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Acoustic Analysis and Mathematical Modelling of a Stiff Spring-Coupled Double-String Banam: A Study of Vibrational Characteristics

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Abstract
Indian musical instruments, such as the Banam, combine strings with a resonating membrane. In this study, we investigate the acoustic properties of the Banam musical in-strument. Initially, the Banam is played, and the resulting audio signal is recorded using an audio recorder. The recorded signal is plotted, and its frequency content is analysed using the Short-Time Fourier Transform (STFT) rather than the Fast Fourier Transform (FFT) to capture its time-varying characteristics. The power spectrum and spectrogram are used to investigate the energy distribution across frequencies and the variation in frequency con-tent over time, respectively. The power spectrum indicates that the signal contains more energy at lower frequencies than at higher frequencies, which is consistent with the spec-trogram, where prominent dark bands appear in the lower-frequency region. Furthermore, a mathematical model of the Banam, represented as a coupled two-string system connected by a bridge, is developed to investigate its natural frequencies and their variation.

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I. Introduction

The Banam is a traditional bowed string instrument associated with the Santal(Santhal) com-munities of eastern India, particularly the regions of Jharkhand, West Bengal and Odisha. It represents important elements in Santal cultural heritage and is traditionally used in musical performances, such as folk songs and community performances. The instrument is commonly classified as a bowed monochord, a single-string bowed chordophone, or a double-string bowed chordophone. Unlike many conventional bowed string instruments, the Banam is generally hand-crafted from a single piece of wood, with a resonating cavity and a skin-covered soundboard. Its distinctive construction and decorative features reflect the traditional craftsmanship and cultural identity of the Santal community [1, 2].

Stringed musical instruments produce sound through the vibration of stretched strings, gen-erating fundamental frequencies and higher-order harmonics. Their acoustic characteristics de-pend on factors such as string length, tension, mass, excitation mechanism, and coupling with the instrument body. In bowed-string instruments, sound production involves frictional interac-tion between the bow and string, resulting in sustained oscillations commonly associated with Helmholtz motion. Early theoretical investigations by Friedlander [5] and Keller [7] established the fundamental principles of bowed-string vibrations. Subsequently, McIntyre and Woodhouse

[9] provided a comprehensive understanding of the acoustics of stringed musical instruments, particularly the relationship between string vibration and sound production.

The acoustic behaviour of bowed strings is significantly influenced by bow velocity, bow force, and contact position. Askenfelt [3] experimentally investigated bow motion and bow force during violin performance, while Schelleng [10] examined the physical interaction between the bowed string and the player. Lawergren [8] studied the motion of bowed violin strings, and Guettler [6] investigated the formation of Helmholtz motion. These studies established important theoretical and experimental foundations for understanding the mechanisms governing sound generation in bowed-string instruments. Apart from string properties, the structural components of an in-strument also influence its acoustic response. The bridge and resonating body affect vibration transmission, natural frequencies, and harmonic characteristics. Cremer [4] provided a detailed theoretical explanation of violin acoustics, while Mandal and Wahi [17] investigated coupled plate–string vibrations and their influence on natural frequencies and

harmonicity. Further de-velopments include the study of modal energy distribution by Mandal and Wahi [16] and the modal behaviour of string instruments across the audible frequency range by Sorge [12].
Recent investigations have increasingly incorporated experimental measurements and numer-ical modelling to understand the dynamic behaviour of bowed strings. Schwarz [15] examined Helmholtz vibrations, while Lampis et al.[11] investigated the playability limits of bowed-string transients. Sueda et al. [13] studied the transient response of bowed strings through numeri-cal analysis, and Lampis et al. examined the influence of cello string types on playability and tonal characteristics. These studies highlight the importance of combining experimental and computational approaches for the systematic investigation of musical acoustics.

