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Water wave scattering in presence of a surface discontinuity over a porous uneven bottom

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***Abstract**—The present study is concerned with water wave scattering by a porous bottom of varying depth in presence of a discontinuity at the upper surface of the ocean. Assuming linear theory and using a perturbation technique in conjunction with the Green's integral theorem, the first order corrections to reflection and transmission coefficients are obtained. By considering two different shape functions describing the bottom undulation, the effect of porosity is investigated numerically and these results are depicted in a number of figures against the wave number of the incident wave.*

***Keywords—** Wave scattering, Surface discontinuity, Porous bottom, Uneven bottom, Shape function, Reflection coefficient, Transmission coefficient.*

I. INTRODUCTION

The problem of water wave scattering due to uneven bottom is an interesting topic of research over a few decades. A class of water wave scattering problems in presence of a small obstacle situated at the bottom was considered earlier by many researchers such as Lamb [1], Kreisel [2], Davies [3-4]. Mei [5] considered the problem of reflection of water waves by periodic sand bars and explained a theory on strong reflection i.e Bragg reflection induced by the sandbars themselves. Mandal and Basu [6] extended this problem to include the effect of surface tension at the free surface for an obliquely incident surface wave train and by employing a perturbational analysis directly to the governing problem to obtain reflection and transmission coefficients up to first-order in terms of integrals involving the shape function describing the bottom topography. Evans and Linton [7] considered the problem of water wave scattering by uneven bottom using step approximation and introduced scattering matrix. Ultimately they reduce this problem to a problem of scattering by a discontinuity at the upper surface in a uniform finite depth water and employed residue calculus technique of complex variable theory to obtain the reflection and transmission coefficients of the incident wave. Martha and Bora [8] also investigated a scattering problem in presence of a small bottom undulation and by using perturbation technique they obtained the reflection and transmission coefficients upto the first order. Recently Mandal and De [9] investigated a wave scattering problem by a free surface discontinuity together with a small bottom undulation. Using Green's integral theorem they obtained reflection and transmission coefficients up to first order in terms of computable integral.

Problem of water wave interaction with a rapidly varying porous bottom is another important aspect in this context. The flow of fluid into the porous media leads to different phenomenon like wave energy dissipation, damping etc. Water wave interaction with the porous media was studied earlier by many scientists like Chwang [10], Chakrabarti [11], Jeng [12] and many others. Zhu [13] studied the wave propagation problem within porous media on an undulating bed by employing a Galerkin eigenfunction expansion technique and investigated the wave reflection coefficient numerically. Gu and Wang [14] also investigated the problem of water wave interaction with rigid porous seabed both theoretically and experimentally. Mase and Takeba [15] focused on the Bragg scattering of gravity wave over a porous sea bed. Also the work of Silva [16] discussed on water-wave reflection and transmission problem where a porous medium was assumed to lie on a rapidly varying sea-bed. Recently, Martha and Bora [17] considered the problem of oblique water wave scatting by a small patch of sinusoidal bottom undulation over a rigid porous bottom and obtained the reflection and transmission coefficients by employing fourier transformation technique.

In the present article, we consider a water wave scattering problem by a rigid porous bottom of variable depth in presence discontinuity at the upper surface of the ocean. The problem is mathematically formulated in case of normally incident wave. The discontinuity at the upper surface may be presented in different form such as two different vast floating inertial surfaces of non-interacting materials or large broken ice sheets in the Antarctic regions etc. Here the upper surface of the ocean is assumed to be covered by two vast floating inertial surfaces of different thickness which produces a discontinuity at the upper surface of water. The effect of porous bottom is taken into account by assuming that the ocean bottom is composed of some specific type of rigid porous material with small undulation which characterized by a known shape function. The bottom boundary condition in this case usually involves a known porosity parameter G representing the typical of porous bottom. The wave train is incident from negative infinity direction which is partially reflected and partially transmitted along the surface of the ocean. The presence of small undulation at the bottom suggests that a perturbation technique can directly be applied to the governing boundary value problem and the associated boundary conditions. By applying perturbation analysis in terms of a small parameter ε characterizing the bottom undulation upto order one, two different boundary value problems(BVP) are generated. They are renamed as BVP-1 and BVP-2 respectively. The BVP for the zero order potential function represents a scattering problem in presence of a discontinuity at the upper surface over a uniform porous bottom. This problem is solved here by using residue calculus technique followed by Evans and Linton [7] and the zero order corrections to the reflection and transmission coefficients are obtained for the purpose. Now without solving the BVP-2 for the first order potential function, the first order corrections to the reflection and transmission coefficient are obtained by using Green's integral theorem. The analytical expressions for these coefficients are presented in terms of computable integrals involving the bottom shape function, zero order potential function and an additional term containing G. We choose two different bottom profile functions, one is exponential and other sinusoidal to describe the undulations at the ocean bottom. These type of bottom profiles can be observed in different regions of an ocean due to growth of sand ripples, sedimentation of different porous materials at the bottom and other naturally occurring phenomenon. For these type of bottom profiles, the first order corrections for reflection and transmission coefficients are depicted graphically against the wave number of the incident wave in a number of figures. The effect of porosity on the first order reflection and transmission coefficients is investigated in presence of surface discontinuity by taking different values of the dimensionless porosity parameter in different cases.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

