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# Synthesis of an optimal controller for gear transmission systems for aviation equipment

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**ABSTRACT**
*Gear transmission systems represent a prevalent type of actuator system found in various machinery sectors, especially within the realm of aerospace engineering. When faced with challenges demanding high precision and the inability to precisely measure frictional forces, shaft torsion, elastic deformations, and gear clearances, alternative mechanical approaches or electronic and electrical control mechanisms become essential. These controllers offer the flexibility to adapt control strategies effectively, compensating for discrepancies beyond the capability of mechanical interventions. In this paper, a novel control technique is proposed to enhance the performance of gear transmission systems by employing an optimal controller. Through simulations conducted using Matlab - Simulink software, the authors aim to showcase the efficacy of the optimization strategy introduced in this study.*

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## I. INTRODUCTION

The gap transmission system is a nonlinear transmission system that is widely used in practice such as gear drives, belt drives, chain drives, screw-nut drives, and screw-wheel drives screws,... (Fig. 1). In the gear system, transmission is achieved by the meshing of gears on the gear or rack. Gear drives are used in various machines and mechanisms to transmit rotary motion from one shaft to another and to convert rotary motion into reciprocating motion and vice versa.

Gear drives are widely used because they have advantages such as large power transmission capacity, large useful coefficient, and smooth transmission. Gear drives are important mechanisms in automobile transmissions, tractors, internal combustion engines, machine tools, agricultural machines, robots, cranes, aviation watches, and many other devices...Speed range The speed and force transmission of the gear is very large. Gear reducers are capable of transmitting power up to tens of thousands of KW. The rotation speed of gears in high-speed motion transmission mechanisms can reach $150\ rpm$.

In a gear transmission system, there are usually active gears, passive gears, and a few intermediate gears. Using gears can transmit rotation between axes that are parallel, diagonal, or perpendicular to each other. For belt drives, due to their simple structure, ease of manufacture, and low cost, they are also used in many systems. The transmission power can reach $3000$ KW, the belt speed can reach $v = 100\ m/s$ and the transmission ratio $I$ can reach $10$. Belt drive has the advantage of stable motion and, able to withstand variable loads and concussion. When overloaded, the belt can slip and reduce danger to the machine. Chain drives are used less due to the disadvantages of having gaps and generating loud noise during work.

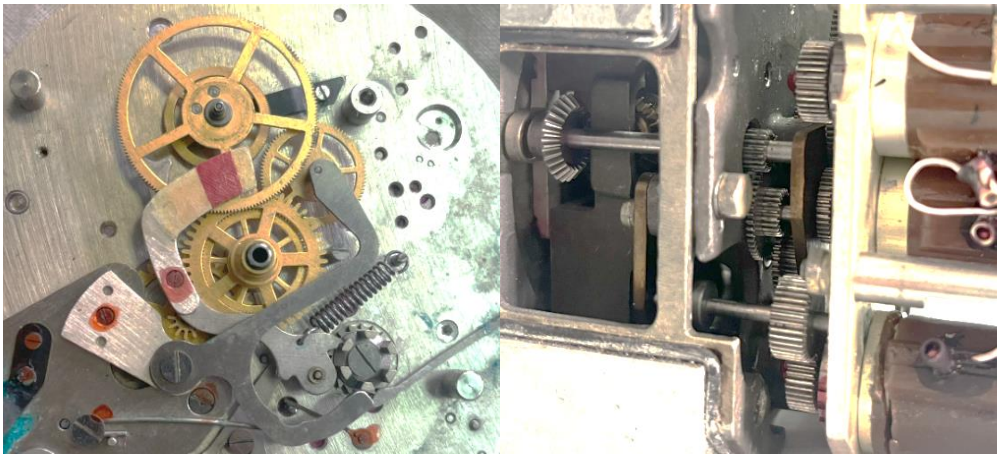

**Figure 1:** The diagram showing the gear transmission systems in aviation equipment

In the above transmission systems, there is always a gap between the active and passive parts, on the other hand, there is a delay between movements, which distorts the transmission and reduces control accuracy. Locations and gaps can reduce the life of mechanical parts, emit noise, cause vibration, and change the stability and performance of the system.

In addition, after a period of working in machine combinations, factors that cause unwanted interference on the transmission system through gears such as friction, gaps between gears, material instability, and abrasion of materials over time..., have led to dynamic instability in the transmission system. Dynamic instability is the most dangerous state that occurs when the frequency of the exciting force is equal to or approximately equal to the natural oscillation frequency of the system. When a machining process falls into an unstable state, the vibration amplitude of the system is very large, causing the system to vibrate strongly, causing noise and reducing the accuracy and quality of the product. To have an overview of the influence of mechanical processing errors on gear transmission systems. Next, we will analyze the dynamics of the gear system taking into account elasticity and clearance factors.

