

A Comprehensive Review on Wet Dark Fluid Models in Modified Gravity Theories

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ABSTRACT

The concept of wet dark fluid has emerged as a promising alternative to conventional dark energy models in modified gravity theories. This review examines the theoretical framework, cosmological implications, and observational constraints of wet dark fluid models within various modified gravity theories including $f(R)$, $f(T)$, $f(Q)$, and $f(R, T)$ gravity. The wet dark fluid equation of state, modeled as $p = \gamma(\rho - \rho_0)$, exhibits liquid-like properties that can effectively describe the transition between different cosmological epochs. Recent investigations demonstrate that wet dark fluid models can successfully explain the late-time cosmic acceleration while maintaining compatibility with observational data from Type Ia supernovae, cosmic microwave background, and baryon acoustic oscillations. This comprehensive review synthesizes current research developments, identifies key challenges, and outlines future research directions in wet dark fluid cosmology within modified gravity frameworks.

Keywords: Wet dark fluid, Modified gravity, $f(R)$ gravity, Dark energy, Cosmological models, Equation of state.

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I. INTRODUCTION

The discovery of cosmic acceleration in the late 1990s through Type Ia supernovae observations fundamentally transformed our understanding of the universe's evolution. This unexpected phenomenon, attributed to dark energy comprising approximately 68% of the cosmic energy budget, presents one of the most significant challenges in modern theoretical physics and cosmology. While the Lambda Cold Dark Matter (Λ CDM) model successfully describes many observational features, it suffers from theoretical difficulties including the cosmological constant problem and fine-tuning issues.

Modified gravity theories have emerged as compelling theoretical frameworks that attempt to explain cosmic acceleration through geometric modifications of Einstein's general relativity rather than introducing exotic dark energy components. These approaches modify the gravitational sector by incorporating functions of curvature scalars, torsion tensors, or non-metricity quantities, leading to field equations that can naturally produce accelerating cosmological solutions.

Among the various dark energy models, wet dark fluid represents a particularly innovative approach that incorporates liquid-like thermodynamic properties into cosmological descriptions. The terminology "wet" derives from the model's equation of state, which mimics the behavior of conventional liquids, particularly water, under different thermodynamic conditions. This approach offers several theoretical advantages, including natural phase transition mechanisms and enhanced flexibility in describing cosmic evolution across different epochs.

The wet dark fluid equation of state is characterized by $p = \gamma(\rho - \rho_0)$, where γ represents a dimensionless parameter governing the fluid's thermodynamic behavior, and ρ_0 denotes a characteristic density scale. This formulation introduces additional degrees of freedom compared to conventional perfect fluid descriptions, enabling more sophisticated modeling of cosmic dynamics and potentially addressing several outstanding cosmological puzzles.

Current observational constraints from diverse sources, including Type Ia supernovae compilations, cosmic microwave background anisotropies, baryon acoustic oscillation measurements, and gravitational lensing surveys, provide stringent tests for alternative dark energy models. These observations reveal that the universe underwent a transition from decelerated to accelerated expansion at redshift $z \approx 0.5 - 0.7$, presenting a crucial benchmark for theoretical models.

II. FUNDAMENTAL THEORETICAL FRAMEWORK

a. Wet Dark Fluid Equation of State Formulation

The wet dark fluid model fundamentally differs from conventional dark energy descriptions through its distinctive equation of state that incorporates liquid-like thermodynamic properties. The basic formulation takes the form:

$$p = \gamma(\rho - \rho_0),$$

where p denotes pressure, ρ represents energy density, γ is a dimensionless thermodynamic parameter, and ρ_0 signifies a characteristic density threshold. This equation captures essential features of liquid behavior, particularly the existence of a critical density below which the fluid exhibits fundamentally different properties.

The parameter γ plays a crucial role in determining cosmological evolution patterns. For $\gamma = 0$, the model reduces to a dust-like scenario with vanishing pressure, while $\gamma = 1$ corresponds to a simple linear relationship between pressure and modified density. Negative values of γ can produce phantom-like behavior with $w < -1$, while positive values typically yield quintessence-like evolution with $w > -1$.

The characteristic density ρ_0 introduces an additional scale that can be related to fundamental physics or fitted to observational constraints. This parameter enables the model to exhibit different behaviors at early and late times, potentially explaining the cosmic coincidence problem through natural evolution mechanisms.

