

Analysis of uncertainties in culvert stability using fuzzy Inference system for the evaluation of Reliability index by Tsukamoto method for Gulakamale watershed

D Kalaswamy Naik ¹, A V Sriram²

¹ Research Scholar, Department of Civil Engineering, UVCE, Bangalore University, Bangalore, Karnataka, India

² Professor, Department of Civil Engineering, UVCE, Bangalore University, Bangalore, Karnataka, India

Abstract

If the design of hydraulic structures in water resources engineering are not adequate and economical sometimes it may lead to difficulties in the vehicular traffic and disruptions in the daily activities of commuters in the region during witnessing of heavy floods which are very frequent nowadays. In the context of such factors the purpose of this research is to introduce the Fuzzy Inference System as an effective method that can assist in determining an optimal result in each fuzzy variable. In the method output is obtained with four stages, namely the formation of fuzzy sets, the establishment of rules, the usage of implicated functions and defuzzification. Fuzzy Inference System, with Tsukamoto method is implemented to facilitate and accelerate the decision-making processes, in the assessment of stability of culvert in an ungauged watershed. Moreover, the fuzzy input variables involved are rainfall intensity, runoff coefficient and manning's coefficient. Output variable is Reliability Index thereby probability of failure can be obtained.

Key Words: Reliability Index, Tsukamoto method, rainfall intensity, runoff coefficient, manning's coefficient

Date of Submission: 03-12-2024

Date of Acceptance: 14-12-2024

I. Introduction

Recently, soft computing technique such as Fuzzy logic and neural networks techniques have been applied successfully to different applications for decision support systems. Fuzzy logic theory provides a useful solution to understanding, quantifying and handling vague, ambiguous and uncertain data (Dutt & Kurian, 2013). Besides that, fuzzy logic is the study of methods which corresponds to a set of principles in giving meaningful information on unconditional or approximate reasoning that can be understood in human languages (Phan & Chen, 2000). Fuzzy logic is the technique that facilitates the control of a complicated system without knowledge of its mathematical description.

Fuzzy set theory (FST) has been used in water resource engineering problems to cope with uncertain data due to lack of precision, incompleteness, vagueness and randomness of the information as well as incorporating subjective judgement from experts into problems analysis.

Fuzzy logic is an extension of Boolean logic dealing with the concept of partial truth. In recent years more and more applications of fuzzy theory to water resource engineering have been reported. Classifications of uniform plant, soil and residue color images were conducted with fuzzy inference systems by Meyer (2004). In this research fuzzy logic-based solution is proposed for reliability analysis for the assessment of stability of culvert in an ungauged watershed. The input parameters are: Rainfall Intensity(i) and Runoff coefficient (c). A detailed reliability fuzzy logic system is developed based on rule-based inferencing to solve reliability of hydraulic structure issues. Analysis of the results obtained using Tsukamoto method Fuzzy Inference System is verified using Monte Carlo Simulation method.

A fuzzy set is an extension of the concept of crisp set. While crisp set only allows full membership or no membership to every element of a universe of discourse, a fuzzy set allows for partial membership. Basically, fuzzy set theory includes fuzzy inference system, fuzzy probability and hybrid fuzzy set.

Some, uncertainties involved such as failure criteria/loads which are not random in nature, may play important roles in the safety assessment of a hydraulic structure/culvert. By use of the theory of fuzzy sets structure stability process can be treated as a fuzzy event. The aim of this research is to implement FIS with the Tsukamoto method and other FIS systems namely Mamdani FIS and Sugeno FIS systems in the evaluation of Reliability Index in an ungauged catchment to help to take decision on selection of minimum Reliability index

keeping in view the stability status of hydraulic structure. A detailed fuzzy logic system is developed based on rule-based inferencing to solve the reliability issues in assessing the stability of the hydraulic structure/culvert. By FIS with the Tsukamoto method, which involves fuzzification, inference and defuzzification the evaluation process of Reliability Index become more accurate with using a weighted average. The factors influencing in the process as input variables are Rainfall Intensity and Runoff coefficient and other variables also. With the decision supporting system based on FIS with Tsukamoto method the calculation of Reliability Index agrees with the other reliability methods used for the analysis.

In the present study the objective in the analysis of uncertainties in culvert stability using fuzzy Inference system for the evaluation of Reliability index by Tsukamoto method for Gulakamale watershed. This paper addresses the effects of input variables in a hydraulic structure culvert responses under various load and Resistance conditions. To compare the results with Probabilistic Monte Carlo Simulation Method which is standard bench mark method.

