International Journal Of Engineering Research And Development
e- ISSN: 2278-067X, p-ISSN: 2278-800X, www.ijerd.com
Volume 21, Issue 1 (January, 2025), PP 143-149

Solving Multi-Objective Transportation Problem by using the Revised Simplex Method

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Abstract: *The Multi-ObjectiveTransportationProblem(MOTP)isachallengingoptimization issuethatencompassesvariousconflicting objectives. This researchinvestigatestheprocessof converting MOTP into a single-objective problem by employing the Weight Sum Method. Three separate conditions wereanalyzed to achievethetransformation from amulti-objective to a single-objective framework. The Revised Simplex Method was then applied to solve the problem under each of these conditions. The results of the study confirm the effectiveness of the Weight Sum Method in reconfiguring multi-objective problems into a solvable single-objective format and emphasize the role of the Revised Simplex Method in yielding optimal solutions for complex transportation challenges.*
Keywords:*Multi-ObjectiveTransportationProblem,WeightSumMethod,RevisedSimplex Method*

Date of Submission: 07-01-2025 Date of acceptance: 18-01-2025

I. Introduction:

One significant area that uses linear programming is the transportation of goods and services from multiple supply areas to multiple demand centers. A TP that is expressed in terms of an LP model can also be resolved using the simplex method. Even if a TP includes a lot of variables and restrictions, solving it with simplex methods takes a long time. The structure of the TP consists of many shipping routes from different supply sites to different demand regions [17, 21]. The goal is to establish shipping routes between supply and demand hubs to fulfill the demand for a certain amount of products or services at each destination location with the supply of those same goods or services at each supply location at the lowest possible transportation expense. Different examples correspond to different types of TP. MOTP is a type of special type of TP. The problem is called a multi-objective transportation problem when it includes multiple objective functions [8]. In the actual world, every company wants to deliver goods while accomplishing several goals, such reducing expenses, time, distance, risk, etc. The first TP model was developed in 1941 by Hitchcock [15].

The intricacy of the social and economic backdrop in real-world situations requires the explicit consideration of aspects other than cost, which can be achieved by redefining classical TP as MOTP models. In 1961, Charnes and Cooper [28] first suggested a number of approaches for addressing management-level problems with numerous conflicting goals. Zangiabadi [29] addressed MOTP in 2007 by using fuzzy goal programming. A fuzzy compromise programming method was put up by Li [20] for MOTP. Kaur [2] offers a simple technique for finding the best compromise solution for the linear MOTP. A fuzzy programming approach was used in the strategy that George [13] suggested. Doke [5] used the arithmetic mean of the global assessment to solve the three objective linear TPs. In 2016, Bharathi [3] employed evolutionary methods for the MOTP. Singh [26] addressed MOTP in a fuzzy environment using geometric methods. The multi-objective transportation problem is studied by Khan [19] using an S-type membership function. Using fuzzy technique and the c-program, Kavita [14] proposed the new row maximum method to MOTP. Singh [27] developed a new method known as the Matrix Maxima Method to solve a MOTP utilizing Pareto Optimality Criteria. In 2022, Ekanayake [11] introduced the ant colony optimization technique and geometric mean method to address MOTP in fuzzy situations. Moreover, many researchers proposed several approaches to solve MOTP. Different type of algorithms was proposed to resolve MOTP [1,4,6,7,9,10,11,16,18,29,24,25].

The goal of this work is to develop a novel alternative algorithm that uses the geometric mean in conjunction with the penalty technique to solve the multi-objective transportation issue. Finally, using illustrated instances of MOTP with multiple targets, the suggested method is compared with several current methods.

II. Preliminaries

In this section some basic definitions are reviewed

Formulation of Transportation problem in Linear Programming Problem

Given m origins and n destinations, the transportation problem can be formulated as the following linear programming problem model:

Minimize: $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$

Subject to constraint:

$\sum_{j=1}^{n} x_{ij} \leq a_i$ i=1, 2, ………..m

$\sum_{i=1}^{m} x_{ij} \geq b_j$ j=1, 2,………..n

$x_{ij} \geq 0$ for all i and j

Where x_{ij} is the amount of units of shipped from origin I to destination j and c_{ij} is the cost of shipping one unit from origin i to destination j. The amount of supply at origin is ai and the amount of destination j is bj. The objective is to determine the unknown xij that will the total transportation cost while satisfying all the supply and demand constraints.

