International Journal of Engineering Research and Development
***e**-ISSN: 2278-067X, **p**-ISSN: 2278-800X, www.ijerd.com*
Volume 3, Issue 10 (September 2012), PP. 47-55

Design of Stabilizing PI Controller for Coupled Tank MIMO Process

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Abstract—In this paper a new method is proposed for calculation of all stabilizing PI controllers for coupled tank multi-input multi-output (MIMO) process. By plotting the stability boundary locus in the (kp,ki) plane for coupled tank multivariable process after applying the decoupling technique, the stabilizing PI controller parameters are computed. The technique proposed does not require sweeping over the parameters and also does not need linear programming t o solve set of inequalities. This method is also used to obtain the all stabilizing PI controllers which achieve user specified gain and phase margins.

Keywords—MIMO, Coupled tank, Boundary locus, Stabilization, Decoupler

I. INTRODUCTION

Designing stabilizing PI, PID and lag/lead controllers is of great importance for researchers since these controllers are popular and extensively used in process industries because of their performance and simplicity. Several methods are available to determine the controller parameters among them Zeigler –Nichols method and Cohen – Coon method are still widely used. After the work of Ho et al. [2], [3] the design of stabilizing P,PI and PID controller get more popular. For design of robust controller by computation of the stability region using the stability boundary locus has been given in [5]. All of these methods have been dealt with controller design applied to single- input single – output (SISO) systems. However in most of the processes more than one variable has to be controlled. These processes are called multi-input multi-output (MIMO) process. The control of these processes is more difficult than SISO processes. Because there is an interaction between other control loops of MIMO processes. Therefore the method used for SISO systems cannot use for control of MIMO systems effectively. Usually two types of control schemes are available to control MIMO processes. The first is decentralized control scheme or multiloop control scheme, where single loop controllers are used (the controller matrix is a diagonal one). The second scheme is a full multivariable controller (known as the centralized controller), where the controller matrix is not a diagonal one. Because of the interaction exist between the loops it is more difficult to design a stabilizing controller for MIMO process.

In this paper computation of stabilizing controller using the stability boundary locus method is applied to coupled tank multivariable process. First the decoupler is designed to eliminate the interaction between the loops and stabilizing values of PI controller is obtained in the parameter plane (K_p, Ki) for coupled tank process. The proposed method is also used for computation of PI controllers for achieving user defined gain and phase margins.

The paper is organized as follows: in section 2 computation of stabilizing PI controllers using stability boundary locus method is described. In section3 introduction about the decoupling technique is given. The coupled tank MIMO process is summarized in section 4. Computation of stabilizing PI controller and response for the coupled tank process is given in section 5.

II. DETERMINATION OF STABILIZING PI CONTROLLERS

A possible approach to calculation of stabilizing PI controllers based on plotting the stability boundary locus is proposed in [Tan & kaya, 2003; Tan et al., 2006]. Consider the single-input single-output process as shown in Fig. 1 where $G_p(s)$ is the process to be controlled by the PI controller $G_c(s)$

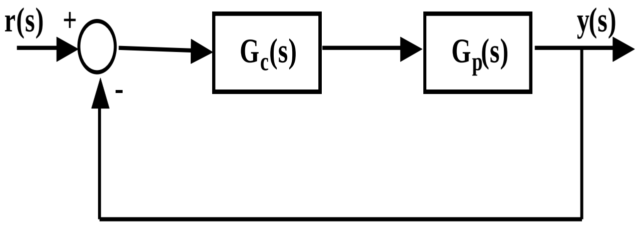

Fig. 1 Block diagram of SISO system

$$G_p(s) = G(s)e^{-\theta s} = \frac{N(s)}{D(s)}e^{-\theta s} \tag{1}$$

$$G_c(s) = k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s} \tag{2}$$

N(s) and D(s) are the numerator and denominator polynomial of the process by decomposing it into their corresponding even and odd part by replacing s by jω gives

$$G(j\omega) = \frac{N_e(-\omega^2)+ j\omega\ N_o(-\omega^2)}{D_e(-\omega^2)+ j\omega\ D_o(-\omega^2)} \quad (3)$$