A right handed regular cartesian co-ordinate system (x, y) is chosen in which $y = 0$ represents the undisturbed upper surface of water and the y-axis is taken vertically downward into the fluid region. The upper surface is covered by two floating inertial surfaces of uniform area densities $\varepsilon_1\rho$ and $\varepsilon_2\rho$ (ρ being the density of water). One of the inertial surface occupies the region $y = 0, x < 0$ and the other occupies the region $y = 0, x > 0$. The wave field is incident from $x \to -\infty$ and the direction of the positive x-axis being opposite to the direction of the incoming incident wave field. Assuming linear theory and irrotational motion, the velocity potential describing the time-harmonic motion of angular frequency σ, can be represented by $Re[\phi(x, y)e^{-i\sigma t}]$, where ϕ satisfies the following two dimensional Laplace equation:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0, \qquad \text{on } 0 < y < h + \varepsilon c(x), -\infty < x < \infty. \tag{1}$$

The upper surface boundary conditions are prescribed by

$$K_1\phi + \frac{\partial\phi}{\partial y} = 0 \qquad on\ y = 0, x < 0, \tag{2}$$

$$K_2\phi + \frac{\partial\phi}{\partial y} = 0 \qquad on\ y = 0, x > 0. \tag{3}$$

This produces a discontinuity in the upper surface boundary conditions at the point (0,0), where $K_1 = \frac{K}{1-\varepsilon_1 K}, K_2 = \frac{K}{1-\varepsilon_2 K}, \varepsilon_1, \varepsilon_2 < \frac{g}{\sigma^2}$ and $K = \frac{\sigma^2}{g}$ (g is the acceleration due to gravity).

The bottom boundary condition in case of uneven porous bottom (Martha, Bora, Chakrabarti [17])is given by

$$\frac{\partial\phi}{\partial\eta} - G\phi = 0\ ony = h + \varepsilon c(x). \tag{4}$$

Here $y = h + \varepsilon c(x)$ describes the bottom of the ocean with variable depth. The positive number ε signifies smallness of the undulation at the bottom and η is the outward normal to the ocean bed. The bottom undulation is characterized by a known shape function $c(x)$ with the property that $c(x) \to 0$ as $x \to \pm\infty$ ensuring that far away from the undulation the ocean is of uniform finite depth of h below the mean free surface.
The far field behavior of $\phi(x, y)$ is described by

$$\phi(x, y) \to \begin{cases} Te^{is_0x}\psi_0^2(y) & asx \to \infty, \\ (e^{ik_0x} + Re^{-ik_0x})\psi_0^1(y) & asx \to -\infty, \end{cases} \tag{5}$$

where

$$\psi_0^1(y) = N_0^1(\cosh k_0(h - y) - \frac{G}{k_0}\sinh k_0(h - y)),$$
$$\psi_0^2(y) = N_0^2(\cosh s_0(h - y) - \frac{G}{s_0}\sinh s_0(h - y))$$

and

$$N_0^1 = \frac{2k_0^{\frac{3}{2}}}{\sqrt{2k_0(G - G^2h + k_0^2h) - 2Gk_0\cosh 2k_0h + (k_0^2 + G^2)\sinh 2k_0h}},$$

$$N_0^2 = \frac{2s_0^{\frac{3}{2}}}{\sqrt{2s_0(G - G^2h + s_0^2h) - 2Gs_0\cosh 2s_0h + (s_0^2 + G^2)\sinh 2s_0h}}.$$

Here, $e^{ik_0x}\psi_0^1(y)$ represents the incident wavefield, R and T are respectively the unknown reflection and transmission coefficients(complex) to be determined. k_0 and s_0 are the unique breal positive roots (Cf. McIver [18]) of the following two transcendental equations in terms of λ:

$$(\lambda + \frac{K_1G}{\lambda})\tanh\lambda h = K_1 + G,$$
$$(\lambda + \frac{K_2G}{\lambda})\tanh\lambda h = K_2 + G.$$

III. METHOD OF SOLUTION

As ε is small, the approximate form of the bottom boundary condition (4) after neglecting $O(\varepsilon^2)$ terms is given by

$$\frac{\partial\phi}{\partial y} - G\phi - \varepsilon\left(\frac{d}{dx}\left\{c(x)\frac{\partial\phi(x,h)}{\partial x}\right\} + Gc(x)\frac{\partial\phi}{\partial y}\right) + O(\varepsilon^2) = 0 \quad on\ y = h. \tag{6}$$

The form of the approximate boundary condition (6) and smallness of the parameter ε suggests that ϕ, R and T has the following perturbational expansion in terms of ε:

$$\phi(x,y;\varepsilon) = \phi_0(x,y) + \varepsilon\phi_1(x,y) + O(\varepsilon^2), \tag{7}$$

$$R(\varepsilon) = R_0 + \varepsilon R_1 + O(\varepsilon^2), \tag{8}$$

$$T(\varepsilon) = T_0 + \varepsilon T_1 + O(\varepsilon^2). \tag{9}$$

Substituting (7)-(9) in (1)-(5), we find that ϕ_0 and ϕ_1 satisfies two different boundary value problems.

BVP-1

The function $\phi_0(x,y)$ satisfies the following boundary value problem:

$$\frac{\partial^2\phi_0}{\partial x^2} + \frac{\partial^2\phi_0}{\partial y^2} = 0\ in\ 0 < y < h, -\infty < x < \infty,$$

$$K_1\phi_0 + \frac{\partial\phi_0}{\partial y} = 0\ on y = 0, x < 0,$$

$$K_2\phi_0 + \frac{\partial\phi_0}{\partial y} = 0\ on y = 0, x > 0,$$

$$\frac{\partial\phi_0}{\partial y} - G\phi_0 = 0\ on y = h,$$

$$\phi_0(x,y) \to \begin{cases} T_0 e^{is_0x}\psi_0^2(y) & as\ x \to \infty, \\ e^{ik_0x}\psi_0^1(y) + R_0 e^{-ik_0x}\psi_0^1(y) & as\ x \to -\infty. \end{cases}$$