Despite considerable developments in string acoustics, indigenous musical instruments have received comparatively limited quantitative investigation, particularly in relation to their struc-tural and acoustic variations. The Banam, a traditional bowed-string instrument associated with the Santhal community of eastern India, provides an important opportunity to investigate the influence of string configuration on sound production. The present study therefore compares the acoustic characteristics of single-string and double-string Banam instruments using digital signal-processing techniques. Spectrogram analysis is employed to examine the temporal evolu-tion of frequency components, while power spectrum analysis investigates dominant frequencies and spectral energy distribution. The study aims to establish a quantitative basis for distin-guishing the acoustic responses of the two configurations while contributing to the scientific documentation and preservation of Jharkhand's indigenous musical heritage.

II. Mathematical Modeling

The present study considers a mechanical system comprising two vibrating strings connected through a common bridge. To represent the elastic behaviour of the bridge, three identical, massless springs, each having stiffness K, are incorporated into the model. The arrangement allows the vibrations of the two strings to interact through the elastic connections.

The first and second strings are characterised by their respective constant tensions, T_1 and T_2, linear mass densities, μ_1 and μ_2, and lengths, L_1 and L_2. One end of each string is connected to the spring-supported bridge, whereas the opposite end is simply supported. The transverse displacements of the two strings are represented by y_1 and y_2, respectively.

The analysis assumes that the string tensions remain constant, the springs have negligible mass, and the transverse displacements are sufficiently small. Under these assumptions, the spring-supported bridge provides an elastic coupling between the two strings, allowing the dy-namic interaction of the system to be investigated.

Total kinetic energy of the system can be written as

$$T = \int_0^l \frac{1}{2}\left[\mu_1\left(\frac{\partial y_1(x,t)}{\partial t}\right)^2 + \mu_2\left(\frac{\partial y_2(x,t)}{\partial t}\right)^2\right]dx \tag{1}$$

The total potential energy can be written as

$$V = \frac{1}{2}Ky_1^2(0,t)+\frac{1}{2}Ky_2^2(0,t)+\frac{1}{2}K\left[y_1(0,t)-y_2(0,t)\right]^2+\int_0^L \frac{1}{2}\left[T_1\left(\frac{\partial y_1}{\partial x}\right)^2 + T_2\left(\frac{\partial y_2}{\partial x}\right)^2\right]dx \tag{2}$$

using

$$\mathcal{L} = T - V \tag{3}$$

From the Hamilton principle,

$$\delta\int_{t_1}^{t_2}\mathcal{L}dt = 0 \tag{4}$$

Equation of motion can be written as for string 1

$$\mu_1 dx\frac{\partial^2 y_1}{\partial t^2} = T_1\left[\left(\frac{\partial y_1}{\partial x}\right)_{x+dx} - \left(\frac{\partial y_1}{\partial x}\right)_x\right] \tag{5}$$

The equation 5 can be simplified as

$$\mu_1\frac{\partial^2 y_1}{\partial t^2} = T_1\frac{\partial^2 y_1}{\partial x^2}, \qquad 0 \le x << L_1 \tag{6}$$

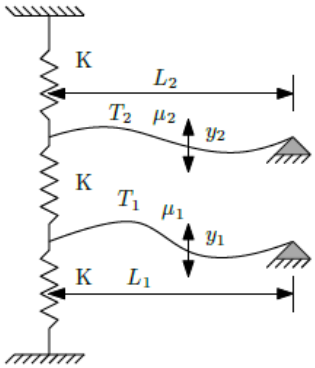
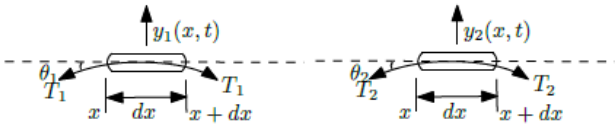


Figure 1: Double string attached through spring



a b

Figure 2: Small part of string a and string b

Similarly we can write the equation of motion of string 2

$$\mu_2 \frac{\partial^2 y_2}{\partial t^2} = T_2 \frac{\partial^2 y_2}{\partial x^2}, \qquad 0 \le x \le L_2 \tag{7}$$

Now we have found the boundary condition from free body diagrams 1 and 2, balancing the forces acting on as follows, Force balance at down junction string 1

$$T_1 \frac{\partial y_1(0,t)}{\partial x} + K[y_2(0,t) - 2y_1(0,t)] = 0 \tag{8}$$