For simplicity ($-\omega^2$) will be neglected in the following equation. The closed loop characteristic polynomial can be written as

$$\Delta(j\omega) = \left[\left(k_i N_e - k_p\omega^2 N_0\right)\cos(\omega\theta) + \omega(k_i N_0 + k_p N_{e)}\sin(\omega\theta) - \omega^2 D_0\right] + j\left[\omega(k_i N_0 + k_p N_e)\cos(\omega\theta) - (k_i N_e - \omega^2 k_p N_0)\sin(\omega\theta) + \omega D_e\right]$$

$$= R_\Delta + jI_\Delta = 0 \quad (4)$$

Equating the real and imaginary parts of Δ (jω) to zero we get

$$k_p[-\omega^2 N_o\cos(\omega\theta) + \omega N_e\sin(\omega\theta)] + k_i[N_e\cos(\omega\theta) + \omega N_o\sin(\omega\theta)] = \omega^2 D_o \quad (5)$$

$$k_p[\omega N_e\cos(\omega\theta) + \omega^2 N_0\sin(\omega\theta)] + k_i[\omega N_0\cos(\omega\theta) - N_e\sin(\omega\theta)] = -\omega D_e \quad (6)$$

Solving equation (5) and (6) for k_p and k_i

$$k_p = \frac{(\omega^2 N_0 D_0 + N_e D_e)\cos(\omega\theta) + \omega(N_0 D_e - N_e D_0)\sin(\omega\theta)}{-(N_e^2 + \omega^2 N_0^2)} \quad (7)$$

And

$$k_i = \frac{\omega^2(N_0 D_e - N_e D_0)\cos(\omega\theta) - \omega(N_e D_e + \omega^2 N_0 D_0)\sin(\omega\theta)}{-(N_e^2 + \omega^2 N_0^2)} \quad (8)$$

The stability boundary locus $l(k_p,k_i,\omega)$ in the(k_p,k_i)-plane can be obtained using equations (7) and (8).
From equations (7) and (8) it is observed that the stability boundary locus depend on the frequency ω which varies from 0 to ∞. Since the controller operates at the frequency range of below the critical frequency $\omega_c$or ultimate frequency , the stability boundary locus can be obtained over the frequency range of ω varies between 0 to ω_c.
The performance of the controller can be evaluated by measuring the phase and gain margin. These two are the important frequency domain performance measure. Let the gain-phase margin tester$G_c(s) = Be^{-j\phi}$, which is connected in the feed forward path of the control system shown in Fig 1. Then the equation for k_p and k_i are

$$k_p = \frac{(\omega^2 N_0 D_0 + N_e D_e)\cos(h) + \omega(N_0 D_e - N_e D_0)\sin(h)}{-B(N_e^2 + \omega^2 N_0^2)} \quad (9)$$

$$k_i = \frac{\omega^2(N_0 D_e - N_e D_0)\cos(h) - \omega(N_e D_e + \omega^2 N_0 D_0)\sin(h)}{-B(N_e^2 + \omega^2 N_0^2)} \quad (10)$$

Where$h = \omega\theta + \phi$
One needs to set $\phi = 0$ in (9) and (10) to obtain stability boundary locus for a given value of gain margin B. On the other hand by setting B=1 in the equation for k_p and k_i one can obtain the stability boundary locus for the given phase margin ϕ

III. DECOUPLING

The problem with control-loop interaction is an important issue in MIMO process. The interaction occurs because one manipulated input affects more than one controlled output. One approach to handling this problem is decoupling.
A simplified decoupling, is shown for a multivariable process is shown in Fig 2. Here the decoupling matrix is restricted with $D_{11}(s) = D_{22}(s) = 1$ to the form

$$D(s) = \begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix} \quad (11)$$

Here we specify a decoupled response and the decoupler with the structure in Equation

$$G_P(s)D(s) = \begin{bmatrix} G_{11(s)} & 0 \\ 0 & G_{22(s)} \end{bmatrix} \quad (12)$$

$$\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}\begin{bmatrix} 1 & D_{12}(s) \\ D_{21}(s) & 1 \end{bmatrix} = \begin{bmatrix} G_{11}^*(s) & 0 \\ 0 & G_{22}^*(s) \end{bmatrix}$$