Multi – objective Transportation Problem (MOTP)

In real life situations, all the transportation problems are not single objective. The transportation problems which are characterized by multiple objective functions are considered here. A special type of linear programming problem in which constraints are of equality type and all the objectives are conflicting with each other, are called MOTP. Similar to a typical transportation problem, in a MOTP problem a product is to be transported from *m* sources to *n* destinations and their capacities are $a_1, a_2, \cdots a_m$ and $b_1, b_2, \cdots b_n$ respectively. In addition, there is a penalty c_{ij} associated with transporting a unit of product from i^{th} andj^{th} destination. This penalty may be cost or delivery time or safety of delivery or etc. A variable x_{ij} represents the unknown quantity to be shipped from i^{th} source to j^{th} destination. A mathematical model of MOTP with *r* objectives, *m* sources and *n* destinations can be written as: Mathematically, the problem can be stated as

$$\text{Minimize } Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{r}.x_{ij} \quad (2.1)$$

$$\left.\begin{aligned} \text{Subject to } & \sum_{j=1}^{n} x_{ij} \geq a_i, \ \ i = 1, 2, \ldots., m \\ & \sum_{i=1}^{m} x_{ij} \leq b_j, \ \ j = 1, 2, \ldots., n \\ & x_{ij} \geq 0 \quad \forall i, j \end{aligned}\right\} \quad (2.2)$$

A transportation problem is said to be balanced if total supply from all sources equals to the total demand in all destinations$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. Otherwise, it is called unbalanced.

Weighted Sum

Weights are assigned $w_1, w_{2,} w_{3,} \cdots w_r$ to each objective to reflect their relative importance according to requirement of customers to change the multiple objectives into the single objective. The weight must be satisfied the following condition:

$$w_1 + w_{2,} + \cdots + w_r = 1$$

i.e

$$\sum_{l=1}^{r} w_r = 1 \quad (2.3)$$

Multiple objective change into the single objective by using assign weights, which know as weighed objective function. The new objective function to be minimized or maximized i.e.

$$Z = w_1.\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{1}q_{ij} + w_2.\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{2}q_{ij} + w_3.\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{3}q_{ij} + \cdots + w_r.\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r}q_{ij} \quad (2.4)$$

Revised Simplex Algorithm:

The revised simplex algorithm can be expressed in the following steps:

Step 1: Express the given problem in standard from:

Express the given problem in the revised simplex from by considering the objective function as one of the constraints and slack and surplus variable, if needed, to inequalities to convert them into equalities.

Step 2: Obtain initial basic feasible solution:

Start with initial basic matrix $B = I_m$ and find B_1^{-1} and $B_1^{-1}b$ to from the initial revised simplex table as shown in Table 1

Table 1

Variable in Basic **B**	Solution Value $b = (x_1^{(1)})$	Basic Inverse B_1^{-1}				$y_k^{(1)}$
		$\beta_0^{(1)}(= Z)$	$\beta_1^{(1)}(= s_1)$	$\cdots$	$\beta_m^{(1)}(= s_m)$	
Z	0	1	0	$\cdots$	0	$c_k - z_k$
$x_{B_1} = s_1$	b_1	0	1	$\cdots$	0	y_{1k}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$x_{B_m} = s_m$	b_m	0	1	$\cdots$	0	y_{mk}

Step 3: Select a variable to enter into the basic (key column):

For each non-basic variable, calculate $c_j - z_j$ by using the formula $c_j - z_j = c_j - c_B B_1^{-1} a_j^{(1)}$

Where, $B_1^{-1} a_j^{(1)}$ represent the product of the first row of B_1^{-1} and successive columns of A not in B_1^{-1}

i) If all $c_j - z_j \leq 0$, then the current basic solution is optimal. Otherwise going to Step 4.

ii) If one or more $c_j - z_j$ are positive, then variable to enter into the basic may be selected by using the formula

$$c_j - z_j = Max\{c_j - z_j; c_j - z_j > 0\}$$

Step 4: Select a variable to leave the basic (Key row)

Calculate $y_k^{(1)} = B_1^{-1} a_k^{(1)} = a_k^{(1)}$; $(k = 1)$ where $a_k^{(1)} = [-c_k, a_k]$. if all $y_{ik} \leq 0$, the optimal solution is unbounded. But if at least one $y_{ik} > 0$, then variable to be removed from the basic is determined by calculating the ratio

$$\frac{x_{Br}}{y_{rk}} = Min_i \left\{ \frac{x_{Bi}}{y_{ik}};\ y_{ik} > 0 \right\}$$

That is, the vector $\beta_r^{(1)}$ is selected to leave the basic and go to step 5.

If the minimum ratio is not unique, i.e. the ratio is same for more than one row, then resulting basic feasible solution will be degenerate. To avoid cycling to occur, the usual method of resolving the degeneracy is applied.

Step 5:Update the current solution

Update the initial table by introducing a non-basic variable x_k $(= a_k^{(1)})$ into basic and removing basic variable x_r $(= \beta_r^{(1)})$ from the basic.
Repeat Steps 3 to 5 until an optimal solution is obtained or there is an indication for an unbounded solution.