## II. MATERIAL AND METHODS

### 2. 1. Mathematical model of gear transmission system

Building mathematical models for gear systems is very important because they can use control measures to improve the quality of the transmission system, reducing the influence of mechanical errors that cannot be overcome by mechanical methods.

Because the transmission system through many pairs of gears always has a reverse transmission structure consisting of many systems of one pair of gears connected in series (Fig 2), when designing a controller for gear transmission systems in general, the authors proposed to design the controller for only one pair of gears.

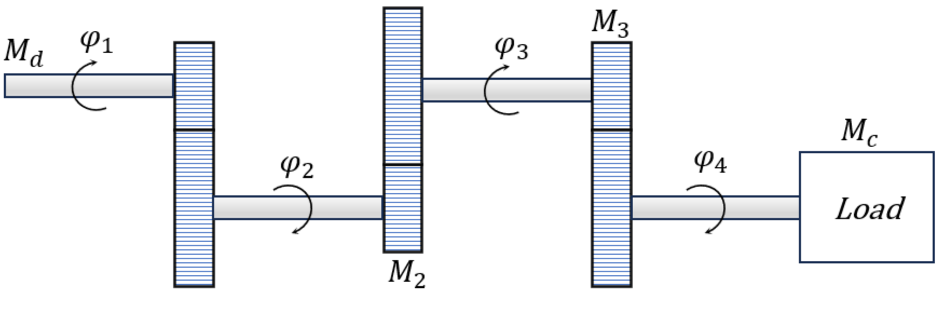


**Figure 2: The diagram showing the multi-pair gear system**

Based on the gear transmission system, we will obtain a dynamic model that takes into account the elastic factor of the gear pair and the friction in the bearings as depicted in Figure 3.

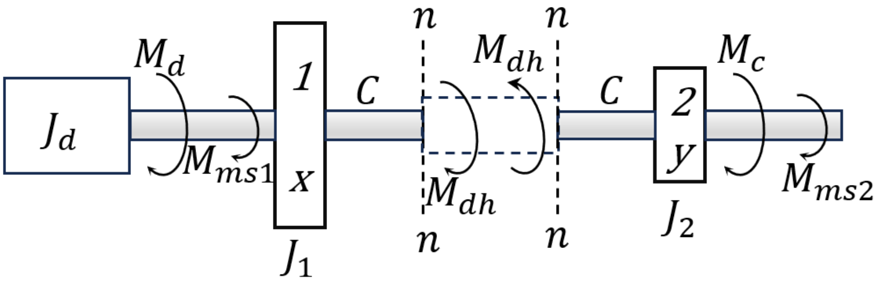


**Figure 3: Dynamic diagram**

**Where:**

$M_d$: the moment starting for gear 1;

$J_d, J_1, J_2$: moment of inertia of the motor, gear 1 and gear 2;

$M_c$: Sisting moment (including load moment)

$M_{ms1}$ và $M_{ms2}$: A frictional moment in gear bearings.

$\omega_1 = \dot{\varphi}_1; \omega_2 = \dot{\varphi}_2$: Angular velocity of the two gears;

$r_{L1}$, $r_{L2}$: Rolling radius of two gears (outer radius)

$r_{01}, r_{02}$: Base radius of two gears (inner radius).

$i_{12}$: Transmission ratio from gear 1 to gear 2

$c$ : Gear hardness;

$\alpha_L$: The meshing angle of two gears.

$M_1, M_2$: Elastic moment on gears 1 and 2.

To be able to establish the equation of motion of the gear transmission system in Fig 3, we use an n-n cross-section, which is subjected to an elastic moment of the two gears as shown in Fig 4.

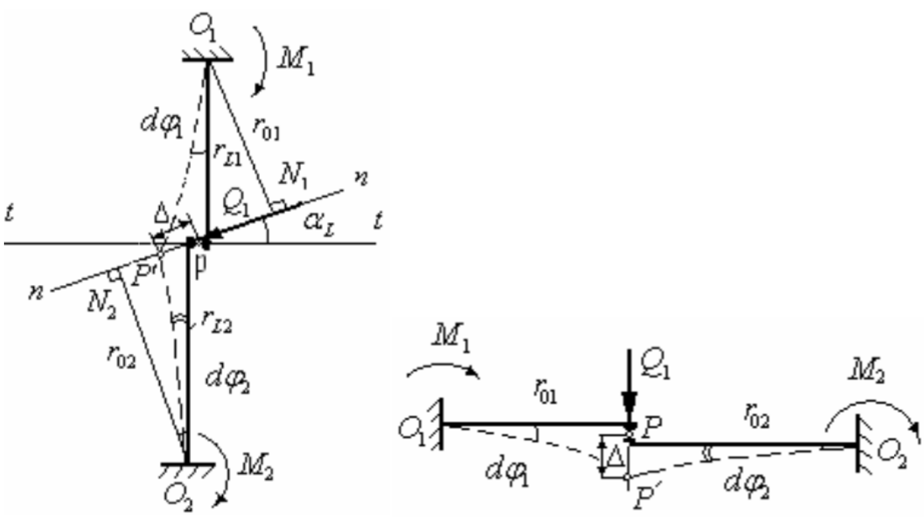

**Figure 4: The diagram showing the force and moment of the gear transmission system**

At this time, the model calculates the operation of a pair of gears, considering the elastic deformation process, under the influence of force, with the following form:

