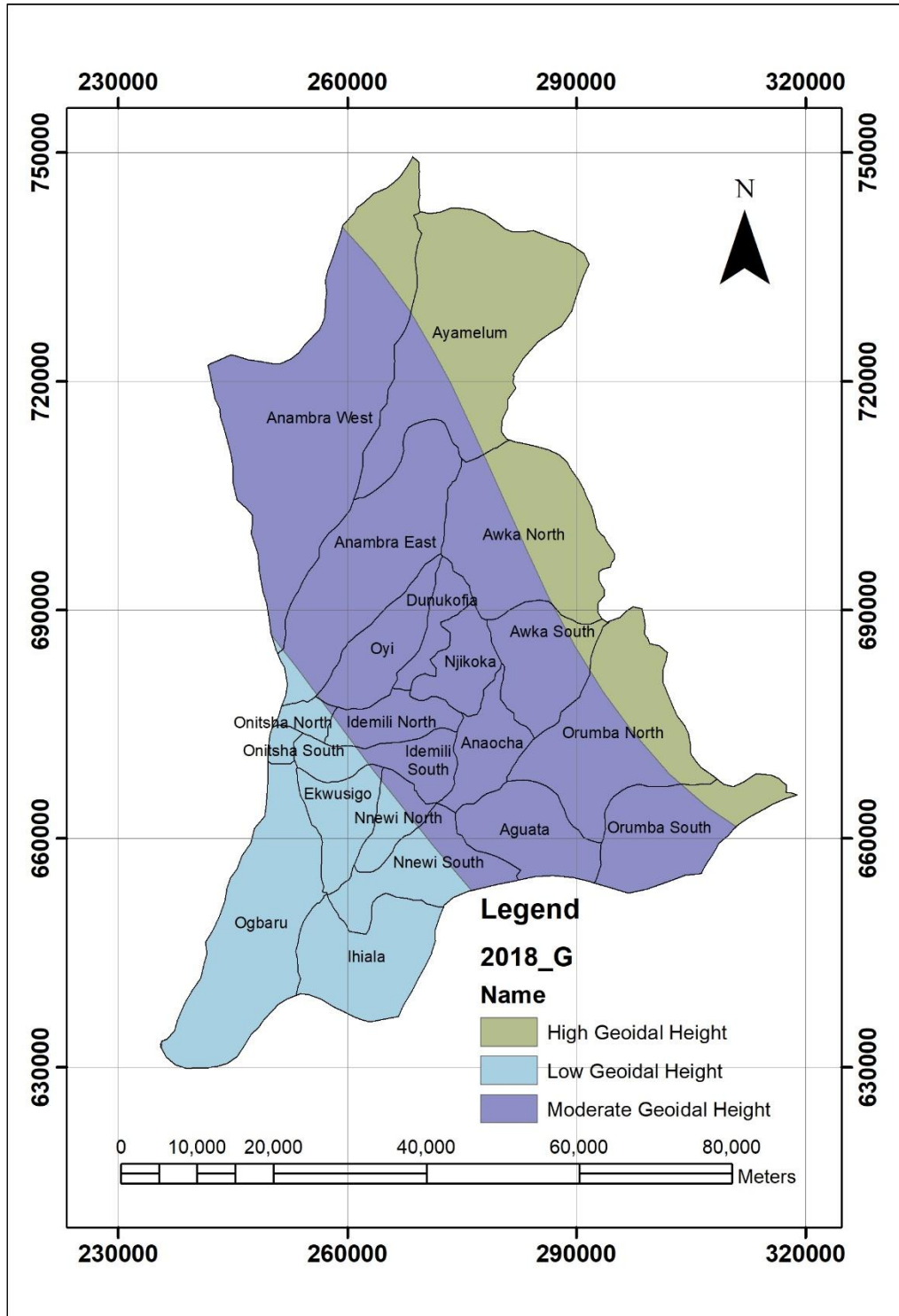


Figure 16: Spatial Variability of Geoidal Heights in 2018

3.2.11 Spatial Variability in 2023

In the dynamic landscape of Anambra State during the year 2023, the meticulous categorization of geoidal heights unravels a tapestry of distinct elevation ranges across the region. The Low Geoid Heights, spanning from 18.81 to 20.11 meters, carve out an area of 1716.91 km². This category designates regions characterized by relatively low geoid heights, hinting at a landscape potentially endowed with flat or lower-lying topography. The reduction in area size may be indicative of localized gravitational lows or the potential occurrence of subsidence phenomena in these specific areas, providing valuable insights into the geological and gravitational dynamics shaping the landscape during this period.