II. Study Area and Data Used

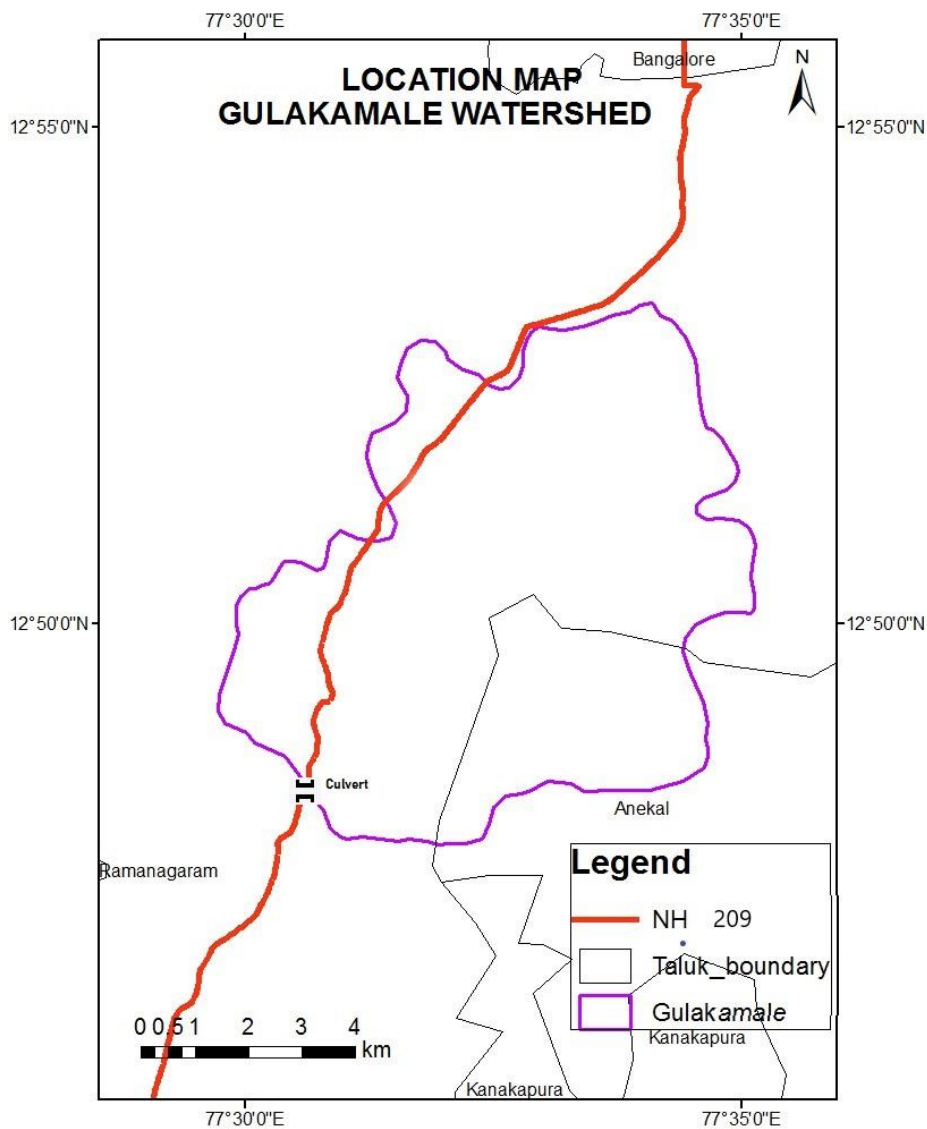


Fig. 1 Location of Gulakamale watershed.

The study area chosen is Gulakamale watershed which lies in Bangalore district. The culvert with its longitude of $77^{\circ} 31' 50.07''$ E and latitude of $12^{\circ} 47' 50.07''$ N is situated over the two-lane district highway road near Kaggalipura village. The nearest national highway passing through to the watershed is NH 209. The location map in Figure 1, showed some of the salient features in the vicinity of Gulakamale watershed.

The following are the input variables considered for the experiment conducted in this study.

The Rational method is adopted to estimate the incoming peak rate of runoff (Load) at a specific watershed location as a function of drainage area. The flow delivery system for the culvert (Capacity) was estimated by Manning's formula. The performance function for the culvert is expressed in terms of Resistance and Loading. The Reliability Index is the number of standard deviations by which the expected value of the performance function exceeds the limit state. Reliability analysis carried out using Monte Carlo Simulation Method.

Table 1: Data of Input variables

Uncertainty Parameter	Values of input variables		
Rainfall Intensity(i) mm/h	9.0	26.82	39
Runoff coefficient(C)	0.25	0.346	0.85
Manning's coefficient	0.025	0.025	0.025

III. Methodology for fuzzy logic system

Fuzzy Logic:

Zadeh states that fuzzy logic is associated with the principles of formal reasoning on unconditional things or approximate reasoning. However fuzzy set theory does not replace the theory of probabilities. In the fuzzy set theory, the most influential component is membership functions. There are several reasons why fuzzy logic is used [Kusumadevi, Purnomo, (2010)].

Mapping input into their grades of membership described as a straight line. It is the simplest form and an excellent option to approach vague concepts. Increasing, linear representation is a set increase started from a domain value whose membership grade is zero, moving to the right to domain values with higher grades of membership. Decreasing linear representation is the opposite to that, wherein a straight line is begun with a domain value with the highest grade of membership on the left, then going down to the domain value with lower grades of membership. Triangular curve representation is a combination of two linear lines shown in the figure below. Shoulder shaped representation is an area located in the middle – between variables represented in a triangle on the right and the left going up and down as shown in the figure below.

Fuzzy System:

Fuzzy theory can be considered as a set of principles within an extension of infinite-valued logic in the sense of incorporating fuzzy sets and fuzzy relations (Maria Bojadziev & George Bojadziev. 1995).

- i) Fuzzy variables are the main elements in a fuzzy system. They are dominant elements which affect the overall system.
- ii) Fuzzy sets are a collection of fuzzy variables which has been directed in a specific state. It enables the description of the variables that can be restricted within a particular range that can be referred.
- iii) Universe sets are entire reasoning values which are permitted to manipulate the variables within them. They are a group of real digits Conventionally increasing from left to right. The values can be either positive or negative.
- iv) The Domain is a subset of the universe set within the fuzzy set. It acts as the overall range for the specific variable in which it can be included in the same series. Similarly, the domain set also increases from left to right and their values can either be presented as positive or negative.

Fuzzy rule-based systems:

Linguistic variable is a numerical interval with linguistic values which is defined by its membership functions. The fuzzy rule-based system consists of three main components as shown in Figure 2.

1. Fuzzification
2. Inference
3. Defuzzification

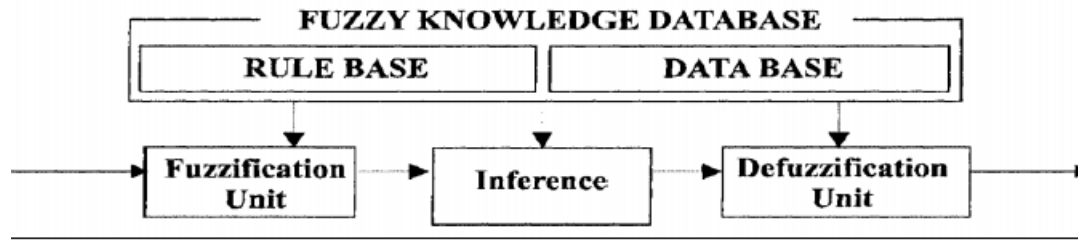


Fig. 2 Basic structure of FIS (Pappis & Siettos, 2005)

Fuzzification

Fuzzification is value mapping process of crisp inputs coming from the system, controlled into a fuzzy set along with its membership functions. The fuzzy set is fuzzy inputs being processed in the next fuzzical process. To change crisp inputs into fuzzy inputs requires determining the membership functions for each crisp input and compare them with the existing membership functions to generate values of fuzzy inputs. Finding the accurate shape and the boundaries for the membership functions increases the accuracy of the results.

Inference

Inference is a processed relationship between values of crisp inputs and values of crisp outputs which are expected by particular rules. These rules will determine the system's response to various conditions of setting points and disruptions in system. The used rules are IF-THEN.

Defuzzification

Defuzzification typically involves weighting and combining a number of fuzzy sets resulting from the fuzzy inference process in a calculation, which gives a single crisp value for each output (Pappis & Seiettos, 2005) also mentioned that the most commonly used defuzzification methods are mean of maximum, centroid and centre of sum of areas. Defuzzification is a stage where minimum values ($\alpha_1, \alpha_2, \dots, \alpha_n$) are defined, then finding the values of z_1 (approximate values), calculating crisp values and outputs.

$$Z = \frac{(\alpha_1 * z_1 + \alpha_2 * z_2 + \alpha_3 * z_3 + \dots + \alpha_n * z_n)}{(\alpha_1 + \alpha_2 + \alpha_3 + \dots + \alpha_n)}$$

Fuzzy Set

Fuzzy logic was born in line with imprecise natural phenomenon, which is reviewed from people's perspectives in which no condition or statement is exactly right or wrong. Lotfi A. Zadeh suggests that fuzzy set is a class of objects with a united series of membership grades. A set is characterized by functions, which gives each object a membership grade from 0 to 1. Ideas of inclusion, union, intersection, complement, relations and convexity are given to the set, and various properties of these ideas, in the context of fuzzy set are constructed.