Here R_0 and T_0 stands for the reflection and transmission coefficient of zero order respectively. The above BVP of $\phi_0(x,y)$ describes a scattering problem by a discontinuity at the free surface boundary condition over a uniform porous bottom. Following Evans and Linton [7], using residue calculus method of complex variable theory the zero order reflection and transmission coefficients are respectively given by

$$R_0 = \frac{k_0 - s_0}{k_0 + s_0} e^{2i\alpha}, \tag{10}$$

$$T_0 = \frac{2k_0}{k_0 + s_0} e^{i(\alpha+\beta)}, \tag{11}$$

Where,

$$\alpha = \sum_{n=1}^{\infty} [\tan^{-1}(\frac{k_0}{s_n}) - \tan^{-1}(\frac{k_0}{k_n})],$$

$$\beta = \sum_{n=1}^{\infty} [\tan^{-1}(\frac{s_0}{k_n}) - \tan^{-1}(\frac{s_0}{s_n})].$$

Here, k_n and $s_n (n = 1,2,3, \ldots)$ are given by positive real roots of the following two transcendental equations in terms of λ:

$$(\lambda - \frac{K_1 G}{\lambda})\tan\lambda h + K_1 + G = 0,$$

$$(\lambda - \frac{K_2 G}{\lambda})\tan\lambda h + K_2 + G = 0.$$

BVP-2

The first order potential function $\phi_1(x,y)$ satisfies the following BVP:

$$\frac{\partial^2\phi_1}{\partial x^2} + \frac{\partial^2\phi_1}{\partial y^2} = 0\ \ in\ \ 0 < y < h, -\infty < x < \infty,$$

$$K_1\phi_1 + \frac{\partial\phi_1}{\partial y} = 0\ on y = 0, x < 0,$$

$$K_2\phi_1 + \frac{\partial\phi_1}{\partial y} = 0\ on y = 0, x > 0,$$

$$\frac{\partial\phi_1}{\partial y} - G\phi_1 = \frac{d}{dx}\{c(x)\frac{\partial\phi_0}{\partial x}\} + Gc(x)\frac{\partial\phi_0}{\partial y} ony = h,$$

$$\phi_1(x,y) \to \begin{cases} T_1 e^{is_0x}\psi_0^2(y) & asx \to \infty, \\ R_1 e^{-ik_0x}\psi_0^1(y) & asx \to -\infty. \end{cases}$$

The above BVP for ϕ_1 represents a radiation problem in presence of an upper surface discontinuity over an uneven porous bottom.R_1 and T_1 are the first order reflection and transmission coefficient respectively. It may be noted that bottom boundary condition for ϕ_1 involves the porosity parameter G and the known bottom shape function $c(x)$. This ensures that the bottom undulation over porous bottom has an effect on the reflection and transmission coefficients at first order.

The eigenfunction expansion of the zero order potential function $\phi_0(x,y)$ is given by

$$\phi_0(x,y) \to \begin{cases} T_0 e^{is_0x}\psi_0^1(y) + \sum_{n=1}^{\infty} B_n e^{-s_nx}\psi_n^2(y) & as\ x > 0, \\ (e^{ik_0x} + R_0 e^{-ik_0x})\psi_0^1(y) + \sum_{n=1}^{\infty} A_n e^{k_nx}\psi_n^1(y) & as\ x < 0, \end{cases} \quad (12)$$

where A_n and $B_n(n = 1,2,\ldots..)$ are the unknown constants. $\psi_n^1(y)$, $\psi_n^2(y)(n = 1,2,\ldots..)$ are the orthogonal depth eigenfunctions given by

$$\psi_n^1(y) = N_n^1(\cos k_n(h-y) - \frac{G}{k_n}\sin k_n(h-y)),$$
$$\psi_n^2(y) = N_n^2(\cos s_n(h-y) - \frac{G}{s_n}\sin s_n(h-y))(n = 1,2,\ldots..)$$

where,

$$N_n^1 = \frac{2k_n^{\frac{3}{2}}}{\sqrt{2k_n(-G+G^2h+k_n^2h) - 2Gk_n\cos 2k_nh + (k_n^2 - G^2)\sin 2k_nh}},$$
$$N_n^2 = \frac{2s_n^{\frac{3}{2}}}{\sqrt{2s_n(-G+G^2h+s_n^2h) - 2Gs_n\cos 2s_nh + (s_n^2 - G^2)\sin 2s_nh}}(n = 1,2,3,\ldots\ldots).$$

Now using the matching condition of $\phi_0(x,y)$ at $x = 0$ and orthogonality of the depth eigenfunctions produces the following system of linear equations

$$\sum_{n=1}^{\infty} \frac{B_n N_n^2 \cos s_n h}{s_n - k_m} = \frac{T_0 N_n^2 \cosh s_0 h}{is_0 + k_m},$$
$$\sum_{n=1}^{\infty} \left(\frac{A_n N_n^1 \cos k_n h}{k_n - s_m}\right) = N_0^1 \cosh k_0 h \left[\frac{R_0}{ik_0 + s_m} - \frac{1}{ik_0 - s_m}\right], (m = 1,2,\ldots.). \quad (13)$$

The unknown constants A_n, $B_n(n = 1,2,3,\ldots\ldots.)$ are to be eliminated numerically from (13) after truncating the infinite sums upto the desired accuracy.