$$Q_1 = \frac{M}{r_{01}} \quad (1)$$

Assuming that there is no impact of force $Q_1$that the two tooth edges contact each other at point $P$, under the influence of force ,$Q_1$ and because the material is elastic, the contact point $P$ will move to point $P'$. So $\Delta = PP'$ is the deformation during the meshing process of two gears under the influence of $Q_1$. If we call $c$ the hardness of the gear pair then:

$$Q_1 = \frac{M}{r_{01}} \quad (2)$$

$$c.\Delta = Q_1 \text{ hay } c.\Delta r_{01} = cr_{01}\Delta = M_1 \quad (3)$$

Therefore, the elastic moments on gears 1 and 2 respectively are shown in equation (4).

$$M_1 = \mathrm{cr}_{01}(\mathrm{r}_{01}\mathrm{d}\varphi_1 + \mathrm{r}_{01}\mathrm{d}\varphi_2); \mathrm{M}_2 = \mathrm{cr}_{02}(\mathrm{r}_{02}\mathrm{d}\varphi_2 + \mathrm{r}_{01}\mathrm{d}\varphi_1) \quad (4)$$

According to Newton's laws of motion, we can write:

$$\begin{cases} J_1\ddot{\varphi}_1 = M_d - (M_{ms1} + M_1) \\ J_2\ddot{\varphi}_2 = M_2 - (M_{ms2} + M_c) \end{cases} \quad (5)$$

$$\begin{cases} J_1\ddot{\varphi}_1 + \mathrm{cr}_{01}(\mathrm{r}_{01}\mathrm{d}\varphi_1 + \mathrm{r}_{01}\mathrm{d}\varphi_2) = M_d - M_{ms1} \\ J_2\ddot{\varphi}_2 - \mathrm{cr}_{02}(\mathrm{r}_{02}\mathrm{d}\varphi_2 + \mathrm{r}_{01}\mathrm{d}\varphi_1) = -M_c - M_{ms2} \end{cases}$$

After transforming by placing $r_{01}^2, r_{02}^2$ outside the brackets and replace:

$$r_{01} = r_{L1}cos\alpha_L; r_{02} = r_{L2}cos\alpha_L; i_{12} = \frac{r_{02}}{r_{01}}; i_{21} = \frac{r_{01}}{r_{02}} \quad (6)$$

When it is assumed that the system is in steady running mode (steady), the bearing is lubricated with oil, at this time the gears are in mesh, that is, when the friction torque is only proportional to the angular velocity of the gear shaft and not also depends on acceleration:

$$M_{ms1} = b_1\dot{\varphi}_1 \text{ và } M_{ms2} = b_2\dot{\varphi}_2 \quad (7)$$

The general equation becomes:

$$\begin{cases} J_1\ddot{\varphi}_1 + \mathrm{cr}_{L1}^2cos^2\alpha_L(\varphi_1 + \varphi_{12}\varphi_2) = M_d - b_1\dot{\varphi}_1 \\ J_2\ddot{\varphi}_2 + \mathrm{cr}_{L2}^2cos^2\alpha_L(\varphi_2 + \varphi_{21}\varphi_1) = -M_c - b_2\dot{\varphi}_2 \end{cases} \quad (8)$$

Besides that:

$$\varphi_{12} = \frac{\varphi_1}{\varphi_2} = \frac{\omega_1}{\omega_2} = \pm\frac{r_2}{r_1} \quad (9)$$

$$\mathrm{cr}_{L1}^2cos^2\alpha_L = c_{z1}; \mathrm{cr}_{L2}^2cos^2\alpha_L = c_{z2}$$

Substituting into the above equation we have:

$$\begin{cases} J_1\ddot{\varphi}_1 + c_{z1}(\varphi_1 + \varphi_{12}\varphi_2) = M_d - b_1\dot{\varphi}_1 \\ J_2\ddot{\varphi}_2 + c_{z2}(\varphi_2 + \varphi_{21}\varphi_1) = -M_c - b_2\dot{\varphi}_2 \end{cases} \quad (10)$$

Thus, the mathematical equation system of the gear transmission system in steady mode is written:

$$\begin{cases} J_1\ddot{\varphi}_1 + b_1\dot{\varphi}_1 + c_{z1}(\varphi_1 + \varphi_{12}\varphi_2) = M_d \\ J_2\ddot{\varphi}_2 + b_2\dot{\varphi}_2 + c_{z2}(\varphi_2 + \varphi_{21}\varphi_1) = -M_c \end{cases} \quad (11)$$

