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A volume pliable sustainable inventory management for deteriorating products with random machine disruption and stochastic restoration time

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Abstract
For deteriorating items with random machine disruption and stochastic restoration time, a sustainable inventory model is developed here. The key conferments in this framework are as follows: the deterministic rate of demand is assumed to be a selling price mathematical function. The selling price is the buyer's biggest benchmark as he/she goes to the market to purchase a specific product. The production rate is finite and regulated by new technologies, investment in capital, and the number of laborers. Demand is altered also by the amount of liability of the inventory and this pliability is accomplished by deciding the optimum ceas- ing period of production, the number of laborers, and unit selling price of the product. The model assumes that the time for machine restoration is autonomous of the rate of machine disruption. The Genetic Algorithm (GA) is proposed to calculate the potential cost of benefit predicted. To illustrate the model, a numerical example and sensitivity analysis are shown.
Keywords: *Genetic algorithm; random machine disruption; stochastic restoration time; deterministic sustainable inventory model; Cobb-Douglas production function.*

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I. Introduction

The business situation today is depicted by rising the variety of product and mounting demand level. With competency decisions regularly frozen well prior to the actual product shipment, misalignments between built competency and realized demand are unavoidable while demand is still uncertain. Manufacturing pliability aims to mitigate the economically significant effect of such as mismatch in demand-supply by enabling businesses to correctly reallocate or customize the expertise in compliance with the demand realized.

The pliability of volume can also be used to combat aggregate demand, ambiguity for a product set, ambiguity in demand for the launch of a whole line of new apparels, while product pliability is often used to minimize the ambiguity of the particular demand, and ambiguity in each type of the apparel being launched.

In recent years, integrating traditional inventory management with other types of decisions made by the firm (e.g., pricing, quality level, guaranty period, etc) has drawn attention of many researchers because these decisions must be agreeable to each other in order to acquire maximal profit. In fact, setting prices and planning for how much inventory to hold are the two most strategic ones among the many decisions made by a manager. Keeping these facts in mind, practitioners and academics have concentrated on determining pricing strategy, which influences demands, and production-inventory decisions, which define the cost of satisfying those demands, simultaneously.

Over time, maximum violent goods undergo rot or stagnation. Direct spoilage occurs during storage in the products such as fruits, vegetables and food products suffer from exhaustion.

Through the evaporation process, highly volatile liquids like oil, alcohol and turpentine experience physical degradation over time. With the passage of time, electrical goods, hazardous contami- nants, photographic film, grain, etc., deteriorate with a steady lack of capacity or utility. Putrefying or stagnating physical items in storage is thus a very practical aspect, and inventory researchers felt the need to take this element into an account.

The classical inventory model of economic development suggests that industrial processes are perfectly efficient. However, for certain actual structures, this assumption does not apply. he condition of sudden machine disruption and the time taken in the manufacturing process is faced by both the best and most advanced production systems and system restoration often also relies on the type of accident that has arisen. Reliability of manufacturing equipment is a critical factor in ensuring synchronization in the manufacturing environment which can be detrimental to the company if the current ambiguity of the manufacturing equipment is not taken into account and is prepared accordingly. The production method is considered as versatile to manufacture as per demand in this study, but is not efficient. At any random time, the production equipment can interact and the restoration time is therefore presumed to be stochastic in nature.

A GA method has been built in this study to find the estimated maximum profit cost of the inventory model proposed.

II. Literature Review

The classical production inventory model assumes that manufacturing system is perfectly reliable. Such an assumption appears impractical in real system. Researchers, therefore, have been attracted towards machine breakdown effects on production inventory problem. The effects of machine breakdown and corrective maintenance were studied by Groenevelt et al. [5]. Sheng et al. [8] discussed an infinite-horizon production-inventory problem with constraints that the production rates shall be restricted to an appropriate interval, and the indeterminacy of the decision system is extracted and analyzed through ambiguity theory.

Batarfi et al. [1] studied a profit maximization for a reverse logistics dual-channel supply chain system, which is comprised of production, refurbishing, collection, and waste disposal processes with a return policy. Uthayakumar and Tharani [10] had flourished an economic production model for deteriorating items and time dependent demand with rework and multiple production setups.

Vokurka and Leary-Kelly had worked a paper to compare analytically two such flexibilities (product and volume) and to adept both at mitigating demand ambiguity also to study the impact of combining them. Dem and Singh [2] developed a production model for imperfect production process under volume flexibility. Goyal et al. [4] explored an inventory system with variable demand as well as production under partially backordered shortages. Panda et al. [7] dealt an inventory model with a single item deteriorating production inventory model with price sensitive demand.

Genetic algorithms were first developed by John Holland. Goldberg [3] published a book explaining the theory and application examples of genetic algorithm in details. Kim and Sarkar

[6] had developed a joint replenishment problem for cleaner production process with complex multi-stage quality improvement and lead time dependent ordering cost. Vokurka and Leary-Kelly

[11] had discussed a comprehensive study at the empirical research pertaining to manufacturing flexibility in the marketplace. Taleizadeh [9] had worked on constrained inventory control systems with stochastic replenishments and fuzzy demand by using a hybrid method of fuzzy simulation and genetic algorithm for optimization.

III. Notation and assumptions

3.1 Notations

We need the following notations and assumptions to develop the mathematical model of the pro- posed model. Additional notations and assumptions will be added up when required.