Fuzzy Operators

Zadeh's basic types of fuzzy set operations are similar to a conventional set. Some operations are specifically defined to combine and modify fuzzy sets. Membership values are the results of two-set operations called fire-strength α -predicates. There are three basic operators suggested by Zadeh, namely: AND, OR and NOT. α -predicate sets are obtained as a result of use of AND operator by taking smallest membership value among elements of the set.

$\alpha = \mu$ runoff coefficient (x) \cap μ rainfall intensity (y) \cap μ Manning's coefficient (z)

Implicative Function

Each proposition on *fuzzy* principles is corresponding to a *fuzzy* relation. The general form of the applied rules in the implicative function is as follows:

IF x is A THEN y is B

x and y are scalar. A and B are fuzzy sets. Proposition following IF is called antecedent. Proposition following IF is called consequence. The propositions can be extended by fuzzy operators as follows.

IF (x_1 is A_1) \circ (x_2 is A_2) \circ (x_3 is A_3) \circ (x_n is A_n) \circ THEN y is B

\circ is an operator. In general, there are two implicative functions.

- Min (minimum) is a function which cuts outputs of a fuzzy set.
- Dot (product) is a function scaling outputs of a fuzzy set.

Fuzzy Inference System (Tsukamoto Method)

In Tsukamoto method each consequence of the IF-THEN rules must be represented by a fuzzy set with monotonous membership function. Consequently, the inference outputs of each rule are crisply presented in line with α -predicate (firestrength). The end result is obtained by a weighted average.

For example, there are two input variables-Variable 1(x) and variable 2(y) and one output variable 3 (z). The variable 1 is divided into two sets, namely A1 and A2. The variable 2 is divided into two sets, namely B1 and B2. The variable 3 is divided into two sets, namely C1 and C2 (C1 and C2 must be monotonous). Two rules are used as follows:

Rule 1 IF (x is A₁) AND (y is B₂) THEN z is C₁.

Rule 2 IF (x is A₂) AND (y is B₂) THEN z is C₂.

First, membership functions of each fuzzy set of each rule is the set A1, B1 and C1 from the fuzzy rule [R1] and set A2, B2 and C2 from the fuzzy rule [R2]. From fuzzy rule R1 and R2 the crisp value Z can be obtained.

Because, in Tsukamoto method, the used set operation is a conjunction (AND), the membership values of antecedents from the fuzzy rule [R1] is a portion of the membership value A1 from variable-1 with the membership value B1 from variable-2.

According to the theory of set operations, the value of antecedent memberships from the conjunction operation (AND) of the fuzzy rule [R1] is the minimum value between the membership value A1 from variable-1 and membership value B1 from variable-2.

Likewise, value of antecedent memberships from the fuzzy rule [R2] is the minimum value between the membership value A2 from variable-1 and membership value B2 from variable-2.

In continuation, membership values of antecedents from the fuzzy rule [R1] and [R2] called α_1 and α_2 are substituted into the set of membership functions C1 and C2 in line with fuzzy rule [R1] and [R2] to obtain the values of z1 and z2 which are the values of Z (approximate values) for the fuzzy rule [R1] and [R2].

To obtain crisp output value Z requires changing the input values which are obtained by the composition of the fuzzy rules into a number is called the defuzzification method.

The defuzzification method used in Tsukamoto method is Weighed Average Defuzzifier formulated in an equation.

Fuzzy set has two attributes (Maria Bojadziev & George Bojadziev 2006), as follows:

1. Linguistic
2. Numeric.

Linguistic variables that are variables whose values are not numbers but words or sentences in a natural or artificial Language such as very, less and moderate etc.

Numeric is a value (number) indicating a size of variables, such as:0.5, -2,10 etc.

Kazeminezhad et al. (2005) mentioned that FIS can be used to predict uncertain systems and its application does not require knowledge of the underlying physical process as a precondition. Moreover, Nauck and Kruse (1999) mentioned that the success of FIS is due to its closeness to human perception and reasoning, as well as its intuitive handling and simplicity, which are important factors for acceptance and usability of the systems. The same methodology can be extended for more than two input variables adopting the modifications using rule-based inferencing.

IV. Results and Discussion:

This study used Tsukamoto fuzzy inference system specification as follows:

The three input variables considered in the evaluation of performance function in the reliability analysis are Rainfall intensity(i), Runoff coefficient(c) and Manning's coefficient(n).

The main idea of the Tsukamoto method is to describe the process states by linguistic variables and to use these variables as inputs to control rules. In Tsukamoto method which is a particular type of fuzzy inference system, in addition to knowledge base and a fuzzy inference engine, there is a fuzzifier that represents inputs numerical as fuzzy set, and a defuzzifier that transforms the output set to crisp.

To apply the technique, a total of 3 situations were selected. For the watershed, membership functions of the rainfall intensity parameter were labeled in four features "Very low, Low, High and Very high" (Fig.3), membership functions of the runoff coefficient parameter were labeled in four features "Low, medium, High and Very high" (Fig.4). and membership functions of the Manning's coefficient parameter were labeled in four features "Smooth, Very smooth, Rough and Very rough" (Fig.5). The largest rainfall intensity is calculated and four levels of [0-15], [10-30], [25-35] and [30-45] were allocated to Very low, Low, High and Very high, range of runoff coefficient is calculated and four levels of [0-0.38], [0.2-0.5], [0.4-0.9] and [0.8-1.0] were allocated to Low, medium, High and Very high classes and range of Manning's coefficient is also calculated and four levels

[0.01-0.02], [0.015-0.035], [0.03-0.045] and [0.04-0.05] were allocated to Smooth, Very smooth, Rough and Very rough. Fig. 6 shows the output of fuzzy system in four reliability index features “Perfectly safe, Safe, Moderately safe and Failure” with levels of [3-5], [0.5-2.5], [(-1) - 1] and [(-3) -0]. Many researchers have investigated techniques for determining rules, and expert knowledge to construct the fuzzy model are explained in the rule definition. The Tsukamoto method used here has $4 \times 4 \times 4 = 64$ rules based on the membership functions considered for inputs. The decision method used for fuzzy logic operators AND (intersection) is “MIN”.

Membership functions for Rainfall intensity as input variable is shown in figure 3.