Transitioning to the Moderate Geoidal Heights, which range from 20.11 to 21.01 meters, this category signifies a moderate range of elevations across Anambra State. Extending its reach over a larger area of 3463.62 km², it hints at a more extensive geographical coverage with a diverse topographical canvas. Regions falling within this classification may exhibit moderate gravitational anomalies or display a variety of terrain features, contributing to the nuanced landscape observed in 2023. This expansive categorization provides a comprehensive view of the varied elevations and geological characteristics within the state during this specific year.

Concluding the categorization, the High Geoidal Heights unfold with elevations ranging from 21.01 to 21.73 meters, covering a substantial area of 4683.61 km². This category designates regions with higher elevations, suggesting potential gravitational highs, elevated terrain, or distinctive geological features contributing to the manifestation of elevated geoidal heights. The significant area size underscores the substantial portion of the analyzed region characterized by heightened geoidal heights, presenting a detailed picture of the geological diversity and gravitational dynamics during the year 2023.

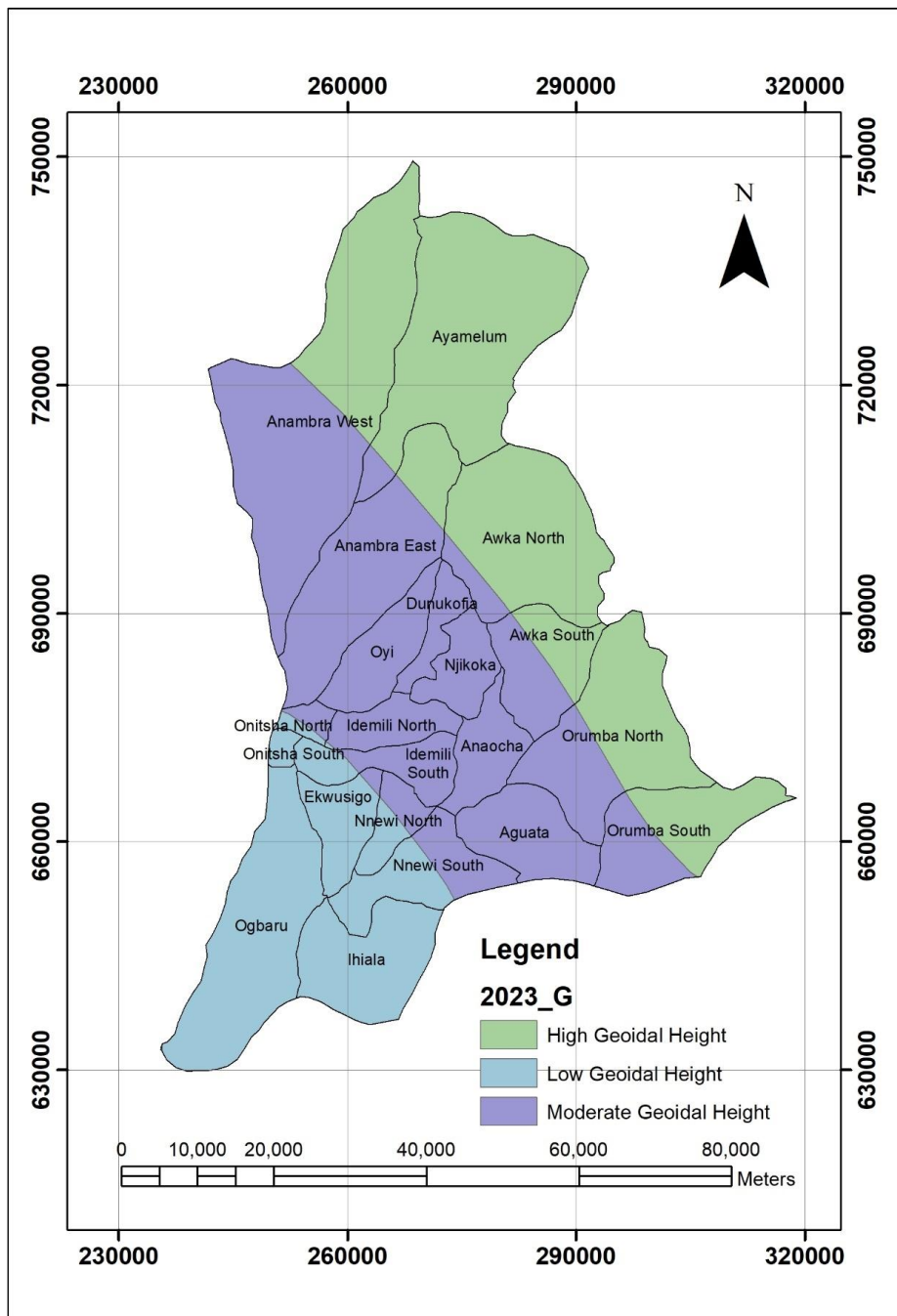


Figure 17: Spatial Variability of Geoidal Heights in 2023

IV. Conclusion

This study has yielded invaluable insights into the geodetic characteristics of Anambra State through the meticulous modeling and analysis of its geoid. The utilization of satellite altimetry has enabled the determination of a confined range of geoidal heights, revealing a relatively stable surface with discernible spatial fluctuations. The examination of geoid slope patterns further emphasized the consistency critical for applications in civil engineering and land use planning.