And we can solve four equations in four unknowns to find

$$D_{12}(s) = -\frac{G_{12}(s)}{G_{11}(s)}$$

$$D_{21}(s) = -\frac{G_{21}(s)}{G_{22}(s)}$$

$$G_{l1}(s) = G_{11}(s) - \frac{G_{12}(s)G_{21}(s)}{G_{22}(s)} \quad (13)$$

$$G_{l2}(s) = G_{22}(s) - \frac{G_{21}(s)G_{12}(s)}{G_{11}(s)} \quad (14)$$

The effect of synthetic inputs is being "feed -forward" to minimize control loop interaction. Similar to feed-forward control, some factorization of the decoupling elements must be performed to make certain they are stable and physically realizable.

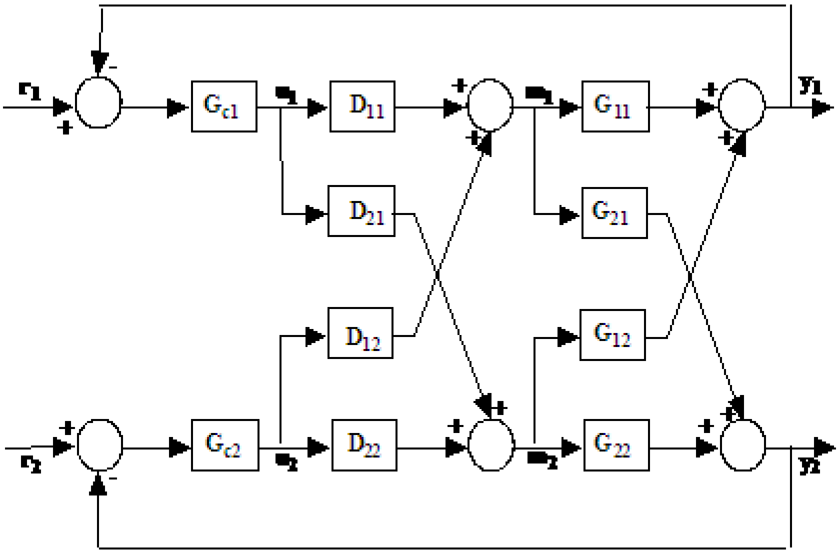

Fig. 2 Decentralized PI controller for MIMO process with decoupler

IV. COUPLED TANK MIMO PROCESS

The schematic diagram of coupled tank MIMO process is shown in Fig.3. The input flows of tank1 and tank2 are F_{in1} and F_{in2}. The controlled variables are level h_1 and h_2 in the tank1 and tank2.
The mass balance and Bernoulli's law yield

$$A\frac{dh_1}{dt} = k_1u_1 - \beta_1 a_1\sqrt{2gH_1} - \beta_x a_{12}\sqrt{2g(H_1 - H_{2)}} \qquad (15)$$

$$A\frac{dh_2}{dt} = k_2u_2 + \beta_x a_{12}\sqrt{2g(H_1 - H_{2)}} - \beta_2 a_2\sqrt{2gH_2} \qquad (16)$$

Where

A_1, A_2 - Cross sectional area of tank1 and tank2 (cm^2)
a_1 - Cross sectional area of output pipe in tank2 (cm^2)
a_2 - Cross sectional area of output pipe in tank2 (cm^2)
a_{12} - Cross sectional area of interaction pipe between tank1 and tank2 (cm^2)
h_1, h_2 - Water level of tank1 and tank2 (cm)
F_{in1} - Inflow of tank1 (cm^3/sec)
F_{in2} - Inflow of tank2 (cm^3/sec)
F_{out1} - Outflow of tank1 (cm^3/sec)
F_{out2} - Outflow of tank2 (cm^3/sec)
u_1 - Input voltage to pump1 (volts)
u_2 - Input voltage to pump2 (volts)
β_x - Valve ratio of jointed pipe between tank1 and tank2
β_1 - Valve ratio at the outlet of tank1
β_2 - Valve ratio at the outlet of tank2
k_1 - Gain of the pump1 (cm^3/v-sec)
k_2 - Gain of the pump2 (cm^3/v-sec)
g - Gravity (cm^2/sec)