Decision variables

n Number of labors.
p Selling price of the product.
T_1 Production period.

Parameters

θ Deteriorating rate
D_c Total deteriorating cost ($ / unit)

A	Manufacturer's set up cost
h	Holding cost per unit in dollars
M	Cost of machine restoration
L_s	Cost of lost sales per unit in dollars.
v_1	Extra production cost of each unit of a product
v_2	Unit variable cost for handling or receiving an item

Variables

P	Sustainable Production rate (unit / year)
R	Sustainable Rework rate (unit / year)
D	Demand rate (unit / year)
T_2	Non-production period
T_3	Shortage period
T_4	Rework production period
T_5	Rework non-production period
T_b	Machine disruption period

3.2 Assumptions

1. Suatainable production rate is greater than demand rate. $P > D$

2. The rate of suatainable production is variable and finite. It is determined by Cobb-Douglas production function which is of the form

$$P = RC^{a}n^{1-a} \quad \text{where} \quad 0 < a < 1 \tag{1}$$

where R is the technology which accelerates the rate of sustainable production, C is the capital investment except the investment in technology and n is the number of labors, a and $(1-a)$ represent the capital and labor elasticity respectively.

3. Demand rate is a decreasing function of the selling price. (ie)

$$D = \alpha p^{-\beta} \quad \alpha > 0,\ \beta > 1 \tag{2}$$

It is commonly known as iso-elastic demand function where α is a scaling factor and β is the index of price elasticity.

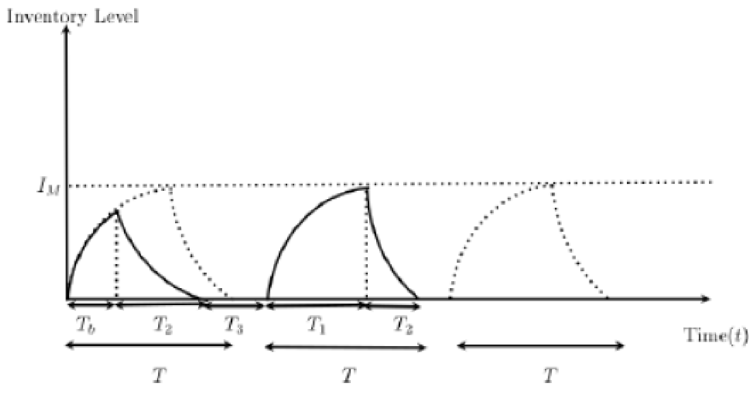

Figure 1: Inventory Management with Lost Sale

4. For each unit of product, the manufacturer spends extra v_1 in sustainable production and v_2 is the unit variable cost for handling an item.

5. A constant fraction $\theta \in [0, 1]$, of the storage inventory is deteriorated per unit time.

6. There is rework or replacement for a deteriorated item and also the deterioration rate involving preservation techniques satisfies the property $\frac{\partial\theta(\gamma)}{\partial\gamma} < 0$ and $\frac{\partial^2\theta(\gamma)}{\partial\gamma^2} > 0$ and is of the form $\theta(\gamma) = \theta_0 e^{\beta\gamma}$, where β is the sensitive parameter of investment in the deterioration rate. .

7. Machine restoration time does not depend upon machine disruption time.

8. Shortages are allowed due to the restoration time of machine.

4 Problem Description

The sustainable inventory model with lost sales case is illustrated in Fig. 1. When the machine does not disrupt during the production period, the production is carried out during T_1 time period. When the inventory attains the maximum inventory level I_M, the production ends and the inventory declines due to the demand and stagnation. The inventory level reaches zero units at time $(T_1 + T_2)$, and the machine begins to produce the product repeatedly. Since there is a possibility of machine disruption, the machine may not run the whole T_1 period. When this disruption happens, the production period time is T_b. When a machine breaks down, it will require some restoration time. Since restoration time is stochastic in $(T_2 + T_3)$ period, production may not always be possible and lost sales may occur during T_3 period.

5 Model development

The inventory level is paraded in the figure 1 . The inventory level grows during the interval $[0, T_1]$. Thus the inventory level in a production period is governed by the differential equations

$$\frac{\mathrm{d}I_1(t_1)}{\mathrm{d}t_1} + \theta I_1(t_1) = P - D, \quad 0 \le t_1 \le T_1 \tag{3}$$

with the initial conditions $I_1(0) = 0$. The inventory level depletes along the interval $[0, T_2]$. The inventory level in a non-production period is represented by the following differential equation

$$\frac{\mathrm{d}I_2(t_2)}{\mathrm{d}t_2} + \theta I_2(t_2) = -D, \quad 0 \le t_2 \le T_2 \tag{4}$$

with the boundary conditions $I_2(T_2) = 0$. By solving the equations 3 and 4 with the initial and boundary conditions, we obtain the inventory level in a production period and non-production period during the interval $[0, T_1]$ and $[0, T_2]$ respectively as

$$I_1(t_1) = \left(\frac{P-D}{\theta}\right)\left(1 - e^{-\theta t_1}\right) \tag{5}$$

$$I_2(t_2) = \frac{D}{\theta}\left(e^{\theta(T_2 - t_2)} - 1\right) \tag{6}$$

When $t_1 = T_1$ and $t_2 = 0$ we can have $I_1 = I_2$.Then we reach T_2 in terms of T_1 by using the Taylors series expansion as

$$T_2 \approx \left(\frac{P-D}{D}\right)\left(T_1 - \frac{\theta}{2}T_1^2\right), \qquad (\because \theta T_2 << 1) \tag{7}$$