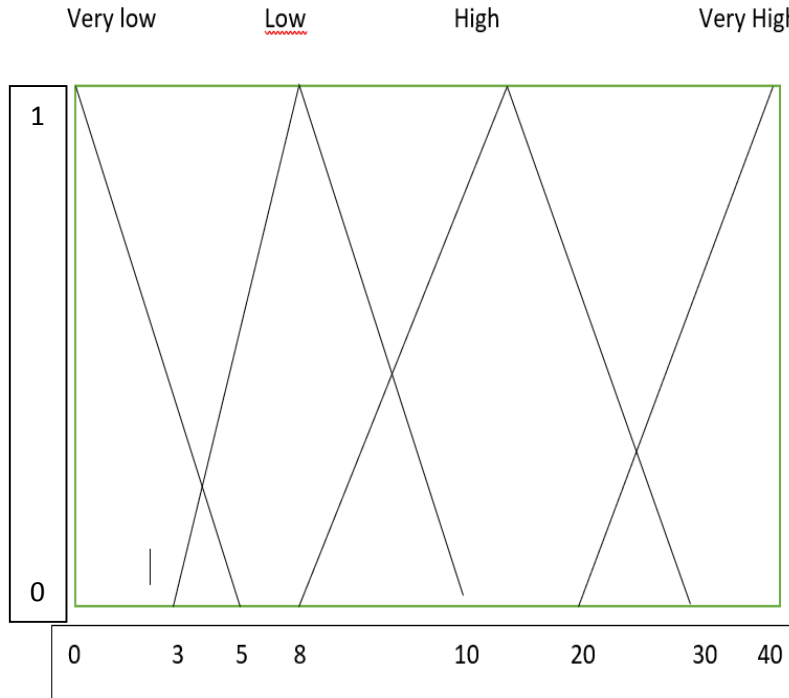


Fig. 3 Input Membership function for Rainfall intensity variable

Membership function for Runoff coefficient as input variable is shown in figure 4.

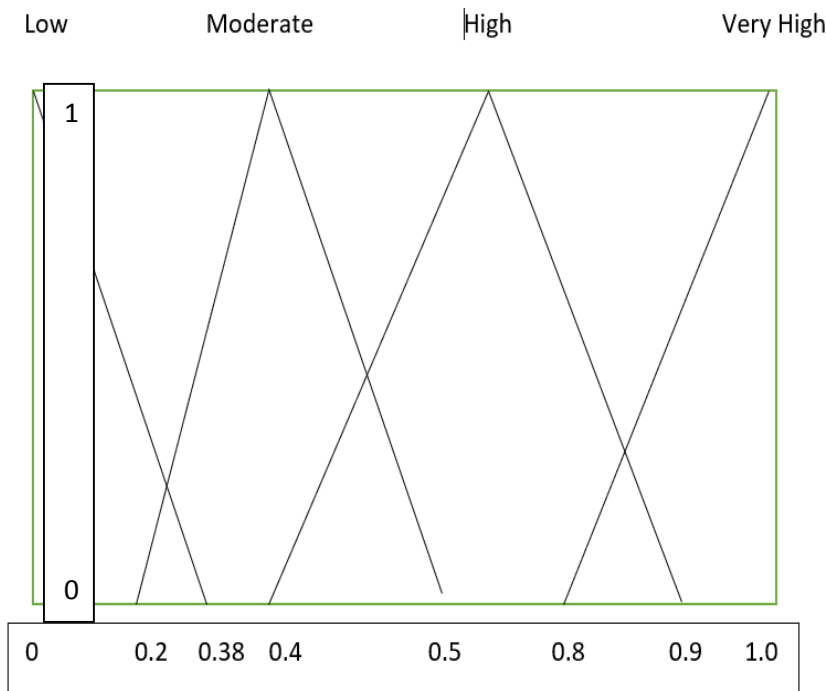


Fig. 4 Input Membership function for Runoff coefficient variable

Membership function for Manning's coefficient as input variable is shown in figure 5.

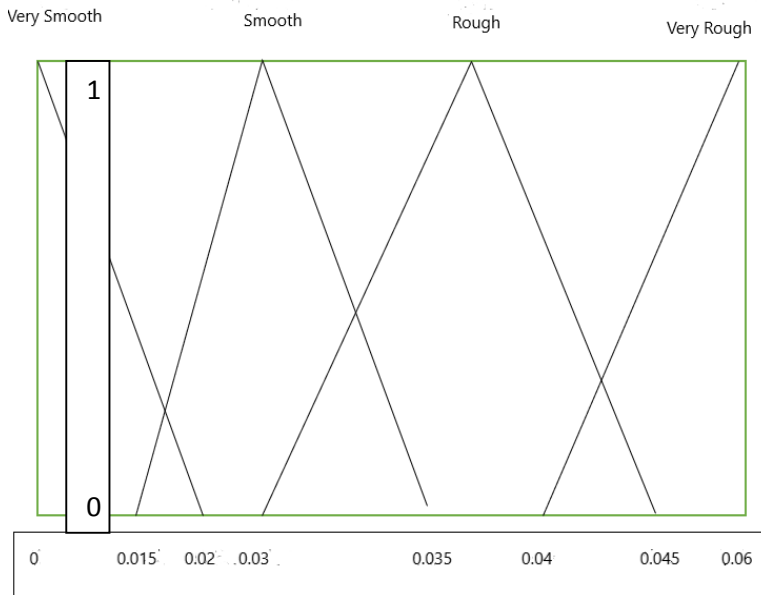


Fig. 5 Membership function for Manning's coefficient as Input variable

Membership function for reliability index as output variable is shown in figure 6.

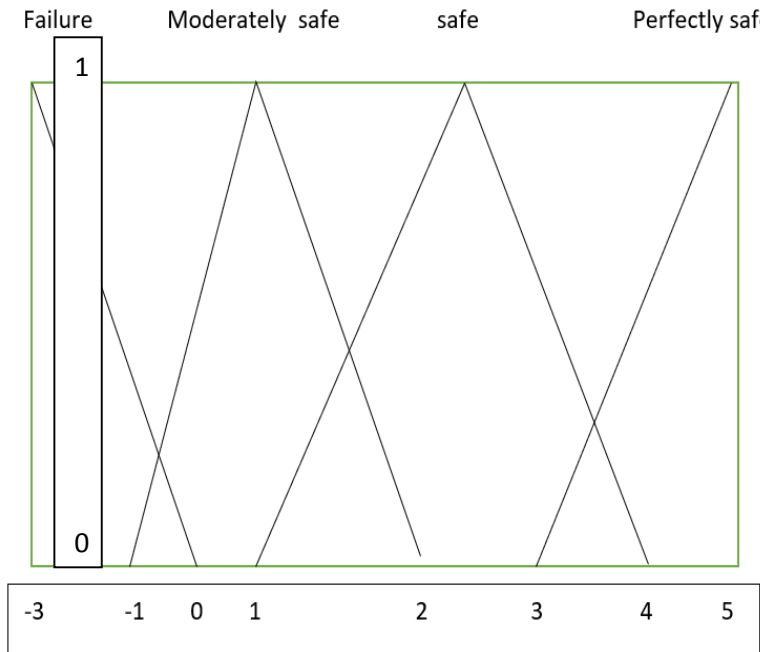


Fig. 6 Membership function for Reliability index as output variable

Determination of membership functions in terms of shape, boundaries and overlapping has a significant effect on the FIS output. This greatly depends on the expert knowledge. Finding the accurate shape and the boundaries for the membership functions increases the accuracy of the results. In this research some properties of applied system, such as membership functions, shape, threshold, which is to determine the overlapping amount and condition among the membership functions, input and output levels, and rules, were tested to find the optimum results. Results showed by applying Triangular membership functions (tmfs) for input and output improved the accuracy. To apply Tsukamoto method to evaluate the value of reliability index in other regions of watersheds the membership functions would need to be tuned to obtain sensible evaluation of results. Statistics of the class population, such as mean, standard deviation and minimum-maximum values, could help the determination of

membership functions (Kavdir and Guyer (2003). For comparison the Monte Carlo Simulation method, is used to estimate the Expected value and variance of performance function, then Reliability index is calculated.