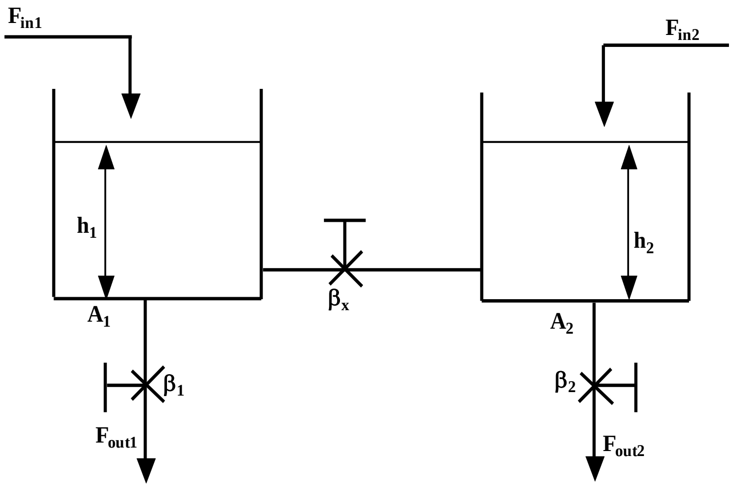

Fig.3 The coupled tank MIMO process

TableI **Parameters of Laboratory Coupled Tank MIMO Process**

A_1, A_2	a_1,a_2, a_{12}	$\square_1$	$\square_2$	$\square_x$
154	0.5	0.7498	0.8040	0.2245

TableII **Operating Conditions of Laboratory Coupled Tank MIMO Process**

u_1	u_2	h_1	h_2	k_1	k_2
2.5	2.0	18.32	12.23	33.336	25.002

The parameter values of the coupled tank process are given in Table I. The nominal operating conditions of the process are shown in Table II.

V. SIMULATION AND EXPERIMENT RESULT

The transfer function model for the coupled tank process is identified using reaction curve method. The levels in the tanks are initially maintained at the operating condition of 18.32 cm and 12.23 cm by giving the input voltage of 2.5 and 2.0 volts to the pump1 and pump2 respectively. Then the input to pump1 is changed from 2.5 to 3.0 voltage by keeping pump 2 input constant and the level in tank 1 and tank2 are recorded. The same procedure is repeated by changing the pump2 input from 2.0 to 2.5 volts by keeping the pump1 input constant. The open loop response for the change in input1 and input2 are shown in Fig.4 and 5.The experimentally identified transfer function model is

$$G_p(s) = \begin{bmatrix} \frac{16.99\, e^{-12.89s}}{(214.03s+1)} & \frac{6.69\, e^{-72.57s}}{(204.93s+1)} \\ \frac{9.23\, e^{-35.01s}}{(256.44s+1)} & \frac{11.38\, e^{-25.04s}}{(169.15s+1)} \end{bmatrix} \tag{17}$$

This transfer function model is used to design a controller. After obtaining the decoupler G_{l1} and G_{l2} are calculated using eq(13) and (14) as

$$G_{l1}(s) = \frac{11.56\, e^{-6.63s}}{(156.18s+1)} \tag{18}$$

And

$$G_{l2}(s) = \frac{7.745\, e^{-24.23s}}{(111.01s+1)} \tag{19}$$

For $G_{l1}(s)$, from equations (7) and(8)

$$k_{p1} = 13.51\omega \sin(6.63\omega) - 0.0865 \cos(6.63\omega) \tag{20}$$

$$k_{i1} = 13.51\omega^2 \cos(6.63\omega) + 0.0865\omega \sin(6.63\omega) \quad (21)$$

Similarly for $G_{l2}(s)$

$$k_{p2} = 14.33\omega \sin(24.23\omega) - 0.129 \cos(24.23\omega) \quad (22)$$
$$k_{i2} = 14.33\omega^2 \cos(24.23\omega) + 0.129\omega \sin(24.23\omega) \quad (23)$$

The stability boundary locus for $G_{l1}(s)$ and $G_{l2}(s)$ is plotted in kp – ki plane for $\omega\epsilon[0\ 1.2]$ and $\omega\epsilon[0\ 0.0702]$respectively, which is shown in Fig .6 and Fig .7.