Force balance at upper junction string 2

$$T_2 \frac{\partial y_2(0,t)}{\partial x} + K[y_1(0,t) - 2y_2(0,t)] = 0 \tag{9}$$

Other boundary conditions can be written as

$$y_1(L_1,t) = 0, \qquad y_1(L_2,t) = 0 \tag{10}$$

From the above expression we have found that both equation are dependent each other so we solve as a coupled differential equation and we have found that

$$A_1 \sin\left(\frac{\omega L_1}{c_1}\right) + B_1 \cos\left(\frac{\omega L_1}{c_1}\right) = 0 \tag{11}$$

similarly,

$$A_2 \sin\left(\frac{\omega L_2}{c_2}\right) + B_2 \cos\left(\frac{\omega L_2}{c_2}\right) = 0 \tag{12}$$

The value of $\frac{\partial y_1}{\partial x}(0,t)$ and $\frac{\partial y_2}{\partial x}(0,t)$ in the equation 8 and, 9 respectively, we get

$$T_1 \frac{A_1 \omega}{c_1} + K(B_2 - 2B_1) = 0 \tag{13}$$

Similarly,

$$T_2 \frac{A_2 \omega}{c_2} + K(B_1 - 2B_2) = 0 \tag{14}$$

Now we have formed a matrix from above equation (11),(12),(13) and (14) , we get

$$\begin{bmatrix} \sin(\frac{\omega L_1}{c_1}) & 0 & \cos(\frac{\omega L_1}{c_1}) & 0 \\ 0 & \sin(\frac{\omega L_2}{c_2}) & 0 & \cos(\frac{\omega L_2}{c_2}) \\ T_1\frac{\omega}{c_1} & 0 & -2K & K \\ 0 & T_2\frac{\omega}{c_2} & K & -2K \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

for non trivial solution above matrix (15), we found the characteristic equation as

$$3\sin\left(\frac{\omega L_1}{c_1}\right)\sin\left(\frac{\omega L_2}{c_2}\right)c_1c_2K^2 + 2\cos\left(\frac{\omega L_1}{c_1}\right)\sin\left(\frac{\omega L_2}{c_2}\right)T_1c_2K\omega + 2\sin\left(\frac{\omega L_1}{c_1}\right)\cos\left(\frac{\omega L_2}{c_2}\right)T_2c_1K\omega + \cos\left(\frac{\omega L_1}{c_1}\right)\cos\left(\frac{\omega L_2}{c_2}\right)T_2T_1K\omega^2 = 0 \tag{16}$$

Table 1: First five roots of the characteristic equation for a coupled string–spring system.

Frequency	Root 1	Root 2	Root 3	Root 4	Root 5
ω_1	2.8064	5.6127	8.4191	11.2255	14.0319
ω_2	2.8793	5.7585	8.6378	11.5171	14.3963