The effective equation of state parameter becomes:

$$\omega_{eff} = \frac{p}{\rho} = \gamma \left(1 - \frac{\rho_0}{\rho}\right)$$

This expression demonstrates the dynamic nature of wet dark fluid, with the equation of state parameter evolving naturally as the cosmic energy density changes over time. During early epochs with high densities ($\rho \gg \rho_0$), the behavior approaches $\omega_{eff} \approx \gamma$, while at later times, significant departures from constant- w evolution emerge.

b. Implementation in Modified Gravity Theories

The incorporation of wet dark fluid models into modified gravity frameworks requires careful consideration of the underlying geometric structure and field equation formulations. Each modified gravity theory provides unique perspectives on gravitational interactions and introduces specific constraints on matter sources.

In $f(R)$ gravity theories, the Einstein-Hilbert action undergoes modification through replacement of the Ricci scalar R with an arbitrary function $f(R)$. The total action becomes:

$$S = \frac{1}{2\kappa^2} \int (f(R) + L) \sqrt{-g} d^4x$$

where $\kappa^2 = 8\pi G/c^4$ represents the gravitational coupling constant, g denotes the metric determinant, and L_{wet} describes the wet dark fluid Lagrangian density. The resulting field equations take the form:

$$\dot{f}(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) \dot{f}(R) = \kappa^2 T_{\mu\nu}^{wet}$$

where prime notation indicates derivatives with respect to R , and $T_{\mu\nu}^{wet}$ represents the wet dark fluid energy-momentum tensor.

Teleparallel $f(T)$ gravity utilizes torsion as the fundamental geometric quantity instead of curvature. The torsion scalar T is constructed from the Weitzenböck connection, and the gravitational action becomes:

$$S = \frac{1}{2\kappa^2} \int (f(T) + L_{wet}) e\sqrt{-g} d^4x$$

where $e = \det(e_\mu^a)$ represents the tetrad determinant. The field equations in teleparallel gravity exhibit second-order derivatives, avoiding the instability issues that can plague higher-order theories.

Symmetric teleparallel $f(Q)$ gravity represents the most recent development in modified gravity, utilizing non-metricity Q as the fundamental geometric variable. The non-metricity tensor encodes information about the failure of parallel transport to preserve vector magnitudes, providing a third geometric foundation for gravitational theories alongside curvature and torsion.

c. Cosmological Applications and Field Equations

For cosmological implementations, wet dark fluid models are typically studied within the framework of homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) spacetimes:

$$ds^2 = -dt^2 + a^2(t)[dr^2/(1 - kr^2) + r^2(d\theta^2 + \sin^2\theta d\phi^2)]$$

where $a(t)$ represents the cosmic scale factor and k denotes the spatial curvature parameter. The energy-momentum tensor for wet dark fluid assumes the perfect fluid form:

$$T_{\mu\nu}^{wet} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$$

where u_μ denotes the four-velocity of the cosmic fluid.

The modified Friedmann equations in various gravity theories with wet dark fluid sources exhibit rich dynamical behavior. In $f(R)$ gravity, these equations become:

$$H^2 = \frac{\kappa^2}{3}(\rho_m + \rho_{wet}) + \frac{1}{6}(f_R - R\dot{f}) + \frac{H}{3}\ddot{f}(R)$$

$$\dot{H} = -\frac{\kappa^2}{2}(\rho_m + \rho_{wet} + p_{wet}) - \frac{1}{6}[\ddot{f}(R) + \dot{f}(R^2)]$$

where $H = \frac{\dot{a}}{a}$ represents the Hubble parameter, dots indicate time derivatives, and subscripts on f denote R -derivatives.

III. DYNAMICAL SYSTEMS ANALYSIS AND PHASE SPACE BEHAVIOR

a. Phase Space Formulation and Dimensionless Variables

Dynamical systems analysis provides a powerful mathematical framework for understanding the global evolutionary behavior of wet dark fluid cosmologies. This approach involves constructing autonomous systems of differential equations through appropriate dimensionless variables that capture the essential physics while remaining tractable for analytical and numerical investigation.

For wet dark fluid models in $f(R)$ gravity, a comprehensive set of dimensionless variables includes:

$$x_1 = \frac{\dot{R}}{6H^2} \text{ (curvaturodynamics)}$$

$$x_2 = \frac{R - f}{6H^2} \text{ (geometricmodification)}$$

$$x_3 = \frac{\kappa^2 \rho_{wet}}{3H^2} \text{ (wetdarkfluidcontribution)}$$

$$x_4 = \frac{\kappa^2 \rho_m}{3H^2} \text{ (matter contribution)}$$

$$x_5 = \frac{\kappa^2 p_{wet}}{3H^2} \text{ (wet dark fluid pressure)}$$

These variables satisfy the constraint $x_3 + x_4 + x_2 = 1$ from the Friedmann equation, reducing the system's dimensionality and enabling systematic phase space analysis.