To find all stabilizing PI controller for the phase margin of the system greater than 70° and gain margin is greater than 5 for both $G_{l1}(s)$ and $G_{l2}(s)$. First set B=1 and $\phi = 70$ in Eqs (9) and (10). Therefore

$$k_{p1} = 13.51\omega \sin(h) - 0.0865 \cos(h) \quad (24)$$
$$k_{i1} = 13.51\omega^2 \cos(h) + 0.0865\omega \sin(h) \quad (25)$$

$$k_{p2} = 14.33\omega \sin(h) - 0.129 \cos(h) \quad (26)$$

$$k_{i2} = 14.33\omega^2 \cos(h) + 0.129\omega \sin(h) \quad (27)$$

Where $h = \omega\theta + 70$. Second to find the stabilizing PI controllers for the gain margin greater than 5 set B=5 and $\phi = 0$ in Eqs (9) and (10). For$G_{l1}(s)$, using (9) and (10) for these values of B and ϕ gives

$$k_{p1} = 2.702\omega \sin(6.63\omega) - 0.0173 \cos(6.63\omega) \quad (28)$$

$$k_{i1} = 2.702\omega^2 \cos(6.63\omega) + 0.0173\omega \sin(6.63\omega) \quad (29)$$

Similarly for $G_{l2}(s)$

$$k_{p2} = 2.866\omega \sin(24.23\omega) - 0.0258 \cos(24.23\omega) \quad (30)$$

$$k_{i2} = 2.866\omega^2 \cos(24.23\omega) + 0.0258\omega \sin(24.23\omega) \quad (31)$$

Thus the stabilizing PI controller's boundary locus of $G_{l1}(s)$ and $G_{l2}(s)$ for $B \geq 5$ and$\phi \geq 70$ are shown in Fig.8 and 9 respectively

