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Lucas Antimagic Labeling Of Triangular and Quadrilateral Snake Graphs

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ABSTRACT
A (p, q) graph G is said to be a Lucas antimagic graph if there exists a bijection $f: E(G) \rightarrow \{L_1, L_2, \cdots L_q\}$ such that the induced injective function $f^*: V(G) \rightarrow \{1,2, \ldots \sum L_q\}$ given by $f^*(u) = \sum_{e \in E(u)} f(e)$ are all distinct (where E(u) is the set of edges incident to u).
In this paper the Lucas Antimagic Labeling of Triangular Snake, Double Triangular snake, Quadrilateral Snake and Double Quadrilateral Snake are found.
KEYWORDS: *Triangular Snake, Double Triangular snake, Quadrilateral Snake, Double Quadrilateral Snake , Lucas Antimagic graph.*

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I. INTRODUCTION

In this paper, we delve into the study of a graph G(V, E) which is characterized as finite, simple and undirected, consisting of p vertices and q edges. Graph labeling, a fundamental concept within graph theory, involves the assignment of integers to the vertices or edges of a graph. This concept has found extensive applications across various fields, including astronomy, coding theory, and numerous other scientific and engineering domains, thereby propelling it to the forefront of contemporary research.

The seminal work of Gallian, as meticulously documented in his comprehensive survey [1], has significantly influenced the trajectory of research in this area. His contributions have provided a robust foundation upon which many subsequent studies have been built. Inspired by Gallian's groundbreaking work, we have embarked on this research endeavor to explore new dimensions of graph labeling.

Furthermore, the innovative concept of Antimagic labeling, introduced by N. Hartsfield and G. Ringel in 1990, has opened up new avenues for exploration within the field. Antimagic labeling involves assigning integers to the edges of a graph such that the sums of the labels of the edges incident to each vertex are distinct. This intriguing concept has spurred a wealth of research, leading to the discovery of numerous interesting properties.

Building upon these foundational contributions, we introduce the concept of Lucas Antimagic labeling. Further Lucas Antimagic labeling has been investigated on Triangular Snake, Double Triangular snake, Quadrilateral Snake, Double Quadrilateral Snake.

II. DEFINITIONS

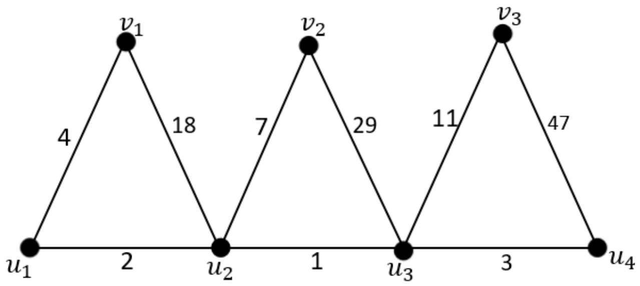
Definition 2.1: Lucas number is defined by
$L_1 = 2, L_2 = 1, L_n = L_{n-1} + L_{n-2}, if\ n > 2$
The first few Lucas numbers are 2,1,3,4,7,11,18,29,47,…
Definition 2.2:[2] A (p, q) graph G is said to be a Lucas antimagic graph if there exists a bijection $f: E(G) \rightarrow \{L_1, L_2, \cdots L_q\}$ such that the induced injective function $f^*: V(G) \rightarrow \{1,2, \ldots \sum L_q\}$ given by $f^*(u) = \sum_{e \in E(u)} f(e)$ are all distinct (where E(u) is the set of edges incident to u).
Definition 2.3:[3] A triangular snake T_n is obtained from a path $u_1, u_2, \ldots u_n$ by joining $u_i\, and\, u_{i+1}$ to new vertices v_i ,$1 \leq i \leq n-1$.

Definition 2.4:[4] A double triangular snake $D(T_n)$ is obtained from a path $u_1, u_2, \dots u_n$ by joining $u_i and\ u_{i+1}$ to new vertices $v_i\ and\ w_i\ for 1 \leq i \leq n-1$.
Definition 2.5:[5] A Quadrilateral snake Q_n is obtained from a path $u_1, u_2, \dots u_n$ by joining $u_i u_{i+1}$ to new vertices $v_i\ and\ w_i$ respectively and adding edges $v_i w_i$ for i=1,2,…n-1.That is every edge of a path is replaced by a cycle C_4
Definition 2.6:[5] A double Quadrilateral snake $D(Q_n)$ is obtained from two Quadrilateral snakes that have a common path.