The evolution equations for these variables can be derived from the modified Friedmann equations and continuity equations, resulting in an autonomous system suitable for critical point analysis. The fixed points of this system correspond to cosmological solutions with time-independent dimensionless variables, representing different evolutionary epochs.

b. Critical Point Analysis and Stability Properties

The critical points of wet dark fluid dynamical systems correspond to specific cosmological scenarios, each characterized by particular values of the dimensionless variables. These equilibrium states can be classified according to their physical interpretation and stability properties.

Matter-dominated fixed points typically appear as saddle-type critical points in the phase space, representing transient evolutionary phases corresponding to structure formation epochs. These solutions are characterized by effective equation of state parameters $\omega_{eff} \approx 0$ and serve as intermediate stages connecting radiation-dominated and dark energy-dominated eras.

Wet dark fluid-dominated solutions exhibit diverse behaviors depending on the model parameters γ and ρ_0 . For negative γ values, these critical points can produce phantom-like acceleration with $\omega_{eff} < -1$, potentially leading to future singularities or Big Rip scenarios. Positive γ values typically yield quintessence-like behavior with $-1 < \omega_{eff} < -\frac{1}{3}$, corresponding to accelerating but non-phantom evolution.

Scaling solutions represent particularly interesting critical points where multiple cosmic components maintain constant density ratios throughout evolution. These solutions are relevant for addressing the cosmic coincidence problem, as they allow for simultaneous presence of matter and wet dark fluid with fixed proportions.

The linear stability analysis of critical points involves computing eigenvalues of the Jacobian matrix evaluated at each fixed point. Stable nodes correspond to future attractors, unstable nodes represent past attractors, and saddle points indicate transient evolutionary phases.

c. Cosmic Phase Transitions and Evolution Pathways

One of the most significant advantages of wet dark fluid models lies in their natural capability to describe phase transitions between different cosmological epochs. The liquid-like equation of state introduces nonlinear dynamics that can drive smooth transitions from decelerated to accelerated expansion phases.

The transition redshift z_t , marking the change from cosmic deceleration to acceleration, can be determined through analysis of the deceleration parameter:

$$q = -1 - \frac{\dot{H}}{H^2}$$

Setting $q = 0$ defines the transition condition, which depends on the wet dark fluid parameters and the specific modified gravity theory under consideration. Current observational constraints suggest $z_t \approx 0.5-0.7$, providing important bounds on model parameters.

The phase space trajectories connecting different critical points reveal the global evolution pathways available to wet dark fluid cosmologies. Generic solutions typically begin near radiation or matter-dominated fixed points at early times and evolve toward dark energy-dominated attractors at late times, potentially passing through intermediate scaling phases.

IV. OBSERVATIONAL CONSTRAINTS AND DATA ANALYSIS

a. Type Ia Supernovae Constraints

Type Ia supernovae serve as standardizable candles for measuring cosmic distances and constraining cosmological parameters. The luminosity distance in wet dark fluid cosmologies is given by:

$$d_{L(z)} = \frac{c}{H_0} (1+z) \int_0^z \frac{dz'}{E(z')}$$

where $E(z) = \frac{H(z)}{H_0}$ represents the normalized Hubble parameter. The specific functional form of $E(z)$ depends on the wet dark fluid equation of state and the underlying modified gravity theory.

Recent analyses using the Pantheon+ supernova compilation, containing over 1700 Type Ia supernovae spanning redshifts $0.001 < z < 2.3$, provide stringent constraints on wet dark fluid parameters. The statistical analysis employs χ^2 minimization:

$$\chi_{SN}^2 = \sum_i \left[\frac{(\mu_{obs}(z_i) - \mu_{th}(z_i))^2}{\sigma_i^2} \right]$$

where μ_{obs} and μ_{th} represent observed and theoretical distance moduli, respectively, and σ_i denotes observational uncertainties.

Bayesian parameter estimation using Markov Chain Monte Carlo (MCMC) methods reveals that wet dark fluid models can achieve competitive fits to supernova data compared to Λ CDM. The optimal parameter values depend on the specific modified gravity framework, with $f(R)$ models typically favoring $\gamma \approx 0.8-1.2$ and ρ_0 values of order H_0^2 .

b. Cosmic Microwave Background Analysis

The cosmic microwave background provides crucial information about the early universe and constrains cosmological models through precise measurements of temperature and polarization anisotropies. Wet dark fluid models primarily affect CMB observables through their influence on the background expansion history and the integrated Sachs-Wolfe effect.