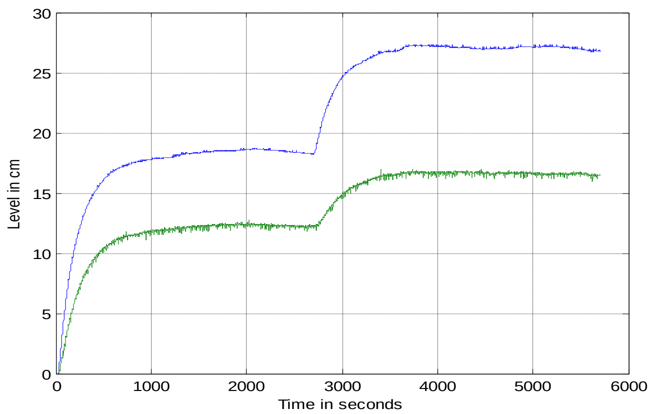

Fig. 4Open loop response for input change in Pump1

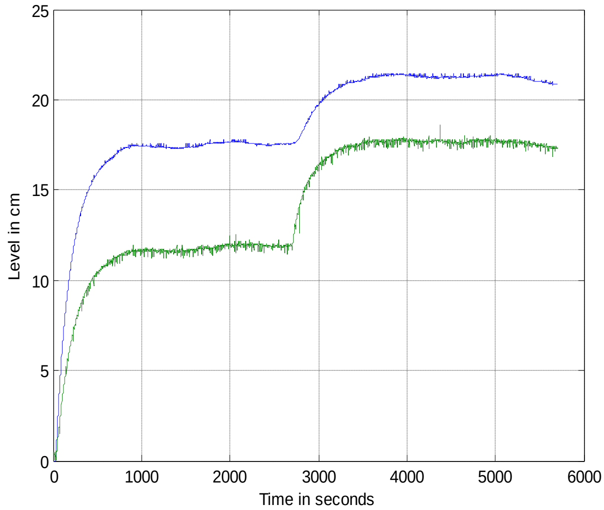

Fig 5. Open loop response for input change in pump2

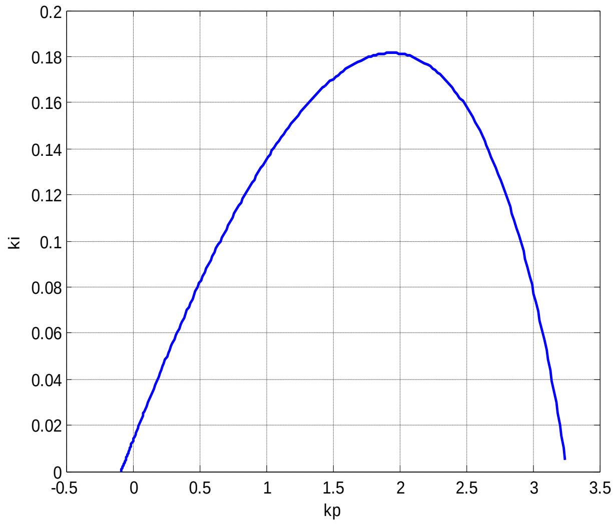

Fig. 6 Stability regions for $G_{l1}(s)$

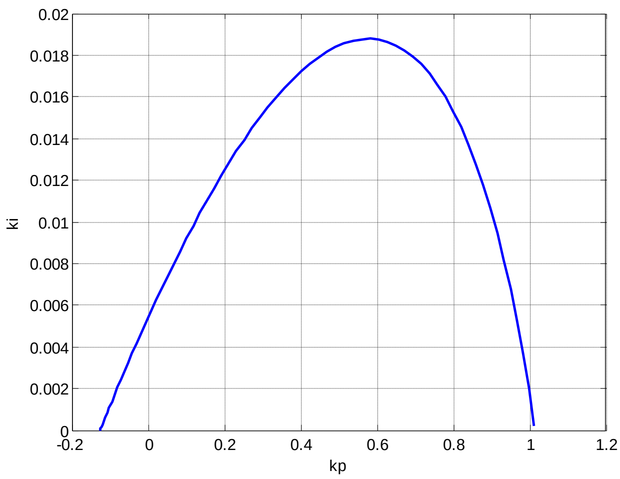

Fig. 7 Stability regions for $G_{l2}(s)$

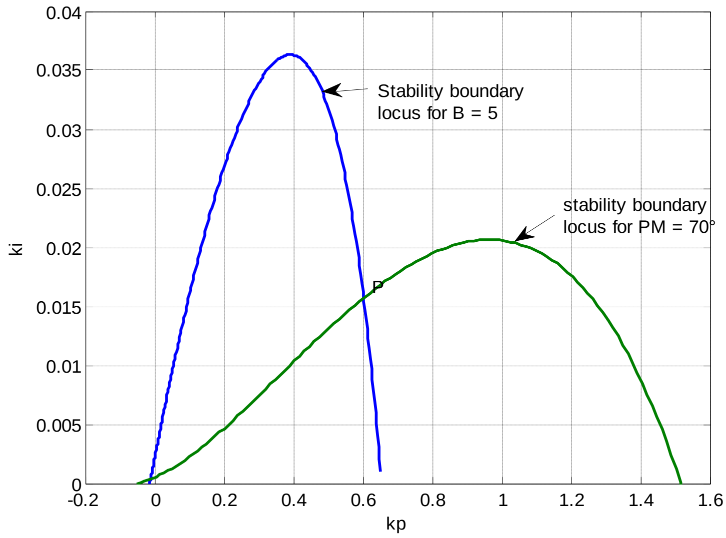

Fig. 8 Stabilizing PI controllers of $G_{l1}(s)$ for specified gain and phase margin