Since there is a possibility of machine disruption we may expect the value of non production period as follows

$$E[T_2] \approx \begin{cases} \left(\frac{P-D}{D}\right)\left(T_b - \frac{\theta}{2}T_b^2\right) & , T_b \le T_1 \\ \left(\frac{P-D}{D}\right)\left(T_1 - \frac{\theta}{2}T_1^2\right) & , T_b > T_1 \end{cases} \tag{8}$$

Though we assert some preservative techniques for the produced items beyond that preservation, we could able to find some items to be deteriorated. Thus the deteriorated inventories are reproduced during the interval $[0, T_4]$. Thus the inventory level in a rework production period is given by the differential equations

$$\frac{\mathrm{d}I_4(t_4)}{\mathrm{d}t_4} + \theta I_4(t_4) = R - D, \quad 0 \le t_4 \le T_4 \tag{9}$$

with the initial conditions $I_4(0) = 0$. The recovered inventories are ready to meet the demand during the interval $[0, T_5]$. The inventory level in a rework non-production period is written as the following differential equation

$$\frac{\mathrm{d}I_5(t_5)}{\mathrm{d}t_5} + \theta I_5(t_5) = -D, \quad 0 \le t_5 \le T_5 \tag{10}$$

with the boundary conditions $I_5(T_5) = 0$. By solving the equations 8 and 9 with the initial and boundary conditions, we obtain the inventory level in a rework production period and rework non-production period during the interval $[0, T_4]$ and $[0, T_5]$ respectively as

$$I_4(t_4) = \left(\frac{R-D}{\theta}\right)\left(1 - e^{-\theta t_4}\right) \tag{11}$$

$$I_5(t_5) = \frac{D}{\theta}\left(e^{\theta(T_5 - t_5)} - 1\right) \tag{12}$$

When $t_4 = T_4$ and $t_5 = 0$ we can have $I_4 = I_5$.Then we reach T_5 in terms of T_4 by using the Taylors series expansion as

$$T_5 \approx \left(\frac{R-D}{D}\right)\left(T_4 - \frac{\theta}{2}T_4^2\right), \qquad (\because \theta T_5 << 1) \tag{13}$$

By integrating the inventory level in a production period, non-production period, rework production period and rework non-production period during the interval $[0, T_1]$, $[0, T_2]$, $[0, T_4]$ and $[0, T_5]$ respectively we can get the total inventory for the whole cycle T as

$$\begin{aligned} I &= \int_0^{T_1}\left(\frac{P-D}{\theta}\right)\left(1 - e^{-\theta t_1}\right)\mathrm{d}t_1 + \int_0^{T_2}\frac{D}{\theta}\left(e^{\theta(T_2 - t_2)} - 1\right)\mathrm{d}t_2 \\ &\quad + \int_0^{T_4}\left(\frac{R-D}{\theta}\right)\left(1 - e^{-\theta t_4}\right)\mathrm{d}t_4 + \int_0^{T_5}\frac{D}{\theta}\left(e^{\theta(T_5 - t_5)} - 1\right)\mathrm{d}t_5 \\ &= \left(\frac{P-D}{\theta^2}\right)\left[e^{-\theta T_1} - 1 + \theta T_1\right] + \frac{D}{\theta^2}\left[e^{\theta T_2} - 1 - \theta T_2\right] \\ &\quad + \left(\frac{R-D}{\theta^2}\right)\left[e^{-\theta T_4} - 1 + \theta T_4 +\right] + \frac{D}{\theta^2}\left[e^{\theta T_5} - 1 - \theta T_5\right] \end{aligned} \tag{14}$$

After applying Taylor's approximation we have the following inventory level

$$I = \left(\frac{P-D}{2}\right)T_1^2 + \frac{D}{2}T_2^2 + \left(\frac{R-D}{2}\right)T_4^2 + \frac{D}{2}T_5^2 \tag{15}$$

After applying Taylor's approximation we have the following inventory level

$$I = \left(\frac{P-D}{2}\right)T_1^2 + \frac{D}{2}T_2^2 + \left(\frac{R-D}{2}\right)T_4^2 + \frac{D}{2}T_5^2 \tag{15}$$

On substituting 7 and 13 in 15, we get the following equation as the inventory level.

$$\begin{aligned} I &= \left(\frac{P-D}{2}\right)T_1^2\left[1 + \left(\frac{P-D}{D}\right)(1 - \theta T_1)\right] \\ &\quad + \left(\frac{R-D}{2}\right)T_4^2\left[1 + \left(\frac{R-D}{D}\right)(1 - \theta T_4)\right], \ \theta^2 T_1^4 << 1, \ \theta^2 T_4^4 << 1 \end{aligned} \tag{16}$$