The angular diameter distance to the surface of last scattering is

$$d_A(z_*) = \int_0^{z_*} \frac{dz'}{H(z')}$$

where $z_* \approx 1090$ represents the recombination redshift. This quantity determines the angular scale of acoustic oscillation peaks in the CMB power spectrum, providing a precise cosmological standard ruler.

Planck satellite observations constrain wet dark fluid parameters through analysis of the CMB temperature and polarization power spectra. The acoustic scale parameter:

$$\ell_A = \frac{\pi d_A(z_*)}{r_s(z_*)}$$

where $r_s(z_*)$ is the comoving sound horizon at recombination, must reproduce the observed value $\ell_A \approx 301.4$ for model viability.

c. Baryon Acoustic Oscillations and Large-Scale Structure

Baryon acoustic oscillations provide complementary constraints on cosmological models through measurements of the characteristic scale imprinted by sound waves in the primordial baryon-photon plasma. The BAO scale is determined by the sound horizon at the drag epoch:

$$r_s(z_d) = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)} dz$$

where $c_s(z)$ represents the sound speed and $z_d \approx 1020$ denotes the drag redshift.

Recent measurements from the Dark Energy Spectroscopic Instrument (DESI) and other large-scale structure surveys provide precise determinations of the angular diameter distance $d_A(z)$ and Hubble parameter $H(z)$ across multiple redshift bins. These model-independent measurements offer stringent tests of wet dark fluid models and their predictions for cosmic expansion.

The combination of BAO, supernova, and CMB constraints through joint likelihood analysis provides comprehensive parameter constraints and enables detailed comparison between wet dark fluid models and the standard Λ CDM framework.

V. APPLICATIONS ACROSS MODIFIED GRAVITY FRAMEWORKS

a. Implementation in $f(R)$ Gravity Models

The $f(R)$ gravity framework provides a natural setting for wet dark fluid implementations, with several well-studied functional forms offering distinct phenomenological predictions. The Hu-Sawicki model, characterized by $f(R) = R - \mu R_0 \left[\frac{R^n}{1 + \frac{R^n}{R_0^n}} \right]$, represents one of the most extensively investigated approaches.

When combined with wet dark fluid sources, Hu-Sawicki $f(R)$ gravity can produce viable cosmological evolution with late-time acceleration. The parameter μ controls the modification strength, while R_0 sets the curvature scale where deviations from general relativity become significant. The exponent n determines the transition behavior between different curvature regimes.

The Starobinsky model, $f(R) = R + \alpha R^2$, offers perhaps the simplest viable $f(R)$ theory with natural connections to inflationary cosmology. The R^2 term dominates during early epochs, potentially driving primordial inflation, while late-time evolution involves interplay between the linear R term and wet dark fluid dynamics.

Exponential $f(R)$ models, such as $f(R) = R - \mu R_0 [1 - \exp(-R/R_0)]$, provide exponential suppression of modifications at high curvatures while maintaining significant effects in low-curvature regimes relevant for late-time cosmology. These models demonstrate excellent compatibility with wet dark fluid descriptions and can successfully reproduce observed cosmic acceleration.

b. Teleparallel $f(T)$ Gravity Applications

Teleparallel gravity theories utilize torsion rather than curvature as the fundamental geometric quantity, offering alternative geometric foundations for gravitational interactions. The torsion scalar T , constructed from the torsion tensor, serves as the basic variable for $f(T)$ modifications.

Power-law $f(T)$ models, exemplified by $f(T) = T + \alpha T^n$, have been extensively studied with wet dark fluid matter sources. These theories naturally accommodate accelerating cosmological solutions without requiring

exotic matter components. The parameter α determines the modification strength, while the exponent n controls the functional dependence on torsion.

Exponential $f(T)$ formulations, including $f(T) = T \exp(\beta T_0/T)$, provide rich phenomenology when combined with wet dark fluid. The exponential structure introduces natural cutoffs preventing pathological behaviors at extreme scales while maintaining cosmologically relevant modifications.

A significant advantage of wet dark fluid in $f(T)$ gravity concerns the absence of higher-order field equations that can plague metric $f(R)$ theories. This simplification leads to more straightforward mathematical implementation while preserving the rich phenomenological features associated with modified gravitational dynamics.

c. Symmetric Teleparallel $f(Q)$ Gravity

Symmetric teleparallel $f(Q)$ gravity represents the newest addition to the modified gravity landscape, employing non-metricity Q as the fundamental geometric variable. The non-metricity tensor encodes information about parallel transport failure to preserve vector lengths, providing a third geometric foundation alongside curvature and torsion.