3 Experimental Investigation and Analysis

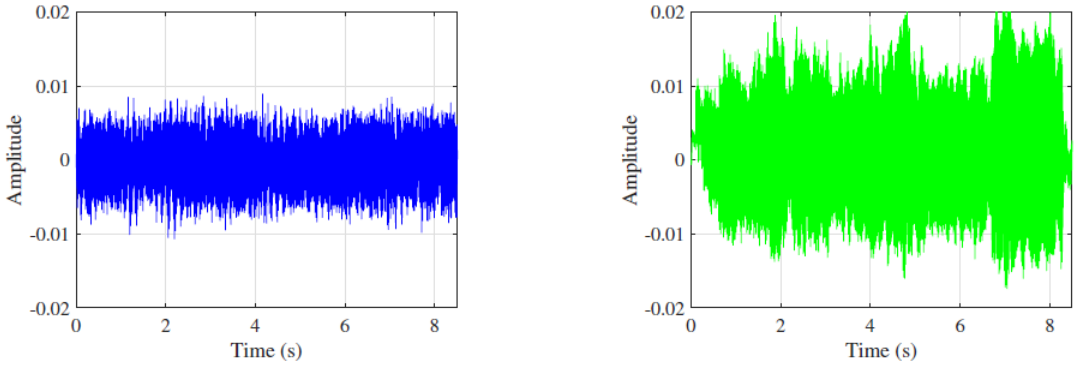
In this section, we have observed the experimental investigation of Banam bowed musical instruments. We have recorded the musical sound through an audio recorder, Audio Tecnica. The Audio Tecnica has a sample frequency of 44400. After recording the sound sample, we have anlyse the sound saples in MATLAB software MATLAB R2020a and extracted the frequency, energy of the sound sample, frequency modulation, and energy contents of frequency with the help of different techniques. We have analysed the qualitative characteristics of the tone of various musical instruments. The techniques used were the raw waveform, Fast Fourier Transform (FFT), Short-Time Fourier Transform (STFT), spectrogram and power spectrum. The given figures (4a) and (4b) show the raw time-domain waveforms of sound signals recorded from the Banam instrument for single-string and double-string configurations. These waveforms provide information on how sound amplitude varies over time and allow us to compare the temporal behaviour of the two configurations. In the single-string configuration, the waveform exhibits relatively consistent amplitude fluctuations throughout the recording duration, indicating comparatively stable temporal amplitude behaviour, and the double-string configuration exhibits more pronounced and irregular amplitude variations, with noticeable fluctuations in the waveform envelope. This behaviour may be attributed to the combined vibrational responses of the two strings and their possible interaction during sound generation. The observed differences suggest that adding a second string affects the temporal characteristics of the generated sound signal. Further, we have conducted frequency-domain and time-frequency analyses to examine the underlying spectral components of musical instrument sound samples. For frequency modulation and time analysis, we have used the FFT (Fast Fourier Transform) technique, but the FFT technique is for steady signals, and our signals are non-stationary, so we use the short-time Fourier Transform. In the above STFT analysis, the variation of frequency components with



(a) (b)

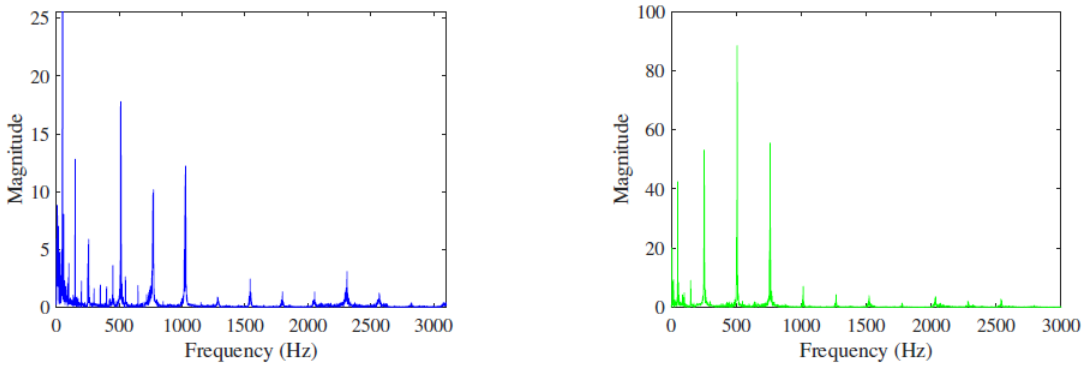
Figure 3: shows(3a) single string and (3b) double string Banam

time cannot be clearly visualised using the frequency spectrum alone. To overcome this limitation, spectrogram analysis is introduced, which provides a time–frequency representation of the



(a) (b)

Figure 4: shows raw signal of sound sample of (4a)single string and (4b) double string Banam



(a) (b)