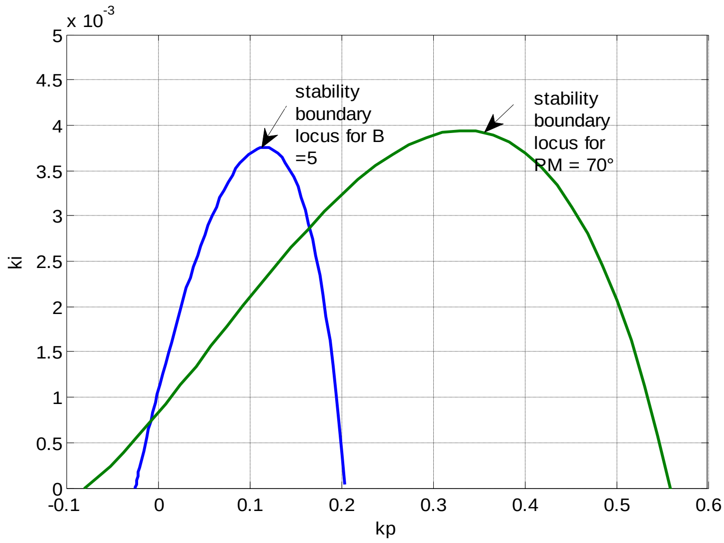

Fig.9 Stabilizing PI controllers of $G_{l2}(s)$ for specified gain and phase margins

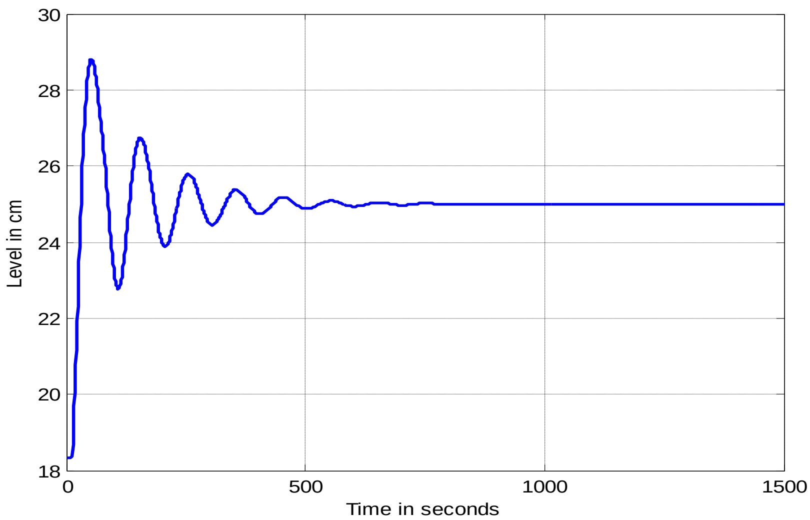

Fig.10 Closed Loop Response for set Point change in Tank1 from 18.32 to 25 cm.

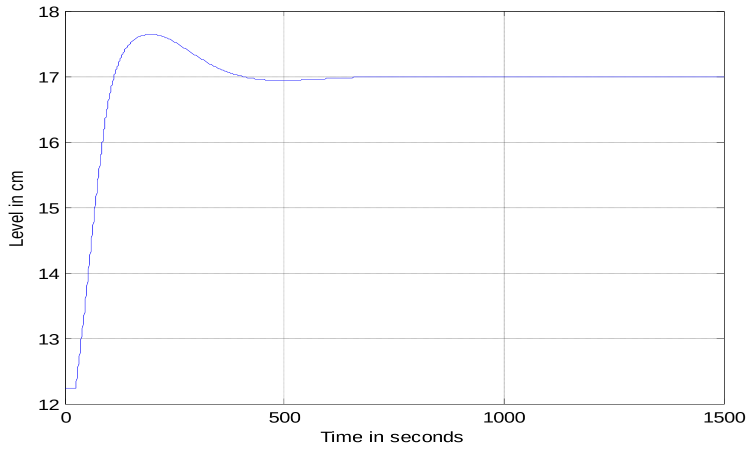

Fig. 11 Closed Loop Response for Tank2 for set Point change in Tank2 from 12.23 to 17 cm.