### 2.2. Build the LQR controller

Linear global control (LQR) is a modern control technique that uses state space to analyze and design a system. State space equations usually have the form:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad (12)$$

Where $x$ is the state vector, $y$ is the output signal vector, u is the control signal vector; $A, B, C, D$ are state matrices. The task is to find a control signal $u(t)$ to adjust the system from any initial state $x(0) = \mathrm{x}_0$ to the final state $x(t_f) = 0$ in such a way that minimize quality criteria in global form:

$$J = \int_0^\infty [x^TQx + u^TRu]dt \quad (13)$$

Where $Q$ is a semi-positive definite weight matrix; $R$ is a positive definite weight matrix. The above problem is called the linear omnidirectional adjustment problem. Thus, with a given quality indicator function, by transforming it into a general form, we can determine the weight matrices of the quality indicator function $Q$ and $R$. Accordingly, $Q$ and $Q$ are matrices diagonal whose diagonal elements are respectively represented by the inverse of the square of the maximum admissible values of the state variable $(x)$ and the input variables $(u)$ . The diagonal values $Q_{ii}$ of the matrix Q are written as follows:

$$Q_{ii} = \frac{1}{x_i^2} \text{ với } i \in (1,2,3 \ldots l) \tag{14}$$

The diagonal values $R_{jj}$ of matrix R can be calculated by the formula:

$$R_{jj} = \frac{1}{u_i^2} \text{ với } j \in (1,2,3 \ldots k) \tag{15}$$

After having the above matrices $Q$ and R, substitute into the Riccati differential equation to find the solution $P$.

$$A.P + PA - PBR^{-1}P + Q = 0 \tag{16}$$

With the $P$ value found, we will calculate to find the optimal control matrix (response gain matrix $K$) according to the formula:

$$K = R^{-1}BP \tag{17}$$

To find the matrix $K$, we use MATLAB software with the following command:

$$K = lqr(A, B, Q, R)$$

The block diagram of the control loop structure using the LQR controller is shown in Figure 5.

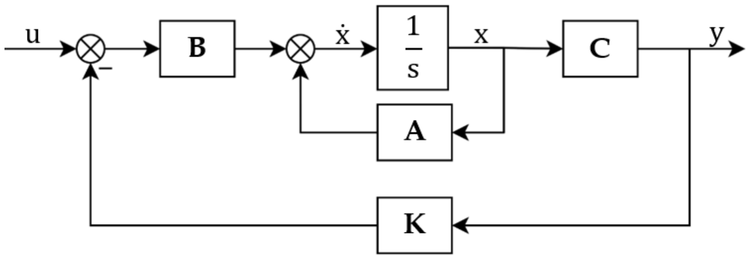


**Figure 5: Control loop structure diagram using LQR controller**

**2.3. Control system structure**

From the mathematical model of the gear system, we build a control structure for the transmission system with clearance as an object consisting of two parts. The drive part uses a 1-way motor to guide the movement of the gear pair. The description of the gear pair is active gear 1 and passive gear 2. As analyzed above, we have a general control structure as shown in Fig. 6.

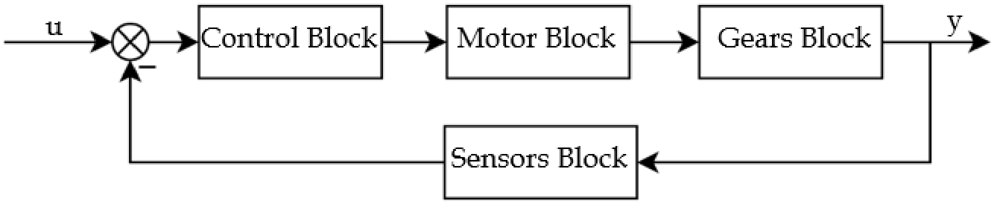


**Figure 6: Structure diagram of gear transmission system control**

The controller is a gear transmission system speed controller. In the article, we will design a PID and LQR controller. From the basic theoretical systems above, we conduct a survey and evaluate the controller's working ability on matlab-simulink.

**3. Results and Discussion**

Based on the mathematical equation system of the gear transmission system in steady mode in part 2. We will build a model of components including motor, gear and sensor on Simulink. From there, build controllers for this gear drive system.