4.1 FUZZIFICATION

In this study three combination of input variables used as shown in table 3.

There are three experiments conducted in the reliability analysis to obtain the value of reliability index using the three input variables Runoff coefficient(C) , Rainfall Intensity(i) and Mannig's coefficient keeping other input variables as constant. The procedure can be extended for more than three input variables also.

Table 3: Data Sampling

Sl.No	Runoff coefficient(C)	Rainfall Intensity(i)in mm/h	Mannig's Coefficient
1	0.25	09	0.025
2	0.346	26	0.025
3	0.85	39	0.025

Fuzzification for **Runoff coefficient** variable with crisp input 0.25 is obtained by

$$\mu[x] = \begin{pmatrix} 0 & x \leq a, x \geq c \\ (x - a)/(b - a) & a < x \leq b \\ (c - a)/(c - b) & b < x \leq c \end{pmatrix}$$

$$\mu_{low}[x] = \begin{pmatrix} 0 & x > 0.25 \\ (0.38 - 0.25)/(0.38 - 0) & 0 < x \leq 0.25 \end{pmatrix}$$

$$\mu_{low}[x] = 0.35$$

$$\mu_{moderate}[x] = \begin{pmatrix} 0 & x > 0.2 \\ (0.25 - 0.2)/(0.5 - 0.2) & 0.2 < x \leq 0.5 \end{pmatrix}$$

$$\mu_{moderate}[x] = 0.33$$

Fuzzification for **Rainfall intensity** variable with crisp input 09 mm/h is obtained by

$$\mu[x] = \begin{pmatrix} 0 & x \leq a, x \geq c \\ (x - a)/(b - a) & a < x \leq b \\ (c - a)/(c - b) & b < x \leq c \end{pmatrix}$$

$$\mu_{low}[x] = \begin{pmatrix} 0 & x \geq 9 \\ (9 - 6.5)/(10 - 6.5) & 6.5 < x \leq 10 \end{pmatrix}$$

$$\mu_{low}[x] = 0.71$$

$$\mu_{high}[x] = \begin{pmatrix} 0 & x \geq 8 \\ (9 - 8)/(11 - 8) & 8 < x \leq 11 \end{pmatrix}$$

$$\mu_{high}[x] = 0.33$$

4.2 INFERENCE

The rule definition is explained below.

R₁ IF x₁ is Low AND x₂ Very Low AND x₃ is Very smooth THEN y is Perfectly Safe

R₂ IF x₁ is Low AND x₂ Very Low AND x₃ is Smooth THEN y is Perfectly Safe

R₃ IF x₁ is Low AND x₂ Very Low AND x₃ is Rough THEN y is Perfectly Safe

- R₄ IF x₁ is Low AND x₂ Very Low AND x₃ is Very rough THEN y is Perfectly Safe
- R₅ IF x₁ is Low AND x₂ Low AND x₃ is Very smooth THEN y is Perfectly Safe
- R₆ IF x₁ is Low AND x₂ Low AND x₃ is Smooth THEN y is Perfectly Safe
- R₇ IF x₁ is Low AND x₂ Low AND x₃ is Rough THEN y is Perfectly Safe
- R₈ IF x₁ is Low AND x₂ Low AND x₃ is Very rough THEN y is Perfectly Safe
- R₉ IF x₁ is Low AND x₂ High AND x₃ is Very smooth THEN y is Safe
- R₁₀ IF x₁ is Low AND x₂ High AND x₃ is Smooth THEN y is Safe
- R₁₁ IF x₁ is Low AND x₂ High AND x₃ is Rough THEN y is Safe
- R₁₂ IF x₁ is Low AND x₂ High AND x₃ is Very rough THEN y is Safe
- R₁₃ IF x₁ is Low AND x₂ Very High AND x₃ is Very smooth THEN y is Moderate Safe
- R₁₄ IF x₁ is Low AND x₂ Very High AND x₃ is Smooth THEN y is Moderate Safe
- R₁₅ IF x₁ is Low AND x₂ Very High AND x₃ is Rough THEN y is Moderate Safe
- R₁₆ IF x₁ is Low AND x₂ Very High AND x₃ is Very rough THEN y is Moderate Safe
- R₁₇ IF x₁ is Moderate AND x₂ Very Low AND x₃ is Very smooth THEN y is Perfectly Safe
- R₁₈ IF x₁ is Moderate AND x₂ Very Low AND x₃ is Smooth THEN y is Perfectly Safe
- R₁₉ IF x₁ is Moderate AND x₂ Very Low AND x₃ is Rough THEN y is Perfectly Safe
- R₂₀ IF x₁ is Moderate AND x₂ Very Low AND x₃ is Very rough THEN y is Perfectly Safe
- R₂₁ IF x₁ is Moderate AND x₂ Low AND x₃ is Very smooth THEN y is Safe
- R₂₂ IF x₁ is Moderate AND x₂ Low AND x₃ is Smooth THEN y is Safe
- R₂₃ IF x₁ is Moderate AND x₂ Low AND x₃ is Rough THEN y is Safe
- R₂₄ IF x₁ is Moderate AND x₂ Low AND x₃ is Very rough THEN y is Safe
- R₂₅ IF x₁ is Moderate AND x₂ High AND x₃ is Very smooth THEN y is Safe
- R₂₆ IF x₁ is Moderate AND x₂ High AND x₃ is Smooth THEN y is Safe
- R₂₇ IF x₁ is Moderate AND x₂ High AND x₃ is Rough THEN y is Safe
- R₂₈ IF x₁ is Moderate AND x₂ High AND x₃ is Very rough THEN y is Safe
- R₂₉ IF x₁ is Moderate AND x₂ Very high AND x₃ is Very smooth THEN y is Moderate Safe
- R₃₀ IF x₁ is Moderate AND x₂ Very high AND x₃ is Smooth THEN y is Moderate Safe
- R₃₁ IF x₁ is Moderate AND x₂ Very high AND x₃ is Rough THEN y is Moderate Safe
- R₃₂ IF x₁ is Moderate AND x₂ Very high AND x₃ is Very rough THEN y is Moderate Safe
- R₃₃ IF x₁ is High AND x₂ Very Low AND x₃ is Very smooth THEN y is Safe