The implementation of wet dark fluid in $f(Q)$ gravity follows analogous principles to other modified theories, with the gravitational action taking the form:

$$S = \frac{1}{2\kappa^2} \int (f(Q) + L_{wet}) \sqrt{-g} d^4x$$

Where L_{wet} represents the wet dark fluid Lagrangian density. The field equations involve the non-metricity tensor and its derivatives, leading to modified cosmological dynamics with additional geometric degrees of freedom.

Recent investigations of power-law $f(Q)$ models with wet dark fluid have revealed stable accelerating solutions capable of explaining late-time cosmic acceleration. The non-metricity-based framework provides additional flexibility for parameter tuning while maintaining theoretical consistency and observational viability.

d. $f(R, T)$ Gravity Extensions

$f(R, T)$ gravity theories represent natural extensions of $f(R)$ models that incorporate explicit coupling between geometric and matter sectors through dependence on both the Ricci scalar R and the energy-momentum tensor trace T . This coupling introduces additional dynamics that can significantly influence cosmological evolution.

The general $f(R, T)$ action is:

$$S = \frac{1}{2\kappa^2} \int (f(R, T) + L_m) \sqrt{-g} d^4x$$

where $f(R, T)$ represents an arbitrary function of R and T . When combined with wet dark fluid, the field equations become:

$$f_R(R, T)R_{\mu\nu} - \frac{1}{2}f(R, T)g_{\mu\nu} - (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square)f(R, T) = \kappa^2 T_{\mu\nu} + f_T(R, T)[T_{\mu\nu} + \Theta_{\mu\nu}]$$

where f_R and f_T denote partial derivatives, and $\Theta_{\mu\nu}$ represents the variation of matter Lagrangian with respect to the metric.

Linear $f(R, T)$ models, characterized by $f(R, T) = R + \alpha T$, have received extensive study with wet dark fluid sources. These models introduce direct coupling between gravitational fields and wet dark fluid energy-momentum, leading to modified conservation equations and dynamics absent in pure $f(R)$ theories.

VI. RECENT ADVANCES AND FUTURE RESEARCH DIRECTIONS

a. Quantum Gravitational Corrections

Recent theoretical developments have begun addressing quantum corrections and ultraviolet behavior in wet dark fluid models within modified gravity frameworks. The incorporation of quantum effects becomes particularly relevant when considering early universe evolution and the transition from quantum to classical cosmological regimes. Loop quantum cosmology provides a natural framework for studying wet dark fluid with quantum corrections. The discrete geometric structure characteristic of loop quantum gravity leads to modified Friedmann equations that can resolve classical cosmological singularities and provide bounce scenarios. Wet dark fluid in loop quantum cosmology exhibits unique features, including possible cyclic universe models and modified early-time dynamics.

Asymptotic safety approaches to quantum gravity have been applied to wet dark fluid models, investigating ultraviolet fixed-point structures and renormalization group flows. These studies suggest that wet dark fluid models may possess improved ultraviolet behavior compared to conventional scalar field dark energy, potentially resolving quantum instability issues.

b. Gravitational Wave Cosmology

The detection of gravitational waves by LIGO-Virgo collaborations has opened unprecedented opportunities for testing modified gravity theories and dark energy models. Wet dark fluid models in modified gravity can affect gravitational wave propagation through background spacetime modifications and potential additional polarization modes. In $f(R)$ gravity with wet dark fluid, gravitational waves experience modified dispersion relations depending on background curvature and wet dark fluid energy density. These modifications can produce detectable deviations from general relativity predictions in gravitational wave observations, particularly for cosmological sources such as merging supermassive black holes.

Multi-messenger astronomy, combining gravitational wave and electromagnetic observations, provides particularly stringent tests of modified gravity theories. The gravitational wave standard siren method can measure luminosity distances independently of the cosmic distance ladder, offering direct tests of wet dark fluid models and their expansion predictions.

c. Machine Learning Applications

The application of machine learning techniques to wet dark fluid cosmology has emerged as a promising research direction. Neural networks and artificial intelligence approaches can efficiently explore complex parameter spaces and identify optimal fits to observational data. Gaussian process regression has been employed to reconstruct wet dark fluid equations of state directly from observational data without assuming specific functional forms. This model-independent approach provides valuable insights into wet dark fluid viability and cosmic acceleration explanations.

Deep learning algorithms have been applied to analyze large-scale structure simulations with wet dark fluid dark energy, identifying characteristic signatures that could distinguish these models from Λ CDM alternatives. These techniques may prove crucial for analyzing data from upcoming surveys including the Vera Rubin Observatory and Euclid mission.

d. Future Observational Prospects

Next-generation cosmological surveys will provide unprecedented precision for testing wet dark fluid models. The Dark Energy Spectroscopic Instrument (DESI) will measure millions of galaxy redshifts, providing precise constraints on expansion history and structure growth across cosmic time.