Since there is some machine disruption we may assume the same probability density function $f(T_b) = \lambda e^{-\lambda T_b}$ for machine disruption time as in Widyadana and Wee. Thus the inventory level in (22) may vary as follows

$$\begin{aligned} E[I] &= \left(\frac{P-D}{2}\right)\int_0^{T_1} T_b^2\left[1+\left(\frac{P-D}{D}\right)(1-\theta T_b)\right]\mathrm{d}T_b \\ &+\left(\frac{P-D}{2}\right)\int_{T_1}^{\infty} T_b^2\left[1+\left(\frac{P-D}{D}\right)(1-\theta T_b)\right]\mathrm{d}T_b \\ &+\left(\frac{R-D}{2}\right)T_4^2\left[1+\left(\frac{R-D}{D}\right)(1-\theta T_4)\right] \\ E[I] &= \left(\frac{P-D}{2}\right)\frac{T_1^3}{12}\left[4-3\lambda T_1+\left(\frac{P-D}{D}\right)(4-3(\lambda+\theta)T_1)\right] \\ &+\left(\frac{R-D}{2}\right)T_4^2\left[1+\left(\frac{R-D}{D}\right)(1-\theta T_4)\right], \end{aligned} \tag{17}$$

The number of deteriorating item is equal to the number of total items produced minus the number of total demands. Since p is selling price of the product, the total deteriorating cost is

$$D_c = \frac{pP}{2}\left(1-\frac{D}{P}\right)\theta T_1^2 \tag{18}$$

Due to machine disruption possibilities, we can have D_c as

$$D_c \approxeq \begin{cases} \frac{pP}{2}\left(1-\frac{D}{P}\right)\theta T_b^2 & , T_b \le T_1 \\ \frac{pP}{2}\left(1-\frac{D}{P}\right)\theta T_1^2 & , T_b > T_1 \end{cases} \tag{19}$$

Thus the expected total deteriorated cost can be formulated as

$$E[D_c] = \frac{pP}{\lambda^2}\left(1-\frac{D}{P}\theta\right)\left(1-e^{-\lambda T_1}-\lambda T_1 e^{-\lambda T_1}\right) \tag{20}$$

The expected corrective cost can be written as

$$E[C_c] = M(1-e^{-\lambda T_b}) \tag{21}$$

Lost sale happens when machine restoration time period is longer than non-production time period. Thus the total sustainable cost is the sum of setup cost, corrective cost, holding cost, deteriorating cost and lost sale cost.

$$\begin{aligned} E[TSC] &= A + M(1-e^{-\lambda T_b}) + hE[I] + E[D_c] \\ &+ L_s D \int\limits_{T_b=0}^{T_1}\int\limits_{t=T_2}^{\infty}(t-T_2)f(t)\lambda e^{-\lambda T_b}\mathrm{d}t\mathrm{d}T_b \end{aligned} \tag{22}$$

The total replenishment time is the sum of production up time period, non-production time period and restoration time probability. Therefore, the expected total replenishment time is represented as

$$\begin{aligned} E[T] &= \int_0^{T_1}\left(T_1+T_2+\int_{T_2}^{\infty}(t-T_2)f(t)\mathrm{d}t\right)\lambda e^{-\lambda T_b}\mathrm{d}T_b \\ &= \frac{P(1-e^{-\lambda T_1})}{\lambda D} + \int_0^{T_1}\int_{T_2}^{\infty}(t-T_2)f(t)\lambda e^{-\lambda T_b}\mathrm{d}T_b\mathrm{d}t \end{aligned} \tag{23}$$

By using the renewal reward theorem, the expected total cost per unit time can be written in the form

$$E[TSCT] = \frac{E[TSC]}{E[T]} \tag{24}$$

From 22 and 23, we have the expected total cost per unit time as

$$\begin{aligned} E[TSCT] &= \frac{A+M(1-e^{-\lambda T_b})+hE[I]+E[D_c]}{\frac{P(1-e^{-\lambda T_1})}{\lambda D}+\int_0^{T_1}\int_{T_2}^{\infty}(t-T_2)f(t)\lambda e^{-\lambda T_b}\mathrm{d}t\mathrm{d}T_b} \\ &+ \frac{L_s D\int\limits_{T_b=0}^{T_1}\int\limits_{t=T_2}^{\infty}(t-T_2)f(t)\lambda e^{-\lambda T_b}\mathrm{d}t\mathrm{d}T_b}{\frac{P(1-e^{-\lambda T_1})}{\lambda D}+\int_0^{T_1}\int_{T_2}^{\infty}(t-T_2)f(t)\lambda e^{-\lambda T_b}\mathrm{d}t\mathrm{d}T_b} \end{aligned} \tag{25}$$

5.1 Machine restoration time follows Uniform distribution

First we assume the machine restoration time t as a uniformly distributed random variable over the interval $[0,\eta]$. Therefore the probability density function is dictated as follows

$$f(t)=\begin{cases}1/\eta, & 0\le t\le \eta,\\ 0, & \text{otherwise}\end{cases} \tag{26}$$

By inserting the uniform probability density function in 25, we have the following expression

$$\begin{aligned} E[TSCT] &= \frac{A+M(1-e^{-\lambda T_b})+hE[I]+E[D_c]}{\frac{P(1-e^{-\lambda T_1})}{\lambda D}+\frac{1}{\eta}\int_0^{T_1}\int_{T_2}^{\eta}(t-T_2)\lambda e^{-\lambda T_b}\mathrm{d}t\mathrm{d}T_b} \\ &+ \frac{\frac{L_sD}{\eta}\int\limits_{T_b=0}^{T_1}\int\limits_{t=T_2}^{\eta}(t-T_2)\lambda e^{-\lambda T_b}\mathrm{d}t\mathrm{d}T_b}{\frac{P(1-e^{-\lambda T_1})}{\lambda D}+\frac{1}{\eta}\int_0^{T_1}\int_{T_2}^{\eta}(t-T_2)\lambda e^{-\lambda T_b}\mathrm{d}t\mathrm{d}T_b} \end{aligned} \tag{27}$$