III. Numerical Example

Here we consider example, A company ships truckloads of grain from three warehouses to four distributed centres. The supply (in Truckloads) and the demand (also in truckloads) together with the unit transportation costs is per Quintals per kilometre on the different routes and transportation time t_{ij} between source and destination are summarized in the transportation model in table.2.

Table - 2

		D_1	D_2	D_3	D_4	Supply
S_1	**Cost**	3	1	4	2	20
	Time	**2**	**3**	**2**	**4**	
S_2	**Cost**	2	3	1	5	30
	Time	**3**	**1**	**5**	**2**	
S_3	**Cost**	4	2	3	3	25
	Time	**1**	**4**	**3**	**2**	
Demand		15	20	25	15	

To change our multiple objectives into the single objective we are going to use weight sum method by considering the three different condition and obtained the following single objective:

CASE I: weightage of the first and second objectives are 0.6 and 0.4 respectively, i.e.

$$w_1 = 0.6 \text{ and } w_2 = 0.4$$

Then by using equation (2.4) we obtained

Table-3

	D_1	D_2	D_3	D_4	Supply
S_1	2.6	1.8	3.2	2.8	20
S_2	2.4	2.2	2.6	3.8	30
S_3	2.8	2.8	3	2.6	25
Demand	15	20	25	15	

CASE II: weightage of the first and second objectives are 0.6 and 0.4 respectively, i.e.

$$w_1 = 0.3 \text{ and } w_2 = 0.7$$

Then by using equation (2.4) we obtained

Table-4

	D_1	D_2	D_3	D_4	Supply
S_1	2.3	2.4	2.6	3.4	20
S_2	2.7	1.6	3.8	2.9	30
S_3	1.9	3.4	3	2.3	25
Demand	15	20	25	15	

CASE II: weightage of the first and second objectives are 0.6 and 0.4 respectively, i.e.

$$w_1 = 0.7 \text{ and } w_2 = 0.3$$

Then by using equation (2.4) we obtained

Table-5

	D_1	D_2	D_3	D_4	Supply
S_1	2.7	1.6	3.4	2.6	20
S_2	2.3	2.4	2.2	4.1	30
S_3	3.1	2.6	3	2.7	25
Demand	15	20	25	15	

To get the optimal solution of all three conditions, firstly we are converting above transportation problem (Table No-3, 4, 5) into the linear programming problem and obtained

Minimize

$$Z_1 = 2.6x_{11} + 1.8x_{12} + 3.2x_{13} + 2.8x_{14} + 2.4x_{21} + 2.2x_{22} + 2.6x_{23} + 3.8x_{24} + 2.8x_{31} + 2.8x_{32} + 3x_{33} + 2.6x_{34}$$

$$Z_2 = 2.3x_{11} + 2.4x_{12} + 2.6x_{13} + 3.4x_{14} + 2.7x_{21} + 1.6x_{22} + 3.8x_{23} + 2.9x_{24} + 1.9x_{31} + 3.4x_{32} + 3x_{33} + 2.3x_{34}$$

$$Z_3 = 2.7x_{11} + 1.6x_{12} + 3.4x_{13} + 2.6x_{14} + 2.3x_{21} + 2.4x_{22} + 2.2x_{23} + 4.1x_{24} + 3.1x_{31} + 2.6x_{32} + 3x_{33} + 2.7x_{34}$$

Subject to constraints

$$x_{11} + x_{12} + x_{13} + x_{14} \le 20$$

$$x_{21} + x_{22} + x_{23} + x_{24} \le 30$$

$$x_{31} + x_{32} + x_{33} + x_{34} \le 25$$

$$x_{11} + x_{21} + x_{31} \ge 15$$

$$x_{12} + x_{22} + x_{32} \ge 20$$

$$x_{13} + x_{23} + x_{33} \ge 25$$

$$x_{14} + x_{24} + x_{34} \ge 15$$

$$x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34} \ge 0$$

Here we observe that all objectives function is different – 2 but subject to constraints are same for all the objective. Now we applying revised simplex method obtained the optimal solution is as follows

Table-6: Optimal Solution by Revised Simplex Method

	Transportation Cost	Transportation Time	Optimal Solution
Case I	0.6	0.4	$Z_1 = 200$
Case II	0.7	0.3	$Z_2 = 168$
Case III	0.3	0.7	$Z_3 = 190.5$

IV. Conclusion:

This study successfully demonstrates the transformation of a Multi-Objective Transportation Problem into a Single Objective Transportation Problem using the Weight Sum Method, considering three distinct weighting conditions. The Revised Simplex Method efficiently solves the resulting problems, providing optimal solutions for each case: $Z_1 = 200, Z_2 = 168$, and $Z_3 = 190.50$. These findings confirm the robustness and adaptability of the combined methods in solving MOTPs, offering valuable insights for practical applications in transportation logistics and decision-making. Future research could explore alternative weighting strategies and extend the methodology to more complex multi-objective optimization problems.

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