- R₃₄ IF x₁ is High AND x₂ Very Low AND x₃ is Smooth THEN y is Safe
- R₃₅ IF x₁ is High AND x₂ Very Low AND x₃ is Rough THEN y is Safe
- R₃₆ IF x₁ is High AND x₂ Very Low AND x₃ is Very rough THEN y is Safe
- R₃₇ IF x₁ is High AND x₂ Low AND x₃ is Very smooth THEN y is Moderate Safe
- R₃₈ IF x₁ is High AND x₂ Low AND x₃ is Smooth THEN y is Moderate Safe
- R₃₉ IF x₁ is High AND x₂ Low AND x₃ is Rough THEN y is Moderate Safe
- R₄₀ IF x₁ is High AND x₂ Low AND x₃ is Very rough THEN y is Moderate Safe
- R₄₁ IF x₁ is High AND x₂ High AND x₃ is Very smooth THEN y is Failure
- R₄₂ IF x₁ is High AND x₂ High AND x₃ is Smooth THEN y is Failure
- R₄₃ IF x₁ is High AND x₂ High AND x₃ is Rough THEN y is Failure
- R₄₄ IF x₁ is High AND x₂ High AND x₃ is Very rough THEN y is Failure
- R₄₅ IF x₁ is High AND x₂ Very High AND x₃ is Very smooth THEN y is Failure
- R₄₆ IF x₁ is High AND x₂ Very High AND x₃ is Smooth THEN y is Failure
- R₄₇ IF x₁ is High AND x₂ Very High AND x₃ is Rough THEN y is Failure
- R₄₈ IF x₁ is High AND x₂ Very High AND x₃ is Very rough THEN y is Failure
- R₄₉ IF x₁ is Very High AND x₂ Very Low AND x₃ is Very smooth THEN y is Moderate Safe
- R₅₀ IF x₁ is Very High AND x₂ Very Low AND x₃ is Smooth THEN y is Moderate Safe
- R₅₁ IF x₁ is Very High AND x₂ Very Low AND x₃ is Rough THEN y is Moderate Safe
- R₅₂ IF x₁ is Very High AND x₂ Very Low AND x₃ is Very rough THEN y is Moderate Safe
- R₅₃ IF x₁ is Very High AND x₂ Low AND x₃ is Very smooth THEN y is Moderate Safe
- R₅₄ IF x₁ is Very High AND x₂ Low AND x₃ is Smooth THEN y is Moderate Safe
- R₅₅ IF x₁ is Very High AND x₂ Low AND x₃ is Rough THEN y is Moderate Safe
- R₅₆ IF x₁ is Very High AND x₂ Low AND x₃ is Very rough THEN y is Moderate Safe
- R₅₇ IF x₁ is Very High AND x₂ High AND x₃ is Very smooth THEN y is Failure
- R₅₈ IF x₁ is Very High AND x₂ High AND x₃ is Smooth THEN y is Failure
- R₅₉ IF x₁ is Very High AND x₂ High AND x₃ is Rough THEN y is Failure
- R₆₀ IF x₁ is Very High AND x₂ High AND x₃ is Very rough THEN y is Failure
- R₆₁ IF x₁ is Very High AND x₂ Very High AND x₃ is Very smooth THEN y is Failure
- R₆₂ IF x₁ is Very High AND x₂ Very High AND x₃ is Smooth THEN y is Failure

R₆₃ IF x₁ is Very High AND x₂ Very High AND x₃ is Rough THEN y is Failure

R₆₄ IF x₁ is Very High AND x₂ Very High AND x₃ is Very rough THEN y is Failure

4.3 DEFUZZIFICATION

The results of Tsukamoto method using crisp input of runoff coefficient of 0.25 and rainfall intensity of 9 mm/h and Manning's coefficient is 0.025 gives the reliability index value of 2.07 which shows that culvert is safe.

1. Rule 1

If runoff coefficient is low and rainfall intensity is low and Manning's coefficient is Low then the reliability index value perfectly safe

$$\begin{aligned} \alpha_1 &= \mu \text{ runoff coefficient low} \cap \mu \text{ rainfall intensity low and Manning's coefficient} \\ &= \min(\mu \text{ runoff coefficient low}(0.25) \cap \mu \text{ rainfall intensity low}(9) \cap \mu \text{ Manning's coefficient}(0.025)) \\ &= \min(0.35, 0.71, 0.6) \\ &= 0.35 \end{aligned}$$

If runoff coefficient is low (0.35) and rainfall intensity is low (0.71) and Manning's coefficient (0.6) then the reliability index value (0.35)

$$Z_1 = \frac{(Z - 3)}{2} = 0.35$$

$$Z_1 = 3.70$$

2. Rule 2

If runoff coefficient is low and rainfall intensity is high and Manning's coefficient is smooth then the reliability index value safe

$$\begin{aligned} \alpha_2 &= \mu \text{ runoff coefficient low} \cap \mu \text{ rainfall intensity high} \cap \mu \text{ Manning's coefficient Smooth} \\ &= \min(\mu \text{ runoff coefficient low}(0.25) \cap \mu \text{ rainfall intensity high}(9) \cap \mu \text{ Manning's coefficient}(0.025)) \\ &= \min(0.35, 0.33, 0.6) \\ &= 0.33 \end{aligned}$$

If runoff coefficient is low (0.35) and rainfall intensity is high (0.33) and Manning's coefficient (0.6) then the reliability index value (0.33)

$$Z_2 = \frac{(Z - (-1))}{1.5} = 0.33$$

$$Z_2 = 1.495$$

3. Rule 3

If runoff coefficient is moderate and rainfall intensity is low and Manning's coefficient is Smooth then the reliability index value safe

$$\begin{aligned} \alpha_3 &= \mu \text{ runoff coefficient moderate} \cap \mu \text{ rainfall intensity low} \cap \mu \text{ Manning's coefficient Smooth} \\ &= \min(\mu \text{ runoff coefficient moderate}(0.25) \cap \mu \text{ rainfall intensity Low}(9) \cap \\ &\mu \text{ Manning's coefficient Smooth}(0.025)) \\ &= \min(0.33, 0.33, 0.6) \\ &= 0.33 \end{aligned}$$

If runoff coefficient is moderate(0.33) and rainfall intensity is low (0.33) and Manning's coefficient (0.6) then the reliability index value (0.33)

$$Z_3 = \frac{(Z - (1))}{1.5} = 0.33$$

$$Z_3 = 1.495$$

4. Rule 4

If runoff coefficient is moderate and rainfall intensity is high and Manning's coefficient then the reliability index value safe

$$\begin{aligned} \alpha_4 &= \mu \text{ runoff coefficient moderate} \cap \mu \text{ rainfall intensity high and} \mu \text{ Manning's coefficient Smooth} \\ &= \min(\mu \text{ runoff coefficient moderate}(0.25) \cap \mu \text{ rainfall intensity high}(9) \cap \\ &\mu \text{ Manning's coefficient}(0.025)) \\ &= \min(0.33, 0.33, 0.6) \\ &= 0.33 \end{aligned}$$