The accuracy and reliability of the determined geoid were assessed employing regression analysis, mean error, root mean square error, and standard deviation. The robust positive correlation, substantial explanatory power, and precision of the model, as indicated by various metrics, underscore its suitability for diverse applications, instilling confidence in its practical utility.

The exploration of spatial-temporal variability spanning the years 2013 to 2023 uncovered nuanced trends, with slight changes in geoidal heights suggesting potential shifts in local geological or environmental factors. The stability observed, coupled with a decrease in standard deviation, indicates a relatively uniform and consistent geoid surface within the region.

Moreover, the study elucidates the broader implications of geoidal variations, ranging from their significance in civil engineering projects to their role in gravitational studies, climate change impact assessments, and land use planning. The continuous monitoring established by this study provides a crucial baseline for future trend analysis, facilitating informed decision-making across diverse domains.

This research not only contributes a geodetic profile of Anambra State but also underscores the practical applications and implications of understanding geoidal variations. The findings serve as a valuable resource for policymakers, researchers, and professionals involved in diverse fields, providing a foundation for informed decision-making and sustainable development practices in the region.

Based on the findings of the study, the following recommendations were made:

- 1. Continued Monitoring:** The study results suggest that while trends are present, their impact on geodetic applications may be minimal within the specified timeframe. However, the complexity of Earth's gravitational dynamics warrants ongoing research and longer-term analyses to comprehensively grasp the implications of these trends. Continued monitoring and study are recommended to refine our understanding of the Earth's geoid and its potential variations.
- 2. Long-Term Trend Analysis:** As the study covers a specific timeframe (2013-2023), it is recommended to extend the temporal analysis for a more extended period. This will facilitate a deeper understanding of long-term trends and variations in the geoid, aiding in more accurate predictions and assessments.
- 3. Investigation into Localized Anomalies:** Areas exhibiting significant changes in geoidal heights should be investigated for localized anomalies. This could involve targeted studies to understand the specific factors contributing to these variations, providing valuable insights into regional geophysical dynamics.
- 4. Refinement of Accuracy Metrics:** Continuously refine and update the accuracy assessment metrics for geoid models. Consider incorporating additional metrics or methodologies to ensure a comprehensive evaluation, keeping pace with advancements in geodetic modeling techniques.
- 5. Comparison with Global Geodetic Trends:** Compare the observed geoidal variations in Anambra State with global geodetic trends. This comparative analysis can contribute to a broader understanding of regional versus global geodetic dynamics and their potential interconnectedness.
- 6. Development of Geoid Prediction Models:** Explore the development of geoid prediction models based on observed trends. These models can serve as predictive tools for anticipating future geoidal variations and their implications, aiding in proactive decision-making.

References

- [1] Adebisi, A., Oyedele, O., and Adekunle, A. (2015). Evaluation of geoid models for Nigeria. *Journal of Surveying Engineering*, 141(3), 1-8.
- [2] Bos, M. S., and Tregoning, P. (2015). Geodetic Earth observation with GRACE: Review and future prospects. *Remote Sensing of Environment*, 163, 117-129.
- [3] Ezeuduji, O. C., and Isioye, A. A. (2018). Regional geoid determination over Nigeria using GOCE gravity field model and satellite altimetry data. *Geocarto International*, 33(5), 498-515. doi: 10.1080/10106049.2016.1260339
- [4] Flechtner, F., Dahle, C., Gruber, C., König, R., Kusche, J., and Mayer-Gürr, T. (2017). AOD1B product description document for GOCE data release 6.0. GFZ Data Services.
- [5] Kirsch, M., Mayer-Gürr, T., Ilk, K. H., and Fecher, T. (2010). Combined satellite gravity field modeling using CHAMP, GRACE and GOCE data. In *Proceedings of the 3rd International GOCE User Workshop*, Munich, Germany.
- [6] Rummel, R. (2011). Gravity field and steady-state ocean circulation explorer (GOCE): ESA's first core earth explorer gravity mission. Springer Science and Business Media.
- [7] Smith, W. H. F., and Sandwell, D. T. (1997). Global sea floor topography from satellite altimetry and ship depth soundings. *Science*, 277(5334), 1956-1962.
- [8] Wunsch, C., and Heimbach, P. (2013). Dynamical balance, the ocean and nonlinear geostrophy. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 371(1991), 20120363.