The Nancy Grace Roman Space Telescope will conduct wide-field imaging and spectroscopic surveys, enabling detailed dark energy studies through multiple complementary probes including supernovae, baryon acoustic oscillations, and weak gravitational lensing. These observations will be particularly sensitive to the dynamic equation of state behavior characteristic of wet dark fluid models.

Future gravitational wave detectors, including the Laser Interferometer Space Antenna (LISA) and third-generation ground-based detectors, will provide novel tests of modified gravity theories through precision measurements of gravitational wave propagation and polarization. These observations may reveal wet dark fluid signatures difficult to detect through electromagnetic observations alone.

VII. CONCLUSIONS

This comprehensive review has systematically examined wet dark fluid models within modified gravity theoretical frameworks, emphasizing their theoretical foundations, cosmological implications, and observational viability. Several significant conclusions emerge from this analysis. Wet dark fluid models provide compelling alternatives to conventional dark energy descriptions through their liquid-like equation of state behavior that naturally accommodates cosmic phase transitions between different evolutionary epochs. The fundamental equation of state $p = \gamma(\rho - \rho_0)$ introduces enhanced flexibility compared to constant- w models while maintaining theoretical elegance and physical motivation grounded in thermodynamic principles. The implementation of wet dark fluid across various modified gravity frameworks, including $f(R)$, $f(T)$, $f(Q)$, and $f(R, T)$ theories, demonstrates remarkable versatility and adaptability. Each modified gravity theory provides unique phenomenological predictions when combined with wet dark fluid, offering multiple observational avenues for distinguishing these models from conventional Λ CDM cosmology.

Dynamical systems analysis conclusively demonstrate that wet dark fluid models can successfully reproduce observed cosmic history, including the crucial transition from decelerated to accelerated expansion. The existence of stable attractor solutions corresponding to late-time acceleration provides robust theoretical support for model viability and cosmological relevance. Observational constraints from Type Ia supernovae, cosmic microwave background radiation, and baryon acoustic oscillation measurements demonstrate that wet dark fluid models achieve data fits comparable to or superior to Λ CDM for appropriate parameter selections. This observational compatibility, combined with theoretical advantages, strongly suggests these models merit continued investigation and development.

Recent advances in quantum corrections, gravitational wave astronomy, and machine learning techniques open exciting new research directions for wet dark fluid cosmology. Future observational programs, including next-generation surveys and gravitational wave detectors, will provide increasingly stringent model tests and observational discrimination capabilities. The field of wet dark fluid research in modified gravity remains highly active and rapidly evolving, presenting numerous opportunities for theoretical advances and observational discoveries. Continued investigation may provide crucial insights into dark energy's fundamental nature and spacetime structure on cosmological scales.

REFERENCES

- [1]. Riess, A. G., et al. (1998). Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astronomical Journal*, 116(3), 1009-1038.
- [2]. Perlmutter, S., et al. (1999). Measurements of Omega and Lambda from 42 high-redshift supernovae. *Astrophysical Journal*, 517(2), 565-586.
- [3]. Weinberg, S. (1989). The cosmological constant problem. *Reviews of Modern Physics*, 61(1), 1-23.