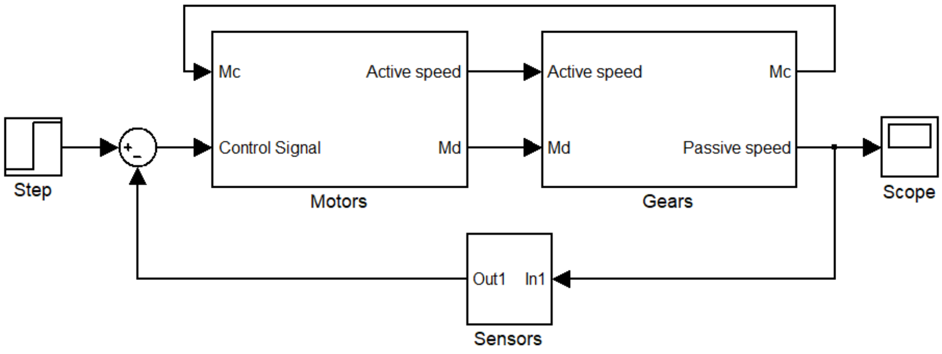


**Figure 7: The diagram of gear transmission system**

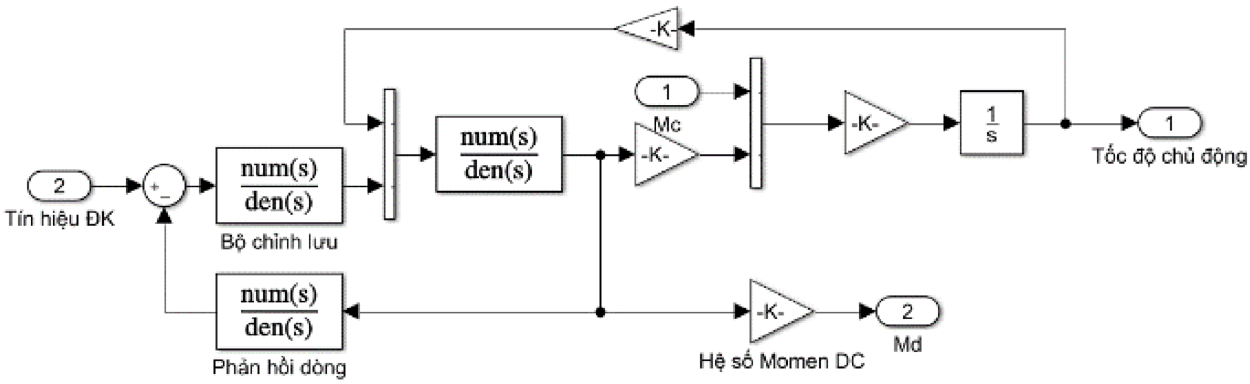


**Figure 8: The diagram of blocks in the motor**

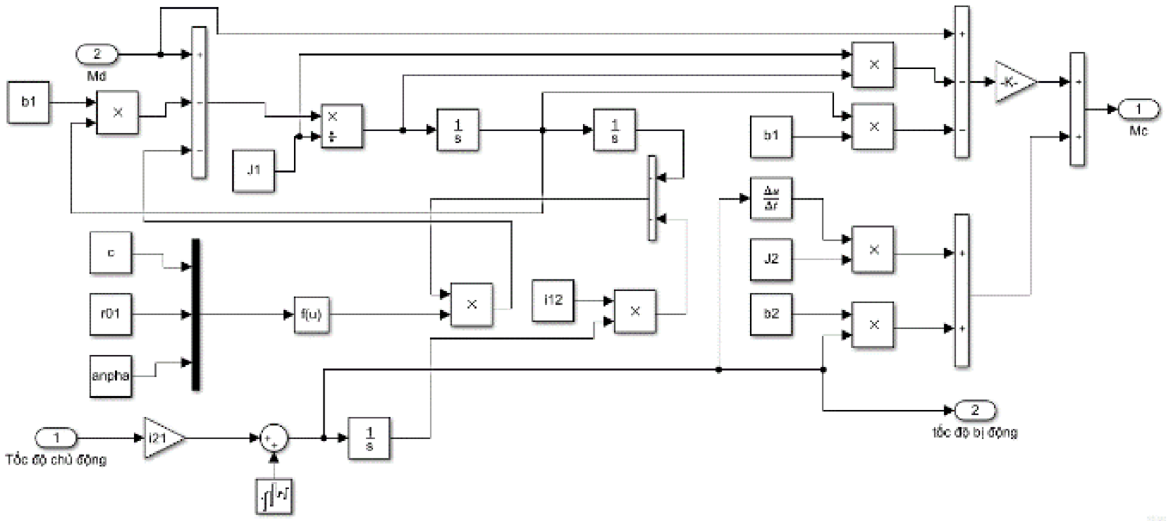


**Figure 9: The diagram of components in the gear block**

First, we compare the speeds of the two gears in the transmission system with the input constant moment $M_d = 400(\text{rpm})$.