Figure 5: Shows STFT of sound sample (5a) single string and (5b) double string Banam

audio signal, illustrating how the frequency components and their corresponding amplitudes vary over time. In our study, spectrograms are used to investigate and compare the acoustic characteristics of two Banam musical instruments: (i) a single-string Banam and (ii) a double-string Banam. Mathematical expression of spectrogram $S(t, f)$ can be written as

$$S(t,f) = |STFT(t,f)|^2 = \left| \sum_{n=-\infty}^{\infty} x[n]w[n-t]e^{-j2\pi fn} \right|^2 \tag{17}$$

where $x[n]$ is the input audio signal in the time domain, $w[n]$ is window function, t is time index, f is frequency bin and ι is imaginary unit. Figure (6) illustrates the spectrograms of the recorded sound signals obtained from the single-string and double-string Banam instruments using the

Short-Time Fourier Transform (STFT). The spectrogram provides a time–frequency representation of the acoustic signals, illustrating the distribution and evolution of frequency components over the recording duration. The horizontal axis represents time, the vertical axis represents frequency, and the colour intensity indicates the relative magnitude of the corresponding frequency components in decibels (dB). Figure (6a) shows that the single-string Banam produces several distinct and relatively continuous horizontal frequency bands throughout the recording duration. These bands indicate the presence of sustained tonal components generated by the vibrating string. Similarly, Figure (6b) presents the spectrogram of the double-string Banam, which exhibits multiple frequency components with variations in their spectral distribution and intensity. The combined vibrations of the two strings may contribute to differences in the resulting frequency content and acoustic response compared with the single-string configuration. Furthermore, the variations in the intensity of the frequency bands indicate changes in the spectral energy distribution over time. Overall, the spectrograms demonstrate differences in the time–frequency characteristics of the two Banam configurations and provide a basis for comparing their acoustic behaviour.

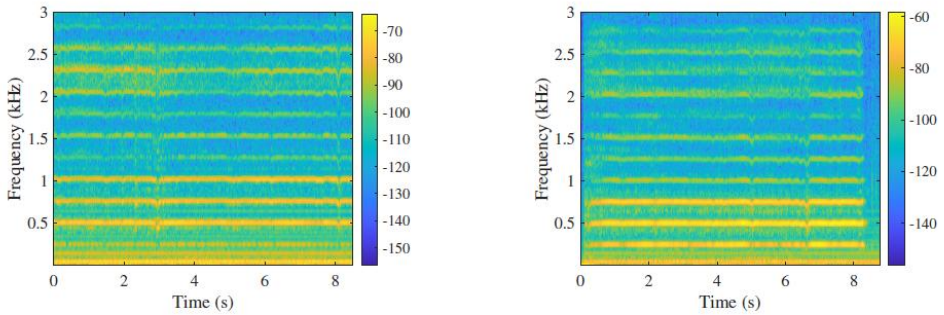


Figure 6: Shows spectrogram of sound sample of (6a) single string and (6b) double string Banam

Unlike the spectrogram, which illustrates how frequency components evolve with time, the power spectrum represents the overall spectral distribution over the analysed signal duration. The power spectrum provides information about the distribution of acoustic power across different frequencies in the sound produced by the single-string and double-string Banam instruments.

Figure (7) presents the power spectra of the recorded sound signals obtained from (7a) the single-string Banam and (7b) the double-string Banam instruments. The power spectrum is used to investigate the distribution of acoustic energy across the frequency range and to identify the dominant frequency components associated with the sound produced by each instrument. The horizontal axis represents frequency in hertz (Hz), while the vertical axis represents power in decibels (dB). The spectral peaks correspond to frequency components with relatively high power, whereas the lower-power regions indicate frequencies with weaker contributions to the recorded sound.

Figure (7a) shows the power spectrum of the single-string Banam. Several distinct peaks are observed across the frequency range, particularly in the lower-frequency region. These peaks in-dicate the presence of dominant tonal components and their associated harmonics in the recorded sound. Additional peaks at higher frequencies contribute to the overall timbre of the instrument. The variation in peak amplitudes suggests that the acoustic energy is not distributed uniformly across the frequency spectrum, with some frequency components contributing more strongly to the resulting sound than others.