- [4]. Copeland, E. J., Sami, M., & Tsujikawa, S. (2006). Dynamics of dark energy. *International Journal of Modern Physics D*, 15(11), 1753-1935.
- [5]. Nojiri, S., & Odintsov, S. D. (2011). Unified cosmic history in modified gravity: From F(R) theory to Lorentz non-invariant models. *Physics Reports*, 505(2-4), 59-144.
- [6]. Capozziello, S., & De Laurentis, M. (2011). Extended theories of gravity. *Physics Reports*, 509(4-5), 167-321.
- [7]. Cai, Y. F., Capozziello, S., De Laurentis, M., & Saridakis, E. N. (2016). f(T) teleparallel gravity and cosmology. *Reports on Progress in Physics*, 79(10), 106901.
- [8]. Harko, T., Lobo, F. S., Nojiri, S., & Odintsov, S. D. (2011). f(R, T) gravity. *Physical Review D*, 84(2), 024020.
- [9]. Singh, J. K., & Sharma, N. K. (2017). Bianchi type-II universe with wet dark fluid in general theory of relativity. *Astrophysics and Space Science*, 362(7), 118.
- [10]. Mandal, S., Sahoo, P. K., & Santos, J. R. L. (2020). Universe with wet dark fluid: A dynamical systems approach. *International Journal of Modern Physics D*, 29(12), 2050127.
- [11]. Mishra, B., Tarai, S., & Tripathy, S. K. (2018). Tilted Bianchi type-I wet dark fluid model in Saez and Ballester theory. *Indian Journal of Physics*, 92(9), 1199-1205.
- [12]. Sahoo, P. K., & Sivakumar, M. (2016). Plane symmetric dark energy models in the form of wet dark fluid in f(R, T) gravity. *Journal of Astrophysics and Astronomy*, 37(2), 1-10.
- [13]. Yadav, A. K., Rahaman, F., & Ray, S. (2011). Dark energy models with variable equation of state parameter. *International Journal of Theoretical Physics*, 50(3), 871-881.
- [14]. Bamba, K., Capozziello, S., Nojiri, S., & Odintsov, S. D. (2012). Dark energy cosmology: The equivalent description via different theoretical models. *Astrophysics and Space Science*, 342(1), 155-228.
- [15]. Planck Collaboration, Aghanim, N., et al. (2020). Planck 2018 results. VI. Cosmological parameters. *Astronomy & Astrophysics*, 641, A6.
- [16]. Scolnic, D. M., et al. (2018). The complete light-curve sample of spectroscopically confirmed SNe Ia from Pan-STARRS1 and cosmological constraints from the combined pantheon sample. *Astrophysical Journal*, 859(2), 101.
- [17]. DESI Collaboration, et al. (2024). DESI 2024 VI: Cosmological Constraints from the Measurements of Baryon Acoustic Oscillations. arXiv preprint arXiv:2404.03002.
- [18]. Hu, W., & Sawicki, I. (2007). Models of f(R) cosmic acceleration that evade solar-system tests. *Physical Review D*, 76(6), 064004.
- [19]. Starobinsky, A. A. (1980). A new type of isotropic cosmological models without singularity. *Physics Letters B*, 91(1), 99-102.
- [20]. Appleby, S. A., & Battye, R. A. (2007). Do consistent F(R) models mimic general relativity plus Λ ? *Physics Letters B*, 654(1-2), 7-12.
- [21]. Bengochea, G. R., & Ferraro, R. (2009). Dark torsion as the cosmic speed-up. *Physical Review D*, 79(12), 124019.
- [22]. Linder, E. V. (2010). Einstein's other gravity and the acceleration of the Universe. *Physical Review D*, 81(12), 127301.
- [23]. Jiménez, J. B., Heisenberg, L., & Koivisto, T. (2018). Coincident general relativity. *Physical Review D*, 98(4), 044048.
- [24]. Nester, J. M., & Yo, H. J. (1999). Symmetric teleparallel general relativity. *Chinese Journal of Physics*, 37(2), 113-117.
- [25]. Lazkoz, R., Lobo, F. S., Ortiz-Baños, M., & Salzano, V. (2019). Observational constraints of f(Q) gravity. *Physical Review D*, 100(10), 104027.
- [26]. Barros, B. J., et al. (2020). Testing F(Q) gravity with redshift space distortions. *Physics of the Dark Universe*, 30, 100616.
- [27]. Sahoo, P. K., Moraes, P. H. R. S., & Sahoo, P. (2018). Wormholes in f(R, T) gravity satisfying the null energy condition with isotropic pressure. *European Physical Journal C*, 78(1), 1-14.
- [28]. Moraes, P. H. R. S., & Sahoo, P. K. (2017). Modeling wormholes in f(R, T) gravity. *Physical Review D*, 96(4), 044038.
- [29]. Yousaf, Z., Bhatti, M. Z., & Naseer, T. (2020). New definition of complexity factor in f(R, T) gravity. *Physics of the Dark Universe*, 28, 100535.
- [30]. Sharif, M., & Zubair, M. (2012). Thermodynamics in f(R, T) theory of gravity. *Journal of Cosmology and Astroparticle Physics*, 2012(03), 028.
- [31]. Chakraborty, S. (2013). Is thermodynamics of the universe a consequence of gravity? *Physical Review D*, 87(8), 084092.
- [32]. Odintsov, S. D., Oikonomou, V. K., & Banerjee, S. (2019). Dynamics of inflation and dark energy from F(R, G) gravity. *Nuclear Physics B*, 938, 935-956.
- [33]. Nojiri, S., Odintsov, S. D., & Oikonomou, V. K. (2017). Modified gravity theories on a nutshell: Inflation, bounce and late-time evolution. *Physics Reports*, 692, 1-104.
- [34]. Sotiriou, T. P., & Faraoni, V. (2010). f(R) theories of gravity. *Reviews of Modern Physics*, 82(1), 451-497.
- [35]. De Felice, A., & Tsujikawa, S. (2010). f(R) theories. *Living Reviews in Relativity*, 13(1), 3.
- [36]. Bamba, K., et al. (2013). Energy conditions in modified f(G) gravity. *General Relativity and Gravitation*, 45(9), 1881-1892.
- [37]. Santos, J., Alcaniz, J. S., Rebouças, M. J., & Carvalho, F. C. (2007). Energy conditions in f(R) gravity. *Physical Review D*, 76(8), 083513.
- [38]. Tripathy, S. K., Mishra, B., & Mudavanhu, P. (2015). Cosmological models with a hybrid scale factor. *International Journal of Modern Physics D*, 24(08), 1550001.
- [39]. Akarsu, Ö., Kumar, S., Myrzakulov, R., Sami, M., & Xu, L. (2014). Cosmology with hybrid expansion law: Scalar field reconstruction of cosmic history and observational constraints. *Journal of Cosmology and Astroparticle Physics*, 2014(01), 022.
- [40]. Kumar, S., & Xu, L. (2014). Observational constraints on variable equation of state parameters of dark energy. *Physics Letters B*, 737, 244-247.
- [41]. Elizalde, E., Nojiri, S., & Odintsov, S. D. (2004). Late-time cosmology in (phantom) scalar-tensor theory: Dark energy and the cosmic speed-up. *Physical Review D*, 70(4), 043539.
- [42]. Chiba, T., Okabe, T., & Yamaguchi, M. (2000). Kinetically driven quintessence. *Physical Review D*, 62(2), 023511.
- [43]. Armendariz-Picón, C., Damour, T., & Mukhanov, V. (1999). k-inflation. *Physics Letters B*, 458(2-3), 209-218.
- [44]. Padmanabhan, T. (2003). Cosmological constant—the weight of the vacuum. *Physics Reports*, 380(5-6), 235-320.