IV. REFLECTION AND TRANSMISSION COEFFICIENT OF FIRST ORDER

The first order corrections for the reflection and transmission coefficients can be obtained by using Green's integral theorem for the two potential function $\phi_0(x,y)$ and $\phi_1(x,y)$ as

$$\oint_L (\phi_0\phi_{1\nu} - \phi_1\phi_{0\nu})dl = 0, \quad (14)$$

where ν is the outward normal to the line element dl and the contour L is specified by the lines

$y = 0\ and\ -X \le x \le X,$

$x = \pm X\ and\ 0 \le y \le h,$

$y = h\ and\ -X \le x \le X(X > 0).$

The outgoing nature of the potential function $\phi_0(x,y)$ as $x \to \infty$ and the boundary conditions for $\phi_0(x,y)$ and $\phi_1(x,y)$ ensures that only contribution to (14) comes from the bottom part $y = hand\ -X \le x \le X$ and $x = -Xand\ 0 \le y \le h$.

Therefore, we obtain,

$$2ik_0R_1 = \int_{-\infty}^{\infty}(c(x)(\phi_{0x}^2(x,h) - G^2\phi_0^2(x,h))dx. \quad (15)$$

The first order transmission coefficient can be obtained by considering $\phi_0(-x,y)$ and $\phi_1(x,y)$ in (14) and arguing in the same manner as above, we obtain,

$$2is_0T_1 = -\int_{-\infty}^{\infty}(c(x)(\phi_{0x}(x,h)\phi_{0x}(-x,h) + G^2\phi_0(x,h)\phi_0(-x,h))dx. \quad (16)$$

The integral representations of R_1 and T_1 given by (15) and (16) contains an additional term of porosity parameter G. However, in case of non-porous bottom(G=0), these forms are similar to the results obtained by Mandal and De [9] for the same type of problem.

V. NUMERICAL RESULTS

In this section, two different bottom shape functions are considered and numerical results for the first order reflection and transmission coefficients are explained briefly.

Case-1:
The first bottom profile function is taken as

$$c(x) = c_0e^{-\mu|x|} - \infty < x < \infty.$$

The above $c(x)$ represents exponentially decaying topography over porous bottom. The expressions of R_1 and T_1 for this $c(x)$ are given in the Appendix. The first order reflection and transmission coefficients are computed numerically for different values of Kh by taking $c_0/h = 0.1, \mu h = 1.0, \varepsilon_1/h = 0.01, \varepsilon_2/h = 0.02$ and the values of the dimensionless porosity parameter is taken as $Gh = 0.00, 0.05, 0.1$ respectively. These numerical results for R_1 and T_1 are depicted against Kh in figure-1 and figure-2 respectively against the wave number of the incoming wave. In the figure-1 the amplitude of R_1 is plotted against Kh for different values of the porosity parameter Gh. It is interesting to note that for each value of Gh, the graph of $|R_1|$ first increases with Kh up to a maximum value and then gradually decreases i.e a Bragg resonance is observed here in all cases.

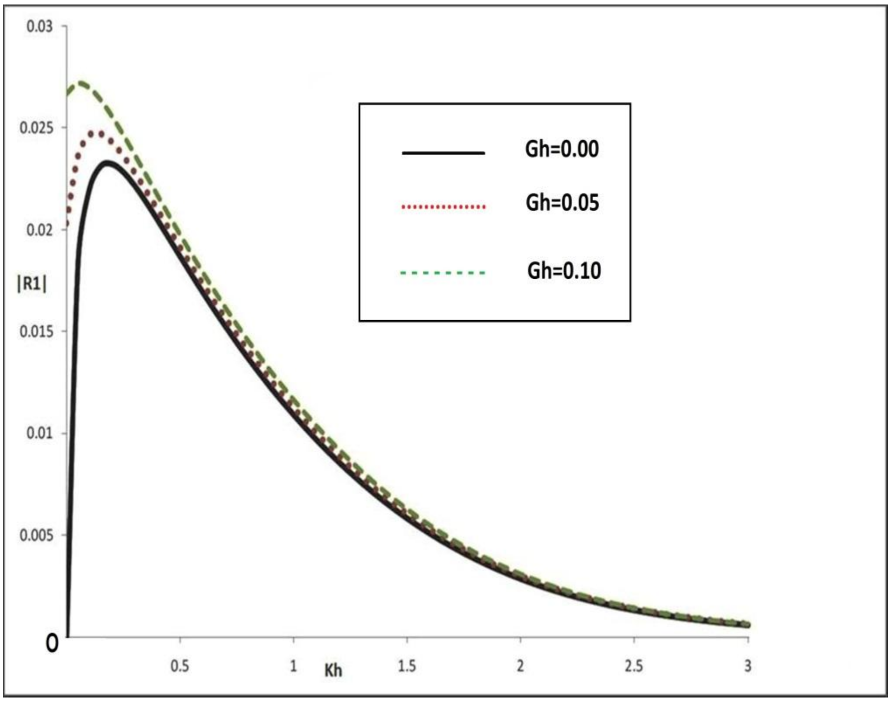

Fig-1

This may be attributed due to exponentially decaying bottom undulation and the nature of the incident wave field. These values of $|R_1|$ are observed to be rapidly decreasing with Kh which is very prominent for $Kh > 1.256$ because far away from the bottom undulation the ocean is of uniform finite depth h below the mean free surface. Moreover, for each Gh, occurrence of zeros of $|R_1|$for certain values of Kh implies that the sinusoidal bottom does not affect the incident waves at first-order for certain frequencies.