The expected shortage can be written as

$$\begin{aligned} E[T_b] &= \frac{1}{\eta}\int_0^{T_1}\int_{T_2}^{\eta}(t-T_2)\lambda e^{-\lambda T_b}\mathrm{d}t\mathrm{d}T_b \\ &= \frac{\left((\eta\lambda D)^2-2\eta(P-D)\lambda D+2(P-D)^2\right)(1-e^{-\lambda T_1})-e^{-\lambda T_1}(P-D)((P-D)(2\lambda T_1)-2\eta\lambda D)}{2\eta\lambda^2D^2} \end{aligned} \tag{28}$$

On substituting 28 in 27, we get

$$\begin{aligned} E[TSCT] &= \frac{A+M(1-e^{-\lambda T_b})+h\frac{P^2}{D\lambda^2}\left(1-\frac{D}{P}\right)\left(1-e^{-\lambda T_1}-\lambda T_1e^{-\lambda T_1}\right)+\frac{pP}{\lambda^2}\left(1-\frac{D}{P}\theta\right)\left(1-e^{-\lambda T_1}-\lambda T_1e^{-\lambda T_1}\right)}{\frac{P(1-e^{-\lambda T_1})}{\lambda D}+\frac{\left((\eta\lambda D)^2-2\eta(P-D)\lambda D+2(P-D)^2\right)(1-e^{-\lambda T_1})-e^{-\lambda T_1}(P-D)((P-D)(2\lambda T_1)-2\eta\lambda D)}{2\eta\lambda^2D^2}} \\ &+ \frac{L_sD\frac{\left((\eta\lambda D)^2-2\eta(P-D)\lambda D+2(P-D)^2\right)(1-e^{-\lambda T_1})-e^{-\lambda T_1}(P-D)((P-D)(2\lambda T_1)-2\eta\lambda D)}{2\eta\lambda^2D^2}}{\frac{P(1-e^{-\lambda T_1})}{\lambda D}+\frac{\left((\eta\lambda D)^2-2\eta(P-D)\lambda D+2(P-D)^2\right)(1-e^{-\lambda T_1})-e^{-\lambda T_1}(P-D)((P-D)(2\lambda T_1)-2\eta\lambda D)}{2\eta\lambda^2D^2}} \end{aligned} \tag{29}$$

Now we can get the following joint total profit which is the function of the three variables p, T_1 and n.

$$\begin{aligned} \max E[TP] &= (p+v_1)P-\frac{A+M(1-e^{-\lambda T_b})+h\frac{P^2}{D\lambda^2}\left(1-\frac{D}{P}\right)\left(1-e^{-\lambda T_1}-\lambda T_1e^{-\lambda T_1}\right)+\frac{pP}{\lambda^2}\left(1-\frac{D}{P}\theta\right)\left(1-e^{-\lambda T_1}-\lambda T_1e^{-\lambda T_1}\right)}{\frac{P(1-e^{-\lambda T_1})}{\lambda D}+\frac{\left((\eta\lambda D)^2-2\eta(P-D)\lambda D+2(P-D)^2\right)(1-e^{-\lambda T_1})-e^{-\lambda T_1}(P-D)((P-D)(2\lambda T_1)-2\eta\lambda D)}{2\eta\lambda^2D^2}} \\ &+ \frac{L_sD\frac{\left((\eta\lambda D)^2-2\eta(P-D)\lambda D+2(P-D)^2\right)(1-e^{-\lambda T_1})-e^{-\lambda T_1}(P-D)((P-D)(2\lambda T_1)-2\eta\lambda D)}{2\eta\lambda^2D^2}}{\frac{P(1-e^{-\lambda T_1})}{\lambda D}+\frac{\left((\eta\lambda D)^2-2\eta(P-D)\lambda D+2(P-D)^2\right)(1-e^{-\lambda T_1})-e^{-\lambda T_1}(P-D)((P-D)(2\lambda T_1)-2\eta\lambda D)}{2\eta\lambda^2D^2}} \end{aligned} \tag{30}$$

Subject to the constraints

$$\begin{aligned} D-P&<0 \\ p&>v_1+v_2 \end{aligned}$$

5.2 Machine restoration time follows Exponential distribution

Next we assume the machine restoration time t as an exponentially distributed random variable. Therefore the probability density function is dictated as follows

$$f(t)=\gamma e^{-\gamma t}\quad \text{for}\quad \gamma>0 \tag{31}$$

By inserting the exponential probability density function in 25, we have the following expression

$$\begin{aligned} E[TSCT] &= \frac{A+M(1-e^{-\lambda T_b})+hE[I]+E[D_c]}{\frac{P(1-e^{-\lambda T_1})}{\lambda D}+\int_0^{T_1}\int_{T_2}^{\infty}(t-T_2)\gamma e^{-\gamma t}\lambda e^{-\lambda T_b}\mathrm{d}t\mathrm{d}T_b} \\ &+ \frac{L_sD\int\limits_{T_b=0}^{T_1}\int\limits_{t=T_2}^{\infty}(t-T_2)\gamma e^{-\gamma t}\lambda e^{-\lambda T_b}\mathrm{d}t\mathrm{d}T_b}{\frac{P(1-e^{-\lambda T_1})}{\lambda D}+\int_0^{T_1}\int_{T_2}^{\infty}(t-T_2)\gamma e^{-\gamma t}\lambda e^{-\lambda T_b}\mathrm{d}t\mathrm{d}T_b} \end{aligned} \tag{32}$$