III. MAIN RESULTS

Theorem 3.1:
The Triangular Snake $T_n (n \geq 3)$ is Lucas antimagic graph.
Proof:
Let G be T_n.
Let $V(G) = \{u_i: 1 \leq i \leq n, v_i: 1 \leq i \leq n-1\}$
$E(G) = \{u_i u_{i+1} : 1 \leq i \leq n-1, u_i v_i : 1 \leq i \leq n-1, v_i u_{i+1} : 1 \leq i \leq n-1\}$
Define a function $f: E(G) \to \{L_1, L_2, \dots L_q\}$ by
$f(u_i u_{i+1}) = L_i$,$1 \leq i \leq n-1$
$f(u_i v_i) = L_{n-1+i}, 1 \leq i \leq n-1$
$f(v_i u_{i+1}) = L_{2n-2+i}$,$1 \leq i \leq n-1$
The induced function $f^*: V(G) \to \{1,2, \dots \sum L_q\}$ is given by
$f^*(u_1) = L_1 + L_n$
$f^*(u_i) = L_i + L_{i-1} + L_{n-1+i} + L_{2n-3+i}, 2 \leq i \leq n-1$
$f^*(u_n) = L_{n-1} + L_{3n-3}$
$f^*(v_i) = L_{n-1+i} + L_{2n-2+i}, 1 \leq i \leq n-1$
We observe that the vertices are all distinct.
Hence G is Lucas antimagic graph.

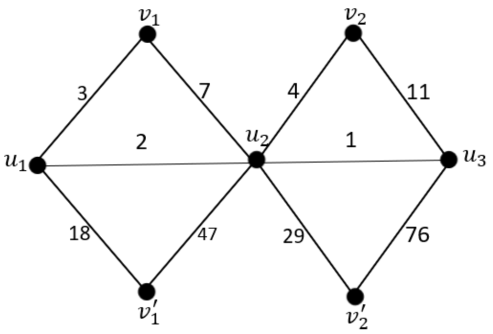
Example 3.1.1: The Triangular snake graph T_4 and its Lucas Antimagic Labeling.



Theorem 3.2:
The Double Triangular Snake $D(T_n)(n \geq 3)$ is Lucas antimagic graph.
Proof:
Let G be $D(T_n)$.
Let $V(G) = \{u_i: 1 \leq i \leq n, v_i, v_i': 1 \leq i \leq n-1\}$
$E(G) = \{u_i u_{i+1}, u_i v_i, v_i u_{i+1}, u_i v_i', v_i' u_{i+1}: 1 \leq i \leq n-1\}$
Define a function $f: E(G) \to \{L_1, L_2, \dots L_q\}$ by
$f(u_i u_{i+1}) = L_i$,$1 \leq i \leq n-1$
$f(u_i v_i) = L_{n-1+i}, 1 \leq i \leq n-1$
$f(v_i u_{i+1}) = L_{2n-2+i}$,$1 \leq i \leq n-1$
$f(u_i v_i') = L_{3n-3+i}$,$1 \leq i \leq n-1$
$f(v_i' u_{i+1}) = L_{4n-4+i}$,$1 \leq i \leq n-1$
The induced function $f^*: V(G) \to \{1,2, \dots \sum L_q\}$ is given by
$f^*(u_1) = L_1 + L_n + L_{3n-2}$
$f^*(u_i) = L_i + L_{i-1} + L_{n-1+i} + L_{2n-3+i} + L_{4n-5+i} + L_{3n-3+i}, 2 \leq i \leq n-1$
$f^*(u_n) = L_{n-1} + L_{3n-3} + L_{5n-5}$
$f^*(v_i) = L_{n-1+i} + L_{2n-2+i}, 1 \leq i \leq n-1$
$f^*(v_i') = L_{3n-3+i} + L_{4n-4+i}, 1 \leq i \leq n-1$
We observe that the vertices are all distinct.

Hence G is Lucas antimagic graph.

Example 3.2.1: The Double Triangular snake graph $D(T_3)$ and its Lucas Antimagic Labeling.



Theorem 3.3:

The Quadrilateral snake $Q_n (n \geq 3)$ is Lucas antimagic graph.

Proof:

Let G be Q_n.