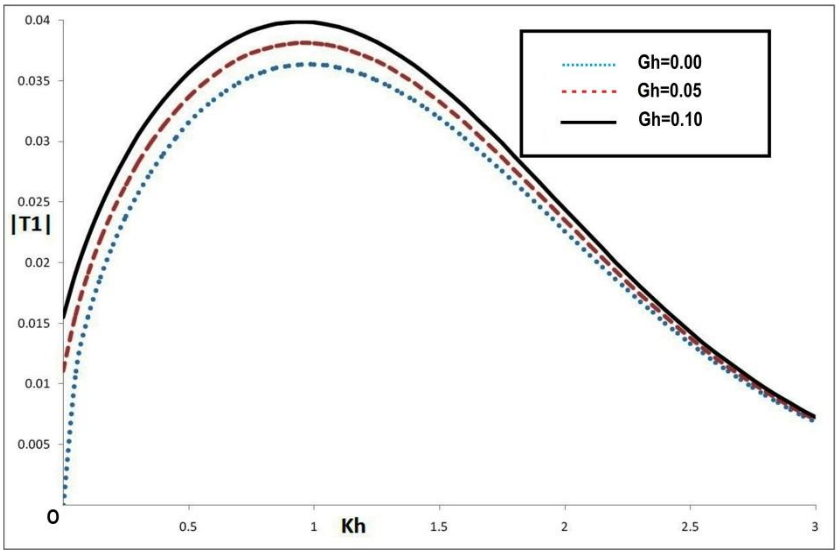

Fig. 2

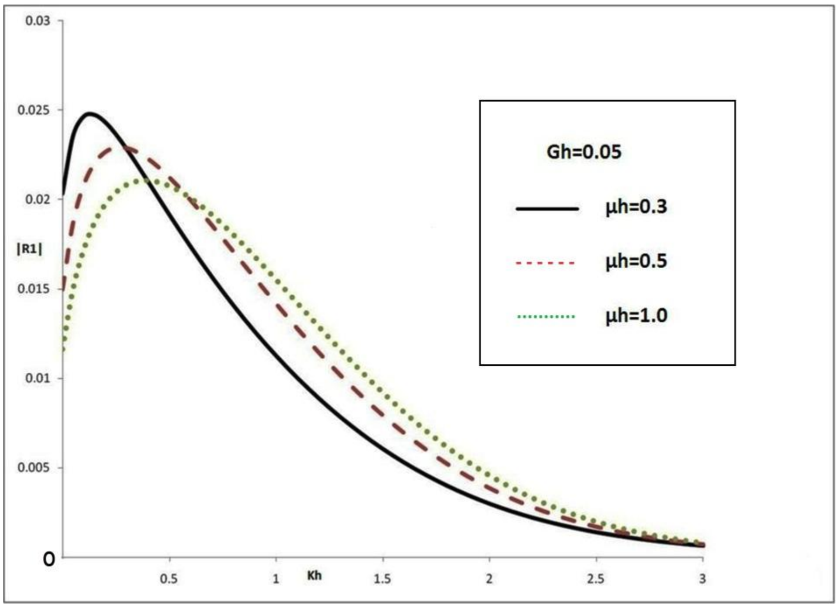

Fig. 3

Another interesting feature of figure-1 is that as numerical value of the porosity parameter Gh increases, the highest amplitude of R_1 also increases with Gh. This can be explained as R_1 in (15) contains $c(x)$ and an additional term containing G^2. Moreover the ocean bed is assumed to be composed of specific type of porous material with porosity parameter G along with the free surface discontinuity has an effect on the reflection and transmission coefficient at first order.

In the figure-2 the amplitude of T_1 is plotted against Kh for different values of Gh. For each Gh, $|T_1|$ picks up a highest value for a certain Kh and gradually decreases with Kh which is prominent for $Kh > 1.5898$ in all cases. The highest amplitude of T_1 is found to be increasing with the increasing value of Gh. Also from the figure-1 and figure-2, it is observed that figures of $|R_1|$ and $|T_1|$ agrees with the results obtained by Mandal and De [9] in case of non-porous bottom($Gh = 0$).