If runoff coefficient is moderate(0.33) and rainfall intensity is high (0.33) and Manning's coefficient (0.8) then the reliability index value (0.33)

$$Z_4 = \frac{(Z - (-1))}{1.5} = 0.33$$

$$Z_4 = 1.495$$

$$\begin{aligned} \text{Reliability index} &= \frac{(\alpha_1 * Z_1 + \alpha_2 * Z_2 + \alpha_3 * Z_3 + \alpha_4 * Z_4)}{(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)} \\ &= \frac{(3.70 * 0.35 + (1.495 * 0.33) + (1.495 * 0.33) + (1.495 * 0.33))}{(0.35 + 0.33 + 0.33 + 0.33)} \end{aligned}$$

$$\text{Reliability index} = 2.07$$

The results of Tsukamoto method using crisp input of runoff coefficient of 0.346 and rainfall intensity of 26 mm/h, and Manning's coefficient of 0.025 gives the reliability index value of 1.259 which shows that culvert is safe.

1 Rule 1

If runoff coefficient is low and rainfall intensity is high and Manning's coefficient is Smooth then the reliability index value safe.

$$\begin{aligned} \alpha_1 &= \mu \text{ runoff coefficient low} \cap \mu \text{ rainfall intensity high} \cap \mu \text{ Manning's coefficient Smooth} \\ &= \min(\mu \text{ runoff coefficient low}(0.346) \cap \mu \text{ rainfall intensity high}(26) \cap \\ &\mu \text{ Manning's coefficient Smooth}(0.025)) \\ &= \min(0.09, 0.17, 0.6) \\ &= 0.09 \end{aligned}$$

If runoff coefficient is low (0.09) and rainfall intensity is high (0.17) Manning's coefficient is (0.6) then the reliability index value (0.09)

$$Z_1 = \frac{(Z - (1))}{1.5} = 0.09$$

$$Z_1 = 1.135$$

2 Rule 2

If runoff coefficient is low and rainfall intensity is very high and Manning's coefficient is Smooth then the reliability index value moderate safe

$$\begin{aligned} \alpha_2 &= \mu \text{ runoff coefficient low} \cap \mu \text{ rainfall intensity very high} \cap \mu \text{ Manning's coefficient Smooth} \\ &= \min(\mu \text{ runoff coefficient low}(0.346) \cap \\ &\mu \text{ rainfall intensity very high}(26) \cap \mu \text{ Manning's coefficient is}(0.025)) \\ &= \min(0.09, 0.341, 0.6) \\ &= 0.09 \end{aligned}$$

If runoff coefficient is low(0.09) and rainfall intensity is very high (0.341) Manning's coefficient is (0.025) then the reliability index value (0.09)

$$Z_2 = \frac{(Z - (1))}{1.0} = 0.09$$

$$Z_2 = 1.09$$

3 Rule 3

If runoff coefficient is moderate and rainfall intensity is high and Manning's coefficient is Smooth then the reliability index value safe

$$\begin{aligned} \alpha_3 &= \mu \text{ runoff coefficient moderate} \cap \mu \text{ rainfall intensity high} \\ &= \min(\mu \text{ runoff coefficient moderate}(0.346) \cap \mu \text{ rainfall intensity high}(26) \cap \mu \text{ Manning's coefficient is Smooth}) \\ &= \min(0.97, 0.17, 0.6) \\ &= 0.17 \end{aligned}$$

If runoff coefficient is moderate(0.97) and rainfall intensity is high (0.17) Manning's coefficient is (0.6) then the reliability index value (0.17)

$$Z_3 = \frac{(Z - (1))}{1.5} = 0.17$$

$$Z_3 = 1.255$$

4 Rule 4

If runoff coefficient is moderate and rainfall intensity is very high and Manning's coefficient is Low then the reliability index value safe

$\alpha_4 = \mu_{\text{runoff coefficient moderate}} \cap \mu_{\text{rainfall intensity very high}}$
 $= \min(\mu_{\text{runoff coefficient moderate}}(0.346) \cap \mu_{\text{rainfall intensity very high}}(26) \cap \mu_{\text{Manning's coefficient is (0.025)}})$
 $= \min(0.97, 0.341, 0.6)$
 $= 0.341$

If runoff coefficient is moderate(0.97) and rainfall intensity is very high (0.341) and Manning's coefficient is (0.6) then the reliability index value (0.341)

$$Z_4 = \frac{(Z - (1))}{1.0} = 0.341$$

$Z_4 = 1.341$

$$\begin{aligned}
 \text{Reliability index} &= \frac{(\alpha_1 * Z_1 + \alpha_2 * Z_2 + \alpha_3 * Z_3 + \alpha_4 * Z_4)}{(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)} \\
 &= \frac{(1.135 * 0.09 + (1.09 * 0.09) + (1.255 * 0.17) + (1.341 * 0.341))}{(0.09 + 0.09 + 0.17 + 0.341)} \\
 \text{Reliability index} &= 1.259
 \end{aligned}$$

The results of Tsukamoto method using crisp input of runoff coefficient of 0.85 and rainfall intensity of 39 mm/h and Manning's coefficient 0.025 gives the reliability index value of -1.243 which shows that culvert is in Failure mode.

1 Rule 1

If runoff coefficient is low and rainfall intensity is very high Manning's coefficient is Smooth then the reliability index value moderate safe

$\alpha_1 = \mu_{\text{runoff coefficient low}} \cap \mu_{\text{rainfall intensity very high}} \cap \mu_{\text{Manning's coefficient Smooth}}$
 $= \min(\mu_{\text{runoff coefficient low}}(0.85) \cap \mu_{\text{rainfall intensity very high}}(39) \cap \mu_{\text{Manning's coefficient Smooth}}(0.025))$
 $= \min(0.2, 0.95, 0.6)$
 $= 0.2$

If runoff coefficient is low (0.2) and rainfall intensity is very high (0.95) and Manning's coefficient(0.6) then the reliability index value (0.2)

$$Z_1 = \frac{(Z - (-3))}{2} = 0.2$$

$Z_1 = -2.6$

2 Rule 2

If runoff coefficient is moderate and rainfall intensity is very high and Manning's coefficient is Smooth then the reliability index value moderate safe

$\alpha_2 = \mu_{\text{runoff coefficient moderate}} \cap \mu_{\text{rainfall intensity very high}} \cap \mu_{\text{Manning's coefficient Smooth}}$
 $= \min(\mu_{\text{runoff coefficient moderate}}(0.85) \cap \mu_{\text{rainfall intensity very high}}(39) \cap \mu_{\text{Manning's coefficient}}(0.025))$
 $= \min(0.5, 0.95, 0.6)$
 $= 0.5$

If runoff coefficient is moderate (0.5) and rainfall intensity is very high (0.95) and Manning's coefficient (0.6) then the reliability index value (0.5)

$$Z_2 = \frac{(Z - (1))}{1.0} = 0.5$$

$Z_2 = -1.50$

$$\begin{aligned}
 \text{Reliability index} &= \frac{(\alpha_1 * Z_1 + \alpha_2 * Z_2)}{(\alpha_1 + \alpha_2)} \\
 &= \frac{(0.2 * -2.6 + (0.5 * -1.5))}{(0.2 + 0.5)} \\
 \text{Reliability index} &= -1.814
 \end{aligned}$$

The Tsukamoto method, is implemented using Rainfall Intensity and Runoff coefficient and Manning’s coefficient as input variables and Reliability Index is obtained as output variable. The input and output variables calculated using Monte Carlo Simulation Method are considered for comparison.