- [45]. Martin, J. (2012). Everything you always wanted to know about the cosmological constant problem (but were afraid to ask). *Comptes Rendus Physique*, 13(6-7), 566-665.
- [46]. Peebles, P. J. E., & Ratra, B. (2003). The cosmological constant and dark energy. *Reviews of Modern Physics*, 75(2), 559-606.
- [47]. Carroll, S. M. (2001). The cosmological constant. *Living Reviews in Relativity*, 4(1), 1.
- [48]. Joyce, A., Jain, B., Khoury, J., & Trodden, M. (2015). Beyond the cosmological standard model. *Physics Reports*, 568, 1-98.
- [49]. Clifton, T., Ferreira, P. G., Padilla, A., & Skordis, C. (2012). Modified gravity and cosmology. *Physics Reports*, 513(1-3), 1-189.
- [50]. Will, C. M. (2014). The confrontation between general relativity and experiment. *Living Reviews in Relativity*, 17(1), 4.
- [51]. Koyama, K. (2016). Cosmological tests of modified gravity. *Reports on Progress in Physics*, 79(4), 046902.
- [52]. Schmidt, F. (2010). Self-consistent cosmological simulations of DGP braneworld gravity. *Physical Review D*, 81(10), 103002.
- [53]. Zhao, G. B., et al. (2012). Probing modifications of General Relativity using current cosmological observations. *Physical Review D*, 81(10), 103510.
- [54]. Huterer, D., & Turner, M. S. (2001). Probing dark energy: Methods and strategies. *Physical Review D*, 64(12), 123527.
- [55]. Linder, E. V. (2003). Exploring the expansion history of the universe. *Physical Review Letters*, 90(9), 091301.
- [56]. Chevallier, M., & Polarski, D. (2001). Accelerating universes with scaling dark matter. *International Journal of Modern Physics D*, 10(02), 213-223.
- [57]. Jassal, H. K., Bagla, J. S., & Padmanabhan, T. (2005). Observational constraints on low redshift evolution of dark energy: How consistent are different observations? *Physical Review D*, 72(10), 103503.
- [58]. Wang, Y., & Tegmark, M. (2005). Uncorrelated measurements of the cosmic expansion history and dark energy from supernovae. *Physical Review D*, 71(10), 103513.
- [59]. Alam, S., et al. (2017). The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: cosmological analysis of the DR12 galaxy sample. *Monthly Notices of the Royal Astronomical Society*, 470(3), 2617-2652.
- [60]. Abbott, T. M. C., et al. (2018). Dark Energy Survey year 1 results: Cosmological constraints from galaxy clustering and weak lensing. *Physical Review D*, 98(4), 043526.