The expected shortage can be written as

$$E[T_b] = \int_0^{T_1}\int_{T_2}^{\infty}(t-T_2)\gamma e^{-\gamma t}\lambda e^{-\lambda T_b}\,\mathrm{d}t\mathrm{d}T_b = \frac{e^{T_1\left(\frac{\gamma(P-D)}{2D}(-2+\theta T_1)+\frac{\gamma P}{D}+\lambda\right)}}{e^{T_1\left(\frac{\gamma P}{D}+\lambda\right)}} \tag{33}$$

On substituting 28 in 27, we get

$$E[TSCT] = \frac{A+M(1-e^{-\lambda T_b})+h\frac{P^2}{D\lambda^2}\left(1-\frac{D}{P}\right)\left(1-e^{-\lambda T_1}-\lambda T_1 e^{-\lambda T_1}\right)+\frac{pP}{\lambda^2}\left(1-\frac{D}{P}\theta\right)\left(1-e^{-\lambda T_1}-\lambda T_1 e^{-\lambda T_1}\right)}{\frac{P(1-e^{-\lambda T_1})}{\lambda D}+\frac{e^{T_1\left(\frac{\gamma(P-D)}{2D}(-2+\theta T_1)+\frac{\gamma P}{D}+\lambda\right)}}{e^{T_1\left(\frac{\gamma P}{D}+\lambda\right)}}} + \frac{L_s D\frac{e^{T_1\left(\frac{\gamma(P-D)}{2D}(-2+\theta T_1)+\frac{\gamma P}{D}+\lambda\right)}}{e^{T_1\left(\frac{\gamma P}{D}+\lambda\right)}}}{\frac{P(1-e^{-\lambda T_1})}{\lambda D}+\frac{e^{T_1\left(\frac{\gamma(P-D)}{2D}(-2+\theta T_1)+\frac{\gamma P}{D}+\lambda\right)}}{e^{T_1\left(\frac{\gamma P}{D}+\lambda\right)}}} \tag{34}$$

Now we can get the following joint total profit which is the function of the three variables p, T_1 and n

$$\max E[TP] = (p+v_1)P - \frac{A+M(1-e^{-\lambda T_b})+h\frac{P^2}{D\lambda^2}\left(1-\frac{D}{P}\right)\left(1-e^{-\lambda T_1}-\lambda T_1 e^{-\lambda T_1}\right)+\frac{pP}{\lambda^2}\left(1-\frac{D}{P}\theta\right)\left(1-e^{-\lambda T_1}-\lambda T_1 e^{-\lambda T_1}\right)}{\frac{P(1-e^{-\lambda T_1})}{\lambda D}+\frac{e^{T_1\left(\frac{\gamma(P-D)}{2D}(-2+\theta T_1)+\frac{\gamma P}{D}+\lambda\right)}}{e^{T_1\left(\frac{\gamma P}{D}+\lambda\right)}}} + \frac{L_s D\frac{e^{T_1\left(\frac{\gamma(P-D)}{2D}(-2+\theta T_1)+\frac{\gamma P}{D}+\lambda\right)}}{e^{T_1\left(\frac{\gamma P}{D}+\lambda\right)}}}{\frac{P(1-e^{-\lambda T_1})}{\lambda D}+\frac{e^{T_1\left(\frac{\gamma(P-D)}{2D}(-2+\theta T_1)+\frac{\gamma P}{D}+\lambda\right)}}{e^{T_1\left(\frac{\gamma P}{D}+\lambda\right)}}} \tag{35}$$

Subject to the constraints

$$D - P < 0$$
$$p > v_1 + v_2$$

The procedure of estimating $\max E[TP]$ for both the cases is done through the genetic algorithm.

5.3 Genetic Algorithm

In the following section, we narrate the genetic algorithm which labours corresponding to postulates of natural genetics and evolution, and which has been demonstrated to solve various search and optimization problems. In the usual form of genetic algorithm (GA), narrated by Goldberg , the finest solution is the winner of the genetic game and any promising solution is presumed a individual determined by different parameters. Several authors have engaged GA to work out complex inventory control problems.

A genetic algorithm is an ergodic search method that mimics some mechanisms of natural development. The algorithm works on a population of designs. The population progresses from generation to generation, gradually improving its modification to the environment through natural selection, fitter individuals have better chances of transmitting their characteristics to later generations.

In the algorithm, the selection of the natural environment is replaced by inorganic selection based on a computed fitness for each design. The term fitness is used to income the chromosome's chances of survival and it is essentially the objective function of the optimization problem. The chromosomes that define characteristics of biological beings are replaced by strings of numerical values representing the design variable.