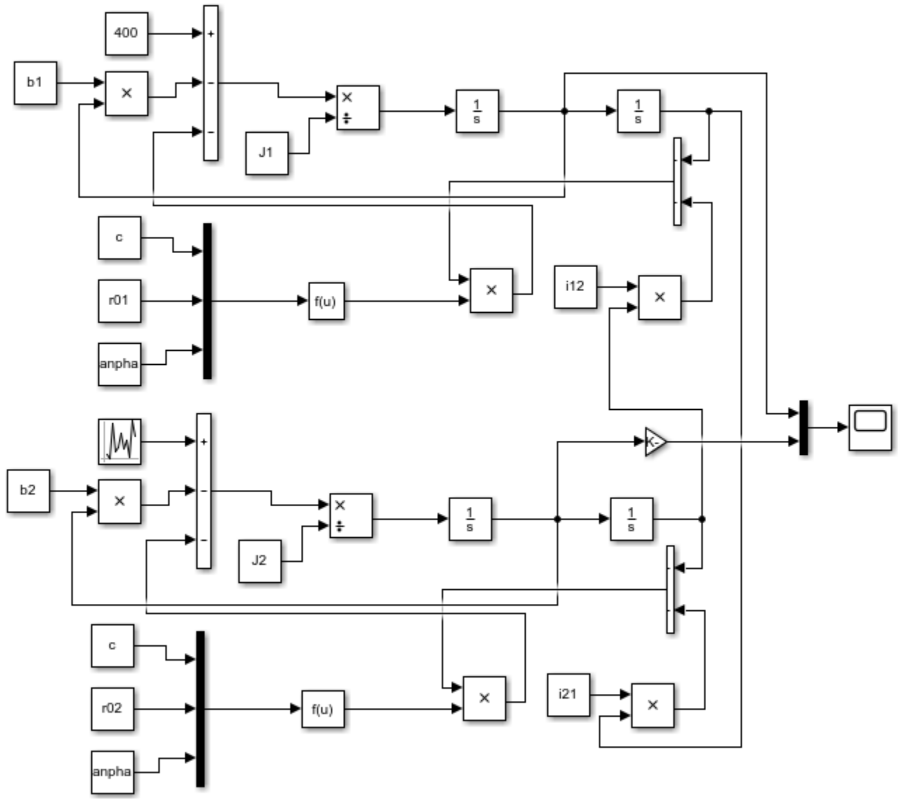


**Figure 10: The diagram of gear system with constant moment $M_d$**

According to the survey results in Figure 11, when the moment $M_d$ is constant and taking into account the effects of clearance and elasticity and without a controller, the speeds of the active and passive gears encounter speed faced with big errors.

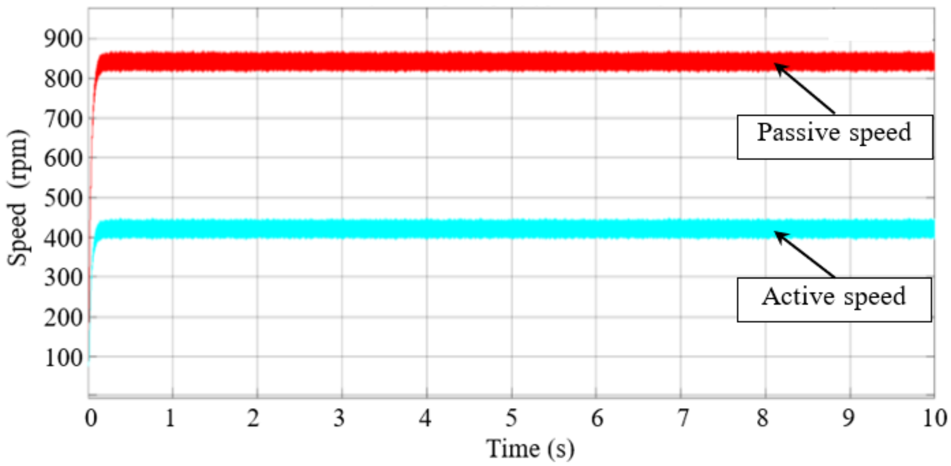


**Figure 11: Speed of active and passive gears when $M_d = const, i = 2$**

On the other hand, when we assign an additional motor to the gear drive system and investigate the system using the step function with the input being the set speed 300 rpm in the first 5 seconds and 500 rpm at 5 seconds later with the assumption that the gap and elasticity are taken into account, with the resistance moment being white noise, the simulation results are shown in Fig.12.