The input and output results for both MCS and Tsukamoto Methods are tabulated in Table 5.

Table. 5 Input and Output variables for MCS and Tsukamoto Method

Input variables			Output Variable/Reliability Index	
			Tsukamoto Method	MCS
Case1	Rainfall Intensity in mm/h	9	2.07	3.5433
	Runoff Coefficient	0.25		
	Manning’s coefficient	0.025		
Case2	Rainfall Intensity in mm/h	26	1.259	1.0329
	Runoff Coefficient	0.346		
	Manning’s coefficient	0.025		
Case3	Rainfall Intensity in mm/h	39	-1.814	-1.2988
	Runoff Coefficient	0.85		
	Manning’s coefficient	0.025		

V. CONCLUSION

It is necessary to provide minimum values of reliability index for culverts under different conditions, considering expected failure mechanism and the potential consequences. Such value would provide guidance for failure of culvert assessment and management. Considerable experience and discussion with other experts would be required in order to justify the individual values.

Based on experiments conducted in this study it can be concluded that the Tsukotomo method proved to be used in reliability analysis to assess the stability of hydraulic structures. The results are comparable with other methods like probabilistic and fuzzy probabilistic methods.

The results of the first experiment using the values of runoff coefficient as 0.25 and rainfall intensity as 9mm/h calculated by using Tsukotomo method shows the value of reliability index 2.07. The results of the second experiment using the values of runoff coefficient as 0.346 and rainfall intensity as 26 mm/h gives the value of reliability index 1.259. The results of the third experiment using the values of runoff coefficient as 0.85 and rainfall intensity as 39 mm/h gives the value of reliability index -1.814. The above values are in concurrence with the other method used for the analysis.

Determination of membership functions in terms of shape, boundaries and overlapping has a significant effect on the Tsukamoto method output. This shows that as the value of variables are increasing, the reliability index is decreasing in case of Rainfall intensity, Runoff coefficient and Manning’s coefficient and comparable with the probabilistic and fuzzy probabilistic methods.

References:

- [1]. Alshala A. Sleeg., and Issmail M. Ellabib. (2013). “Comparison of Mamdani and Sugeno Fuzzy Inference Systems for the Breast Cancer Risk.” International Journal of Computer and Information Engineering, Vol 7, No.10, 2013.
- [2]. Bojadziev George., Maria Bojadziev. (1995). “Fuzzy sets, Fuzzy logic applications.” World Scientific Publishing Co., Singapore.
- [3]. Bojadziev George, and Maria Bojadziev. (2006). “Fuzzy logic for Business Finance and Management.” World Scientific Publishing Co., Kanda.
- [4]. Chen, G., and Pham, T. T. (2001). “Introduction to fuzzy sets, fuzzy logic and fuzzy control systems.” 1st edition, United States of America, CRC Press LLC.
- [5]. Dutt, L. S., and Kurian, M. (2013). “Handling of Uncertainty- A Survey.” International Journal of Scientific and Research Publications, 469.
- [6]. Dubois, D., and Pride, H. “Fuzzy Sets and Systems, Theory and Applications.” Academic Press, NY, 1980.
- [7]. Fenty., Ariani., Robby., and Yuli Endra. (2013). “Implementation of Fuzzy Inference System with Tsukamoto method for study programme selection.”, 2nd International Conference on Engineering and Technology Development , (ICETD 2013).
- [8]. Jafar Khademi Hamidi., Kourosh Shahriar., Baahram Rezai., and Hadi Bejari. (2010). “Application of Fuzzy Set Theory to Rock Engineering Classification Systems: An Illustration of the Rock Mass Excavability Index.” Rock Mech Rock Eng., (2010) 43, 335-350.
- [9]. Kavdir, I., and Guyer, D. E. (2003). “Apple grading using fuzzy logic.” Turkish Journal of Agriculture 27, 375-382.

- [10]. Kazeminezhad, M. H., Etemad-Shahidi, A., and Mousavi, S. J. (2005). "Application of fuzzy inference system in the prediction of wave parameters." *Ocean Engineering*, 32(14), 1709-1725.
- [11]. Kusumadewi, S., and Purnomo, H. (2010). "Fuzzy logic applications for Decision Support." Graha Science, Yogyakarta.
- [12]. Madsen, H. O., kernk, S., and N. C. Lind, N. C. (2006) "Methods of structural safety." New York , Dover publications, 2006.
- [13]. Melchers R. E. (1999). "Structural reliability analysis and prediction." 2nd Ed. Chichester John Wiley and sons., 1999.
- [14]. Meyer, G. E. (2004). "Digital camera operation and fuzzy logic classification of uniform plant, soil and residue color images." *Applied Engineering in Agriculture*, 20(4), 519-529.
- [15]. Nauck, D., and Kruse, R. (1999). "Obtaining interpretable fuzzy classification rules from medical data." *Artificial intelligence in medicine*, 16(2), 149-169.
- [16]. Ozgur Kisi. (2014). "Applicability of Mamdani and Sugeno fuzzy genetic approaches for modeling reference evapotranspiration." *Journal of Hydrology*, 160-170.
- [17]. Pappis, C. P., and Siettos, C. I. (2005). "Fuzzy reasoning, in Search Methodologies", 437-474. Springer, US.
- [18]. Rahaizan Ramian., Ang Poh Cheng. (2016). "The conceptual framework of production planning optimization using Fuzzy Inference System with Tsukamoto." *International Journal of Industrial Management*, Volume 2, 80-91.
- [19]. Weimin DONG., and Haresh C. SHAH. (1987). "Vertex Method for computing functions of Fuzzy Variables." *Fuzzy Sets and Systems* 24, 65-78. Elsevier Science Publishers, B. V. (North Holland).
- [20]. Zadeh L. A. (1965). "Fuzzy sets, Information, and control." 8(3), 338-353.
- [21]. Zadeh L. A. (1978) "Fuzzy sets as a basis for a theory of possibility." *Fuzzy Sets Systems*, 1978, 13-28.