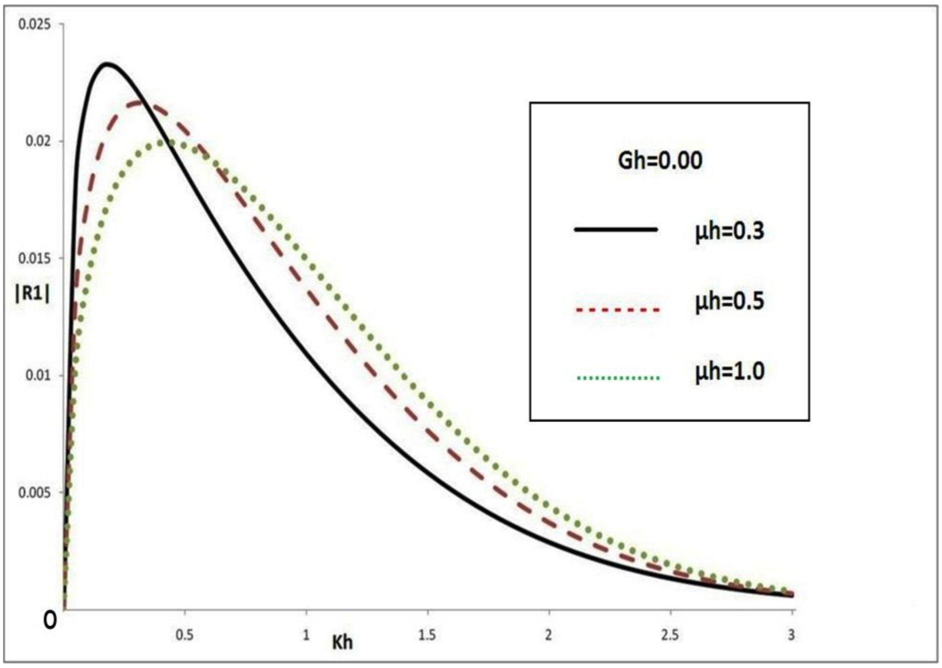

Fig-4

In figure-3, the amplitude of R_1 is shown against Kh for different values of μh when $c_0/h = 0.1, \varepsilon_1/h = 0.01, \varepsilon_2/h = 0.02$ and value of porosity parameter is taken as $Gh = 0.05$. It is seen that for each value of μh, the nature of the graph of $|R_1|$ is almost similar to case of figure-1. In this case, the maximum amplitude of R_1 is found to be decreasing with the increasing value of μh. This fact can be explained according to the form of R_1 given in the Appendix which shows that R_1 is inversely related to μh. In figure-4 the same figure is plotted in case of non-porous bottom ($Gh = 0$) and this matches with the figure obtained by Mandal and De [9] for the exponential bottom profile.

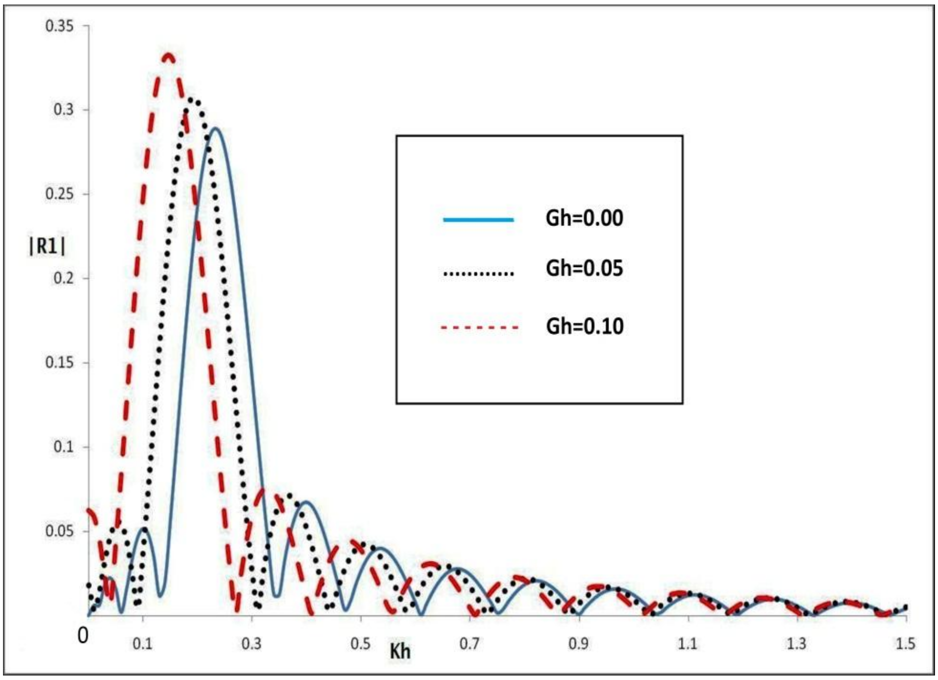

Fig-5

Case-2: The sinusoidally varying bottom profile is taken as

$$c(x) = c_0 \sin\lambda x \qquad \frac{-m\pi}{\lambda} \le x \le \frac{m\pi}{\lambda}$$

$$= 0 \ otherwise\,,$$

where ripple amplitude c_0 and ripple wave number λ. The expression of R_1 is given in the Appendix for the above $c(x)$. By (17) we get $T_1 = 0$. In figure-5, $|R_1|$ is plotted against Kh by taking $Gh = 0.00, 0.05, 0.1$ when $\varepsilon_1/h = 0.01, \varepsilon_2/h = 0.02$ and $\lambda h = 1$. This figure shows the changes of $|R_1|$ for different value of the porosity Gh. The graph of $|R_1|$ is found to be oscillatory in nature for each Gh and the oscillation gradually decreases with Kh which is prominent for $Kh > 1.1$. The oscillation of $|R_1|$ picks a maximum value of for certain Kh hence a Bragg resonance is observed here in all cases. It is also found that as the value of Gh is increasing, the highest amplitude of R_1 is also found to be increasing and has shifted to the left. This may be attributed due to a sinusoidally varying porous bottom and the presence of surface discontinuity at the upper free surface. In figure-5, for case of non-porous bottom($Gh = 0$), the graph of R_1 is almost similar to the results obtained by Mandal and De [9].