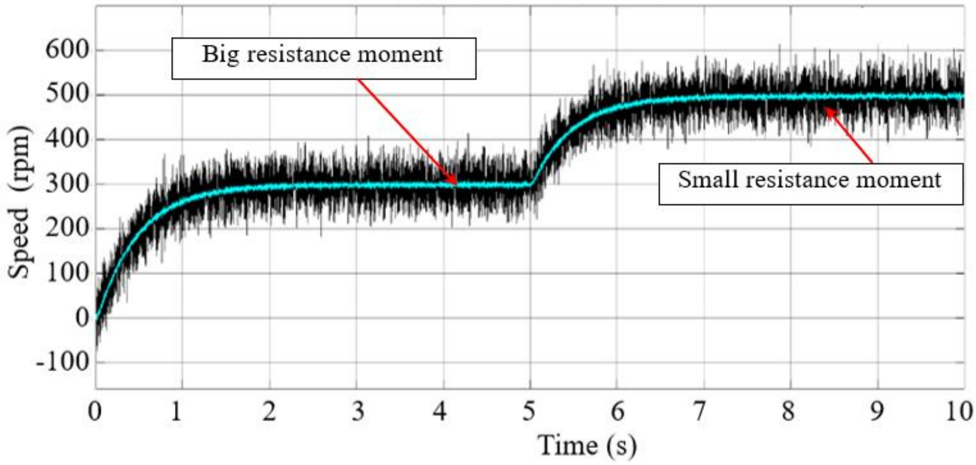


**Figure 12: The system response when the resisting moment is white noise**

Looking at the survey results, we see that the system is affected by noise due to taking into account gaps and elasticity, so the speed error is large. Therefore, we need to build PID, PID-FLC and LQR controllers to ensure noise reduction and steady-state errors.

Next, we examine the system in turn with and without input disturbances, and without calculating and taking into account gaps and elasticity. The simulation results are shown in Fig.13.