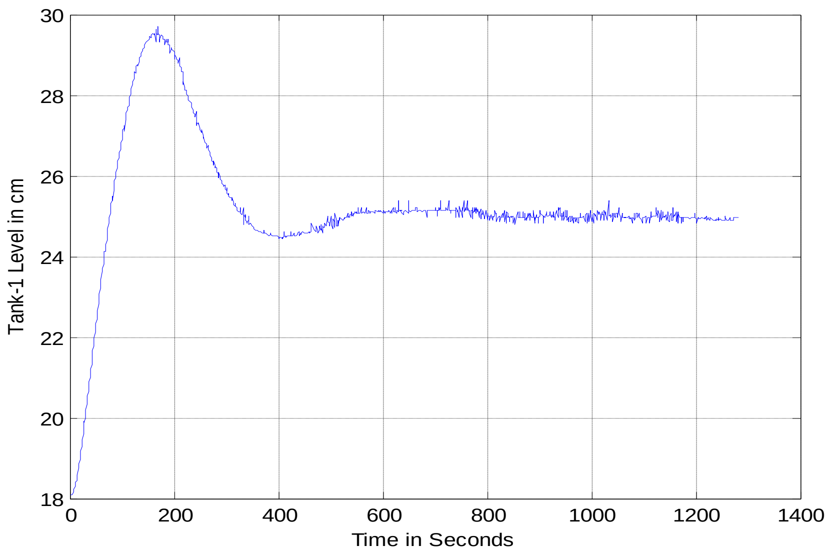

Fig. 12 Real Time Closed Loop Response for set Point change in Tank1 from 18.32 to 25 cm.

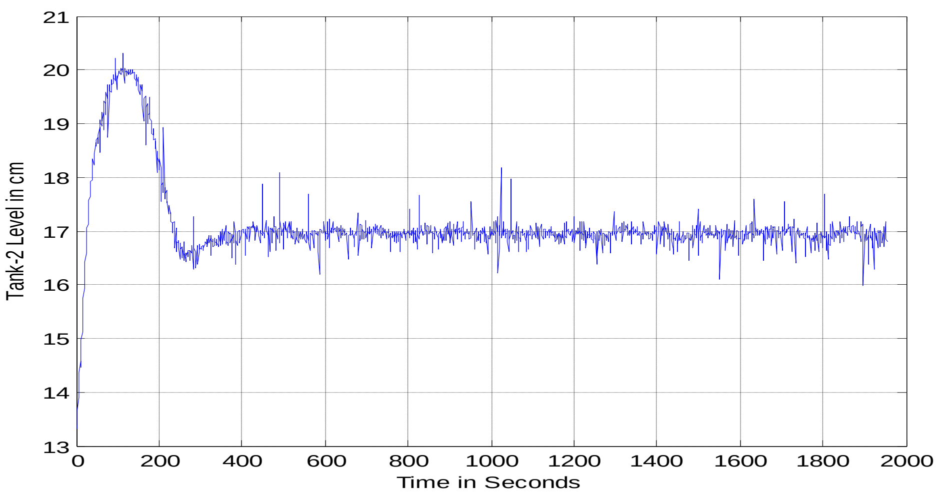

Fig.13 Real Time Closed Loop Response for Tank2 for set Point change in Tank2 from 12.23 to 17 cm.

The closed loop simulation response for setpoint change in tank1 and tank2 are shown in Fig. 11 and Fig.12with the controller parameters of k_{p1}=0.606, k_{i1}=0.0157 and k_{p2}=0.1647, k_{i2}=0.00287 .The real time responses are obtained using VDPID card for interface with the process with sampling time of 0.1 seconds and the closed loop responses are shown in Fig. 13 and 14.

VI. CONCLUSIONS

In this paper, a method is proposed to design a PI controller for a coupled-tank multivariable process by plotting stability boundary locus. The PI controller is designed by equating the real and imaginary part of the characteristic equation of both the loop after applying decoupling to zero. The value for the controller is obtained with ease by plotting the stability boundary locus in K_p-K_i plane, without doing the linear programming to solve the set of inequalities. And also the PI controller parameters for the coupled -tank is obtained for the user defined gain and phase margin. The simulation and experimentation results presented clearly shows the value of the method.

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