Algorithm of the basic GA is given as follows:

Chromosomes

A chromosome, an immodest part of GA, is a twine or series of genes that is appraised as the coded figure of an achievable solution (proper or improper). In this paper, the chromosomes are twines of the decision variables i.e., the solution of two cases is a 1×3 matrix with $[n, p, T_1]$ form.

Evaluation

In GA, the fitness value is allotted for each chromosomes in the population, as soon as it is generated. Since we have a contrived non-linear problem, some of generated chromosomes may not be feasible. In such situation the infeasible chromosomes are detached from the population.

Population

Each population or generation of chromosomes has the same size which is known as the popu- lation size denoted by N. Similar to Taleizadeh et al.[9] 50 are selected as different population sizes of the GA algorithm of the proposed model.

Crossover

In a crossover operation, mating pairs of chromosomes creates offspring. Crossover operates on the parents' chromosomes with the probability of P_c. If no crossover occurs, the offspring's chromosomes will be the same as their parents'. There are many different crossover operators such as constraint dependent, scattered, single point, two point, intermediate, heuristic, arithmetic. Also we can assign the probability distribution for crossover operator. In this research, we use the constrained dependent crossover operator.

Mutation

Mutation is the second operation in GA to explore new solutions by operating on each chro- mosome resulted from the crossover operation, where genes are replaced with randomly selected numbers within the boundaries of the parameter. To do this, a random number RN between (0,1) is generated for each gene. If RN is less than a predetermined mutation probability P_m, then the mutation occurs in the gene. Otherwise, it does not. The usual value of P_m is 0.1 over the numbers of genes in a chromosome.There are many mutation operations are available such as constraint dependent, uniform, Gaussian, adaptive feasible. In this research, we choose uniform mutation operation with probability rate Pm. Infeasible chromosomes of mutation operation does not move to the next population.

Objective function evaluation

In a maximization problem, the more adequate the solution, the greater the objective function (fitness value) will be. Therefore, the fittest chromosomes will take part in offspring generation with a larger probability. Simple Simulation is used to evaluate the objective function of this research.

Selection

Selection enacts a central role in GAs by determining how individuals compete for survival. Selection weeds out the bad solutions and keeps the good ones. This can be dispatched by pro- portional fitness selection that assigns a selection probability in proportion to the fitness of the given individual. Remainder, uniform, roulette, tournament are some of the selection operations in GA. In this research we employed roulette wheel selection because of its better solutions get higher chance to become parents of the next generation.

Stopping criterion

The last step in a GA is to verify whether the algorithm has found a solution that is good enough to meet the user's suppositions. Stopping criterion is a set of conditions such that when satisfied, hints at a good solution. Number of generations, time limit, fitness limit, maximum number evaluations, no improvement in the objective functions are some of stopping criterion in GA. In this research we use the maximum number generations as a stopping criterion. The steps involved in the proposed GA are:

(i) Set the parameters P_c, P_m, and N initialize the population randomly (the individuals should satisfy the constraints)

(ii) Evaluate the objective function for all the chromosomes.

(iii) Select an individual for mating pool by roulette wheel selection.

(iv) Apply crossover to each pair of chromosomes with probability P_c.

(v) Apply mutation to each chromosome with probability P_m.

(vi) Replace the current population by the resulting mating pool.

(vii) Evaluate the objective function.

(viii) If stopping criterion is met, then stop the algorithm. Otherwise, go to step(iv).

6. Numerical Analysis

In this section, we conduct numerical analysis to illustrate the solution procedure. The values of the following parameters which are almost similar to those used in Widyadana and Wee [12]:

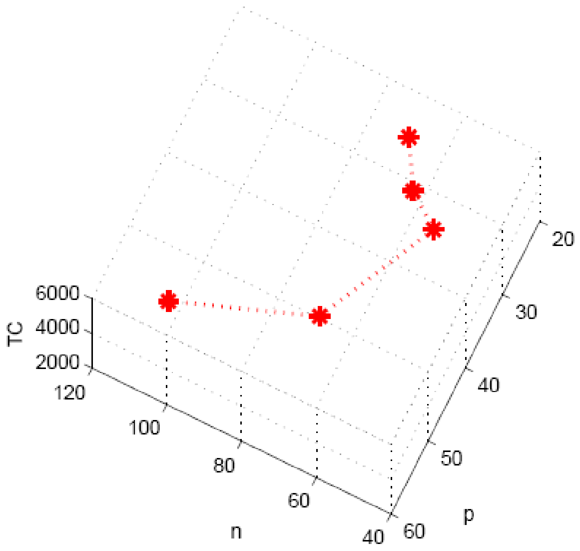

Figure 2: Convex Graph on Uniform Distribution Case

A = \$50 per cycle, M = \$200 per restoration, h = \$1 per unit per unit time, L_s = \$5 per unit, θ = 0.3, λ = 0.2, α = 12500 per unit per unit time, β = 1.25 per unit per unit time, R = 0.9, a = 0.6, C = \$3000 per cycle, v_1 = \$2 per unit, v_2 = \$1.5 per unit.

Uniform distribution case

For this we have considered η as 0.2. Accordingly, when both constraints are ignored, we can choose the optimal values for selling price of the the product p = \$30.5, number of labors n = 58, production period T_1 = 0.008 and the corresponding maximum expected total profit $E[TP]$ = \$4070.