Figure (7b) presents the power spectrum of the double-string Banam. Similar to the single-string instrument, multiple spectral peaks are observed across the analysed frequency range. These peaks reflect the combined acoustic contributions of the vibrating strings, together with possible contributions from the instrument body and other components of the sound-generation process. The relative amplitudes and distribution of the peaks differ from those observed for the single-string Banam, indicating differences in their spectral characteristics. The interaction between the two strings may contribute to the observed spectral variations, depending on their tuning, vibration characteristics, and coupling with the instrument body.

A comparison of Figures (7a) and (7b) indicates that both Banam configurations exhibit complex frequency distributions rather than a single dominant frequency. The differences in the locations and relative amplitudes of the spectral peaks provide a basis for comparing their acoustic characteristics. However, the power spectra alone do not establish whether individual peaks originate from separate strings, harmonics, or structural resonances. Therefore, the ob-served differences should be interpreted in

conjunction with the spectrograms presented in Figure (6), which provide additional information about the temporal persistence and evolution of the frequency components. Together, the power spectrum and spectrogram provide complementary information for characterising and comparing the acoustic behaviour of the single-string and double-string Banam instruments.

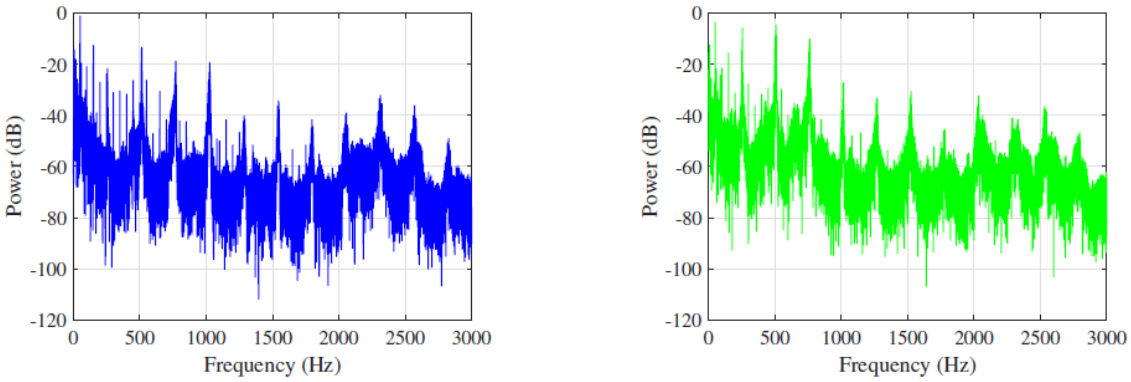


Figure 7: Shows spectrogram of sound sample of (7a) single string and (7b) double string Banam

III.Conclusion

The present study provides a comparative acoustic characterisation of single-string and double-string Banam instruments using spectrogram and power spectrum analyses. The results reveal differences in the temporal distribution, persistence and relative intensity of frequency com-ponents, demonstrating distinct time–frequency and spectral characteristics between the two configurations. The spectrograms illustrate the evolution of frequency components over time, while the power spectra identify the distribution of acoustic energy and the relative prominence of individual spectral peaks. Together, these analyses provide complementary insights into the tonal characteristics and frequency composition of the sound produced by each configuration. The principal research contribution of this study is the systematic application of quantitative signal-processing techniques to compare the acoustic characteristics of two Banam configurations, establishing a measurable basis for investigating the relationship between string configuration and sound production. The observed differences in spectral patterns suggest that the number of strings, their vibrational behaviour and their interaction with the instrument body may influence the resulting acoustic response. The study also demonstrates the value of combining time–frequency analysis with spectral analysis to develop a more comprehensive understanding of indigenous musical instruments. This approach provides a foundation for future investigations involving fundamental frequency, harmonic structure, spectral centroid and other acoustic pa-rameters. Furthermore, the findings contribute to the scientific documentation of the Banam by establishing an analytical framework that can be extended to other traditional stringed instru-ments of Jharkhand. Ultimately, this research bridges traditional musical knowledge and modern acoustic analysis, supporting the systematic documentation, comparative study and preservation of the region's indigenous musical heritage.

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