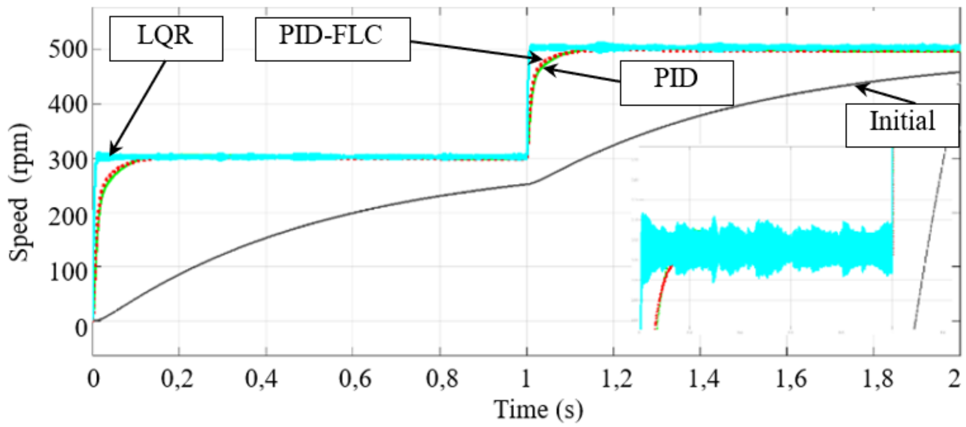


**Figure 13: The system response with no input noise and not take into account the elastic gap**

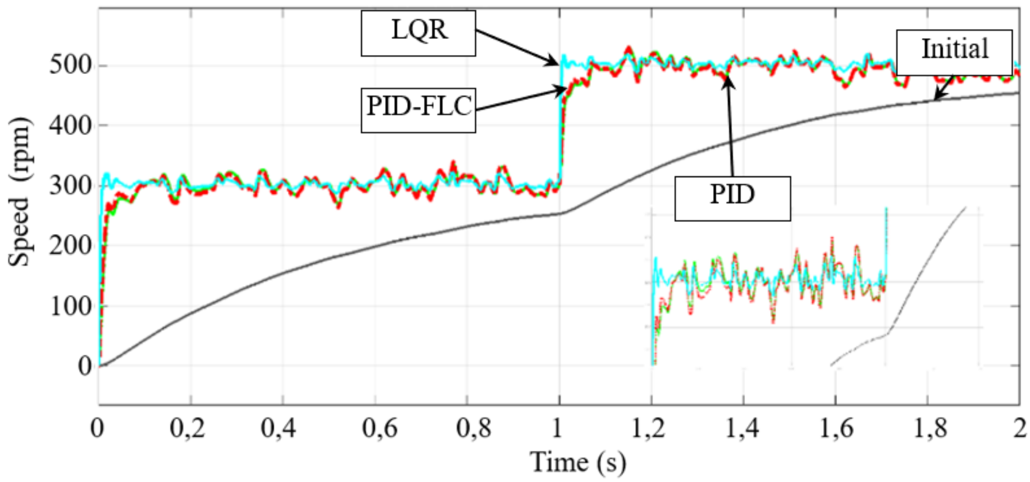


**Figure 14 : System response with input noise, not taking into account the elastic gap**

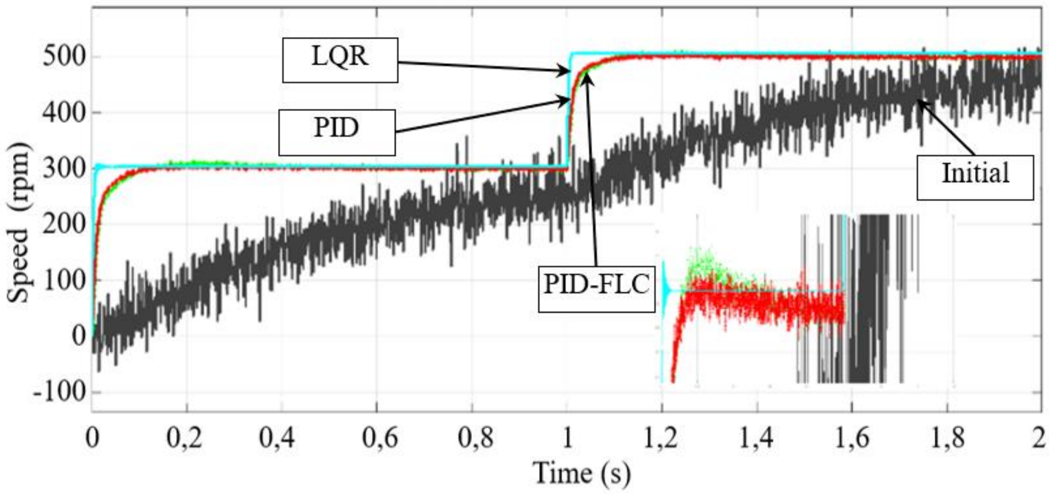


**Figure 15 : System response with no input noise, not taking into account the elastic gap**

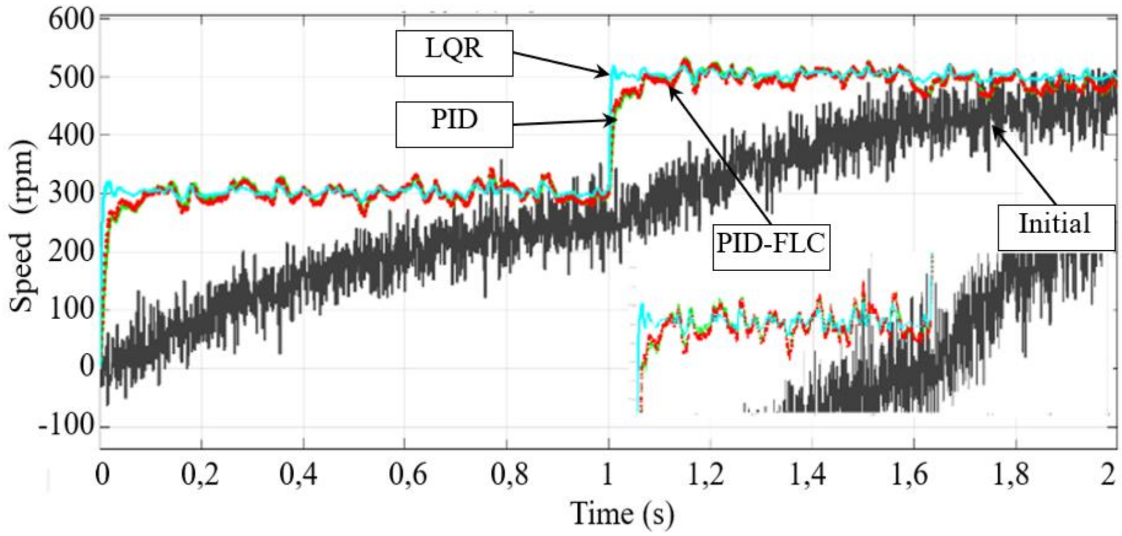


**Figure 16: System response with input noise, taking into account the elastic gap**

## III. DISCUSSION AND CONCLUSION

If the system lacks an electrical controller, the passive shaft speed experiences significant fluctuations due to the combined effects of clearance, elasticity, and friction. Conversely, when the clearance, elasticity, and friction are increased, the system exhibits more pronounced vibrations. Moreover, these fluctuations occur randomly and are contingent upon the operational speed of the system. Notably, when subjected to an input change characterized by a step function of 300 rpm for the initial 5 seconds followed by 500 rpm for the subsequent 5 seconds, the system demonstrates persistent oscillations with a substantial amplitude (Fig. 12).

When there is a classic PID controller and transmission ratio $i_{12} = 2$ in steady mode, the oscillation between the active and passive axes is significantly reduced, and the over-adjustment is very small but the time increases and response time is low. In turn, survey with the step function with the input speed 300 rpm in the first 5 seconds and 500 rpm in the next 5 seconds, with and without input noise (Fig. 13, 14) , when not calculated and taking into account gaps and elasticity (Fig. 15, 16); using PID, PID-FLC, LQR controllers, we see that the old criteria remain the same, response time increases significantly. Specifically, when gaps and elasticity are not taken into account, PID and PID-FLC controllers are more optimal than LQR. But when there is noise affecting the input, taking into account the gap and elasticity, the LQR controller is more optimal than PID, and PID-FLC. However, the LQR oscillator is still affected by noise.

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