Exponential distribution case

For this we have considered γ as 0.2. Accordingly, when both constraints are ignored, we can choose the optimal values for selling price of the the product p = \$17.4, number of labors n = 31, production period T_1 = 0.658 and the corresponding maximum expected total profit $E[TP]$ = \$4613.

From the above cases, we have observed that the model under exponential machine restoration time is considered to be better economically.

7. Sensitivity Analysis

The sensitivity study of the key parameters of each case is performed with changes -20%, -10%, +20%, +10%. At an instant of time one parameter can be changed and all the others should be kept unchanged. The results are revealed in Table (1) and (2).

1. As the demand rate D is based on the selling price, therefore, increase in demand rate increases the sale revenue. But the demand rate depends upon another two factors α and β. Due to this dependency, we are in urge to scrutinize those factors. Thus increase in the factor α increases the sales profit in a greater extent in both the cases and increase in the factor β decreases the sales profit in a greater extent in both the cases.
2. As there is an increase in the production setup cost A, there is a gradual decrease in the expected profit for both the cases.

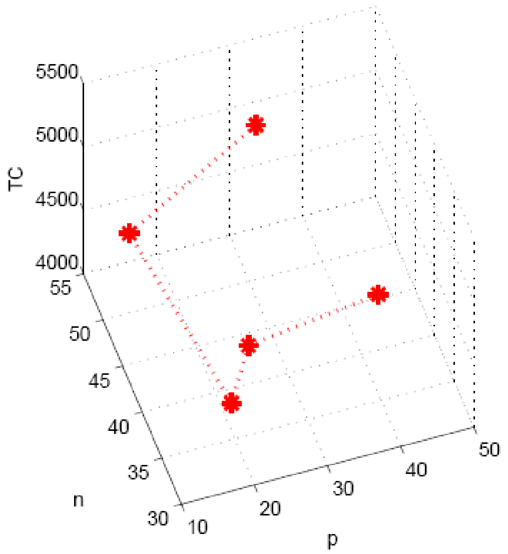

Figure 3: Convex graph on Exponential distribution case

3. The production rate *P* is based on *R*, the technology which accelerates the rate of production, *C*, the capital investment except the investment in technology and *n*, the number of labors, *a* and (1 *a*), the capital and labor elasticity respectively. Hence higher values of *R* and *C* lead to higher the sales revenue slightly in both the cases but increase in *a* gives a moderate increase in the sales profit under the two different cases.

4. The increase in restorating cost of the machine does not affect the sales revenue but sometimes it increases the sales profit slightly for both the cases.

5. As *h* increases, the expected total profit remains unchanged in uniform distribution case but decreases gradually in the exponential distribution case. The increase in lost sale L_s gives a moderate decrease in sales revenue.

6. Due to the increase in the rate parameter corresponding to each distribution, there is a moderate decrease in the expected profit.

8. Conclusion

An EPQ model has been developed in this paper for a deteriorating product to be manufactured on a machine that is subject to random disruptions and stochastic restoration time. The objective is that an evolutionary optimization technique has been used to determine the expected maximum profit cost. It is seen that the genetic algorithm approach seems to be a powerful approach for present type problem. GA approach is particularly suited for problems where there does not exist a well-defined mathematical relationship between the objective function and the design variables. The results prove the success of the application of GAs for the fixture layout optimization problems. Furthermore, the applicability of the proposed methodology along with a sensitivity analysis on its parameters has been shown by numerical example. Hence the model under exponential machine restoration time is considered to be better economically. Some avenues for future works follow:

The demand or other parameters of the problem may take ambiguity forms (stochastic or rough) as well.

Table 1: Sensitivity of Profit in case 1

Parameter	-20%	-10%	+10%	+20%
A	4080	4075	6064.9	4059.9
M	4070	4070	4069.9	4069.9

h	4070	4070	4070	4070
L_s	4239.7	4154.8	3985.1	3900
θ	4070	4070	4070	4070
η	4075.6	4072.8	4067.2	4064.5
λ	4075.6	4072.8	4067.2	4064.4
α	3263.3	3667.4	4470.7	4869.1
β	8873.4	6171.9	2664	1740.3
R	4033.9	4056.6	4078.4	4083.9
α	3926.5	4030.6	4086.7	4094.2
C	4052	4062.6	4075.3	4079.4
v_1	4139.7	4104.8	4035.1	4000.3
v_2	4070	4070	4070	4070

Table 2: Sensitivity of Profit in case 2

Parameter	-20%	-10%	+10%	+20%
A	4618	4615.5	4610.4	6407.9
M	4615.4	4614.2	4611.7	4610.5
h	4643.8	4628.4	4597.5	4582.1
L_s	4674.3	4643.6	4582.3	4551.6
θ	4674.5	4643.7	4582.2	4551.4
γ	4626.4	4619.6	4606.6	4600.4
λ	4631.9	4622.4	4603.6	4594.4
α	3630.2	4121.2	5105.4	5598.5
β	4667.1	4640	4586	4559
R	4667.1	4640	4586	4559
α	4687.1	4652.8	4566.3	4511.5
C	4646.8	4629.5	4597.1	4581.8

v_1	4753.3	4683.1	4542.8	4472.6
v_2	4612.9	4612.9	4612.9	4612.9

Some other probability density functions rather than uniform and exponential may be con- sidered for the time between replenishments.

- One can consider preventive maintenance and normal distribution corrective time.
- One can choose other distributions for stagnation and time-to-disruption.

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