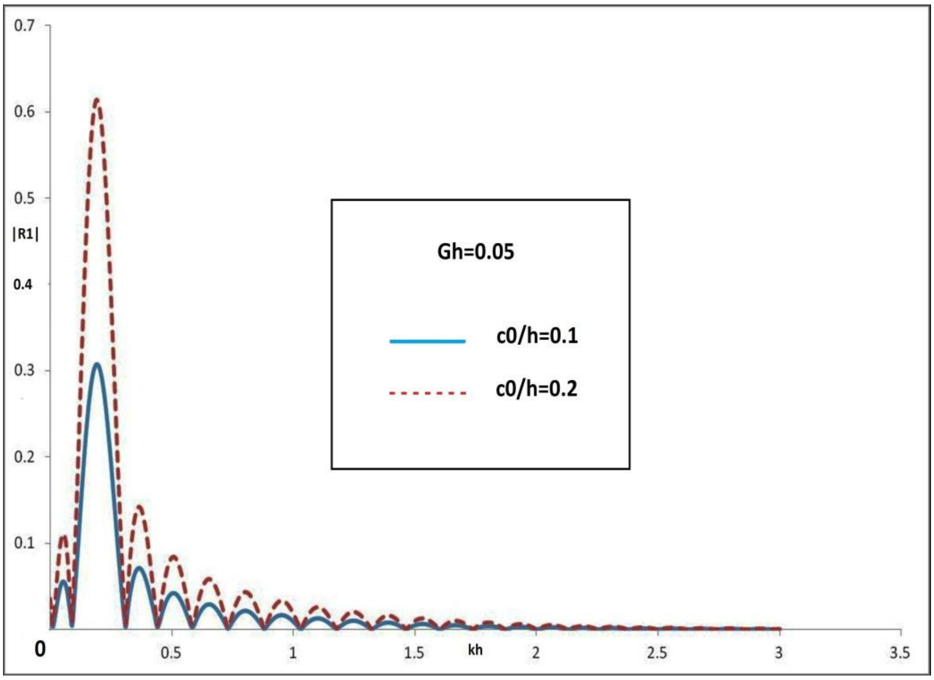

Fig-6

The figure-6 is plotted for $|R_1|$ against Kh by taking two different values of the ripple amplitude $c_0/h = 0.1, 0.2$ over porous bottom(Gh=0.05) while $\varepsilon_1/h = 0.01, \varepsilon_2/h = 0.02$ and $\lambda h = 1$. It is observed that overall values of $|R_1|$ are increasing with the value of c_0/h. The same figure for $|R_1|$ in case of non-porous bottom($Gh = 0$) is plotted in figure-7, agrees with the result obtained by Mandal and De [9].

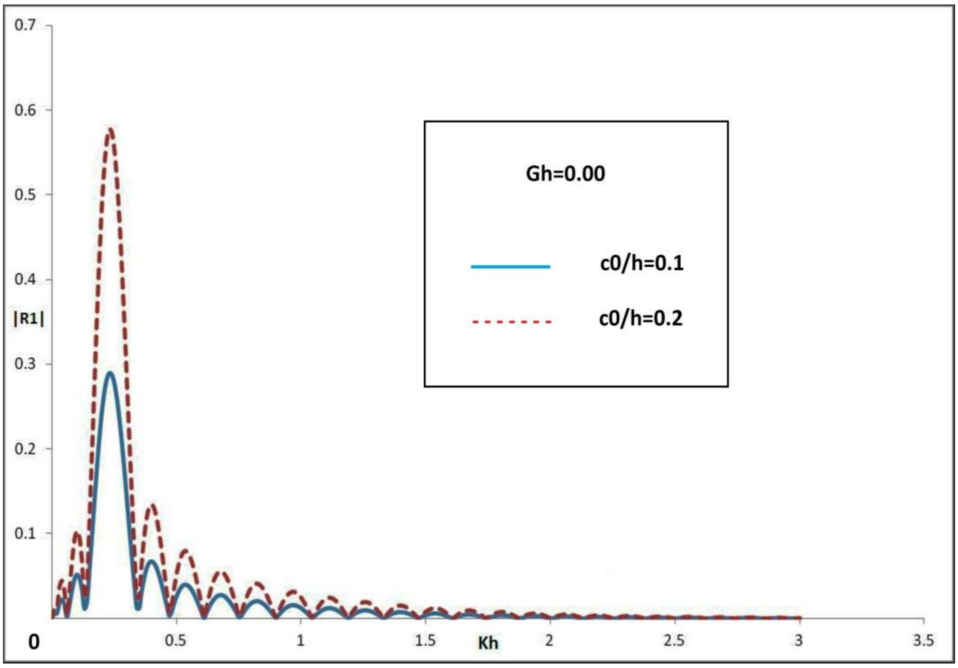

Fig-7

VI. CONCLUSION

A simplified perturbation technique and Green's integral theorem are employed to obtain the first order reflection and transmission coefficient. From the numerical results and figures, it is observed that for a sinusoidal patch and exponentially decaying porous bottom with an upper surface discontinuity has a significant impact on the reflection and transmission coefficients at first order in different cases. The analytical and numerical results agrees with the results obtained by Mandal and De [9] in the case of non-porous bottom($G = 0$).

Problems of this type relating to porous bo ttom is found to be important in different branch of coastal science and engineering over a few decades. Understanding the effect of porous bottom on the wave characteristics in presence of a surface discontinuity over uneven bottom has gained considerable amount of interest to the scientists due to its various potential applications in different research areas of marine science and oceanography.

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APPENDIX

Case-1 (for sinusoidally varying bottom topography $c(x) = c_0\sin\lambda x$):

$$R_1 = \frac{c_0}{2ik_0}[-(N_0^1)^2\{((k_0^2+G^2)\frac{\lambda}{\lambda^2-4k_0^2}\left((-1)^m e^{\frac{-2ik_0m\pi}{\lambda}}-1)\right)+\frac{(k_0^2+G^2)\lambda R_0^2}{\lambda^2-4k_0^2}\left((-1)^m e^{\frac{2ik_0m\pi}{\lambda}}-1\right)+2R_0(k_0^2 + G^2)\frac{((-1)^m-1)}{\lambda}\}+\int_{-m\pi/\lambda}^{0}\left((\sum_{n=1}^{\infty}k_nA_n e^{k_nx}N_n^1)^2-G^2(\sum_{n=1}^{\infty}k_nA_n e^{k_nx}N_n^1)^2\right)\sin\lambda x dx$$

$$+2ik_0N_0^1\sum_{n=1}^{\infty}\frac{k_nA_nN_n^1\lambda}{\lambda^2+(k_n+ik_0)^2}\left((-1)^m e^{\frac{m\pi(k_n+ik_0)}{\lambda}}-1\right)-2ik_0N_0^1R_0\sum_{n=1}^{\infty}\frac{k_nA_nN_n^1\lambda}{\lambda^2+(k_n-ik_0)^2}\left((-1)^m e^{\frac{m\pi(k_n-ik_0)}{\lambda}}-1\right)$$