Let $V(G) = \{u_i : 1 \leq i \leq n, v_i, w_i : 1 \leq i \leq n-1\}$

$E(G) = \{u_i u_{i+1}, u_i v_i, v_i w_i, u_{i+1} w_i : 1 \leq i \leq n-1\}$

Define a function $f : E(G) \rightarrow \{L_1, L_2, \ldots L_q\}$ by

$f(u_i u_{i+1}) = L_i, 1 \leq i \leq n-1$

$f(u_i v_i) = L_{n-1+i}, 1 \leq i \leq n-1$

$f(v_i w_i) = L_{2n-2+i}, 1 \leq i \leq n-1$

$f(u_{i+1} w_i) = L_{3n-3+i}, 1 \leq i \leq n-1$

The induced function $f^* : V(G) \rightarrow \{1, 2, \ldots \sum L_q\}$ is given by

$f^*(u_1) = L_1 + L_n$

$f^*(u_i) = L_i + L_{i-1} + L_{n-1+i} + L_{2n-3+i}, 2 \leq i \leq n-1$

$f^*(u_n) = L_{n-1} + L_{3n-3}$

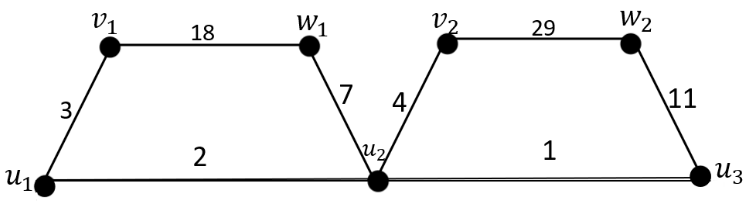
$f^*(v_i) = L_{n-1+i} + L_{3n-3+i}, 1 \leq i \leq n-1$

$f^*(w_i) = L_{3n-3+i} + L_{2n-2+i}, 1 \leq i \leq n-1$

We observe that the vertices are all distinct.

Hence G is Lucas antimagic graph.

Example 3.3.1: The Quadrilateral snake graph Q_3 and its Lucas Antimagic Labeling.



Theorem 3.4:

The Double Quadrilateral Snake $D(Q_n)(n \geq 3)$ is Lucas antimagic graph.

Proof:

Let G be $D(Q_n)$.

Let $V(G) = \{u_i : 1 \leq i \leq n, v_i, v_i', w_i, w_i' : 1 \leq i \leq n-1\}$

$E(G) = \{u_i u_{i+1}, u_i v_i, w_i u_{i+1}, v_i w_i, u_i v_i', w_i' u_{i+1}, v_i' w_i' : 1 \leq i \leq n-1\}$

Define a function $f : E(G) \rightarrow \{L_1, L_2, \ldots L_q\}$ by

$f(u_i u_{i+1}) = L_i, 1 \leq i \leq n-1$

$f(u_i v_i) = L_{n-1+i}, 1 \leq i \leq n-1$

$f(w_i u_{i+1}) = L_{2n-2+i}, 1 \leq i \leq n-1$

$f(v_i w_i) = L_{3n-3+i}, 1 \leq i \leq n-1$

$f(u_i v_i') = L_{4n-4+i}, 1 \leq i \leq n-1$

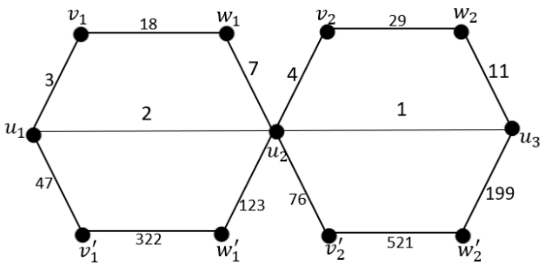
$f(w_i' u_{i+1}) = L_{5n-5+i}, 1 \leq i \leq n-1$

$f(v_i' w_i') = L_{6n-6+i}, 1 \leq i \leq n-1$

The induced function $f^* : V(G) \rightarrow \{1, 2, \ldots \sum L_q\}$ is given by

$f^*(u_1) = L_1 + L_n + L_{4n-3}$
$f^*(u_i) = L_i + L_{i-1} + L_{n-1+i} + L_{2n-3+i} + L_{4n-4+i} + L_{5n-6+i}, 2 \le i \le n-1$
$f^*(u_n) = L_{n-1} + L_{3n-3} + L_{6n-6}$
$f^*(v_i) = L_{n-1+i} + L_{3n-3+i}, 1 \le i \le n-1$
$f^*(w_i) = L_{2n-2+i} + L_{3n-3+i}, 1 \le i \le n-1$
$f^*(v_i') = L_{4n-4+i} + L_{6n-6+i}, 1 \le i \le n-1$
$f^*(w_i') = L_{5n-5+i} + L_{6n-6+i}, 1 \le i \le n-1$
We observe that the vertices are all distinct.
Hence G is Lucas antimagic graph.

Example 3.4.1: The Double Quadrilateral snake graph $D(Q_3)$ and its Lucas Antimagic Labeling.



IV. CONCLUSION

In this paper, We have proved Triangular Snake, Double Triangular snake, Quadrilateral Snake, Double Quadrilateral Snake graphs are Lucas antimagic. Similar investigations are in process.

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