$$-2G^2N_0^1\sum_{n=1}^{\infty}\frac{A_nN_n^1\lambda}{\lambda^2+(k_n+ik_0)^2}\left((-1)^m e^{\frac{m\pi(k_n+ik_0)}{\lambda}}-1\right)-2G^2N_0^1R_0\sum_{n=1}^{\infty}\frac{A_nN_n^1\lambda}{\lambda^2+(k_n-ik_0)^2}\left((-1)^m e^{\frac{m\pi(k_n-ik_0)}{\lambda}}-1\right)$$

$$+\int_0^{\frac{m\pi}{\lambda}}\left((\sum_{n=1}^{\infty}s_nB_n e^{-s_nx}N_n^2)^2-(\sum_{n=1}^{\infty}B_n e^{-s_nx}N_n^2)^2\right)\sin\lambda x dx-((s_0^2+G^2)T_0^2\frac{\lambda}{\lambda^2-4s_0^2}\left((-1)^m e^{\frac{-2is_0m\pi}{\lambda}}-1)\right)$$

$$-2is_0N_0^2T_0\sum_{n=1}^{\infty}\frac{s_nB_nN_n^2\lambda}{\lambda^2+(s_n-is_0)^2}\left((-1)^m\mathrm{e}^{\frac{m\pi\,(s_n-is_0)}{\lambda}}-1\right)-2G^2T_0N_0^2\sum_{n=1}^{\infty}\frac{B_nN_n^2\lambda}{\lambda^2+(s_n+is_0)^2}\left((-1)^m\mathrm{e}^{\frac{m\pi\,(s_n+is_0)}{\lambda}}-1\right)]$$

and

$$T_1=0.$$

Case-2 (for exponential bottom topography $c(x)=c_0e^{-\mu|x|}$):

$$R_1=\frac{c_0}{2ik_0}[-(N_0^1)^2\{\frac{(k_0^2+G^2)}{2ik_0+\mu}+\frac{(k_0^2+G^2)R_0^2}{\mu-2ik_0}-\frac{2(k_0^2-G^2)R_0}{\mu}\}+\int_{-\infty}^{0}((\sum_{n=1}^{\infty}k_nA_n\mathrm{e}^{k_nx}N_n^1)^2$$

$$-G^2(\sum_{n=1}^{\infty}A_n\mathrm{e}^{k_nx}N_n^1)^2)e^{-\mu x}\,dx+2ik_0N_0^1\sum_{n=1}^{\infty}\frac{k_nA_nN_n^1}{\mu+(k_n+ik_0)}-2ik_0N_0^1R_0\sum_{n=1}^{\infty}\frac{k_nA_nN_n^1}{\mu+(k_n-ik_0)}$$

$$-2G^2N_0^1\sum_{n=1}^{\infty}\frac{A_nN_n^1}{\mu+(k_n+ik_0)}-2G^2N_0^1R_0\sum_{n=1}^{\infty}\frac{A_nN_n^1}{\mu+(k_n-ik_0)}+\int_0^{\infty}((\sum_{n=1}^{\infty}s_nB_n\mathrm{e}^{-s_nx}N_n^2)^2$$

$$-G^2(\sum_{n=1}^{\infty}B_n\mathrm{e}^{-s_nx}N_n^2)^2)\mathrm{e}^{-\mu x}\,\mathrm{d}x+(N_0^2)^2T_0^2\frac{(s_0^2+G^2)}{2is_0+\mu}-2is_0N_0^2T_0\sum_{n=1}^{\infty}\frac{s_nB_nN_n^2}{\mu+(s_n-is_0)}$$

$$-2G^2T_0N_0^2\sum_{n=1}^{\infty}\frac{B_nN_n^2}{\mu+(s_n+is_0)}]$$

and

$$T_1=-\frac{c_0}{is_0}[-2s_0k_0T_0N_0^1N_0^2\{\frac{1}{\mu+ik_0+is_0}-\frac{R_0}{\mu+ik_0-is_0}\}$$

$$+G^2T_0N_0^1N_0^2\{\frac{1}{\mu+ik_0+is_0}+\frac{R_0}{\mu+ik_0-is_0}\}$$

$$-is_0T_0N_0^2\sum_1^{\infty}\frac{A_nk_nN_n^1}{is_0-k_n-\mu}-ik_0N_n^1\sum_1^{\infty}\frac{B_nk_nN_n^2}{ik_0+s_n+\mu}+R_0\sum_1^{\infty}\frac{B_nk_nN_n^2}{ik_0-s_n-\mu}$$

$$G^2T_0N_0^2\sum_1^{\infty}\frac{A_nN_n^1}{is_0-k_n-\mu}+G^2N_n^1\sum_1^{\infty}\frac{B_nN_n^2}{ik_0+s_n+\mu}+R_0\sum_1^{\infty}\frac{B_nN_n^2}{ik_0-s_n-\mu}$$

$$-\int_0^{\infty}(\sum_{n=1}^{\infty}k_nA_n\mathrm{e}^{-(k_n+\mu)x}N_n^1)(\sum_{n=1}^{\infty}k_nB_n\mathrm{e}^{-s_nx}N_n^2)dx]$$

$$+G^2\int_0^{\infty}(\sum_{n=1}^{\infty}A_n\mathrm{e}^{-(k_n+\mu)x}N_n^1)(\sum_{n=1}^{\infty}B_n\mathrm{e}^{-s_nx}N_n^2)dx].$$