

## Parametric study of cone angle variation on extrusion complexity & dead metal zone using FEM

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**Abstract**—In extrusion, the desired shape is obtained by plastic deformation of the billet material achieved by forcing it through a die. In any simple homogeneous and uniaxial compression (or tension), the metal flows plastically when the stress reaches the value of the flow stress. Once the applied force exceeds the shear strength of the material, sticking friction at the container surface becomes the dominant friction mode, and deformation takes place through shear in the bulk of the material. Non homogeneous metal flow through an extrusion die can directly affect product quality, productivity and die life. The complexity of a die profile is an indicator of how difficult it is to extrude the profile. Some investigations about the effect of die complexity on extrusion pressure, product quality and die life can be found in this published literature. However the effect of profile complexity on metal flow through the extrusion die has not been explored much. such a study can directly contribute toward die design improvement and reduction of extrusion defects related to metal flow. This article investigates the effect of shape complexity on the dead metal zone(DMZ) and metal flow through cold extrusion experiments and finite element simulations on some solid profiles. Experiments were performed using flat face dies & conical dies of different complexities and different billet materials. A 2D & 3D finite element simulations were carried out to perform these operations. One significant conclusion is that currently existing definitions of extrusion shape complexity cannot satisfactorily explain the variation in DMZ size under different conditions. The factors such as die profile symmetry and extrusion ratio may play a significant roles in the formation of DMZ and distortion of metal flow through the die.

**Keywords**—Cold extrusion, solid profiles, shape complexity, Dead Metal zone(DMZ),metal flow simulation(FEM)

### I. INTRODUCTION

Extrusion is an often-used forming process among the different metal forming operations and its industrial history dates back to the 18th century. A billet is placed in the container and pressed by the punch, causing the metal to flow through a die with an opening. In the process of extrusion, a billet is placed in an enclosed chamber. The chamber has an opening through which the excess material escapes as the volume of chamber is reduced when pushed by ram.

The escaped material has a uniform cross section identical with that of the opening. In general extrusion is used to produce cylindrical bars or hollow tubes. A large variety of irregular cross sections are also produced by this process using dies of complex shapes. The process has definite advantage over rolling for production of complicated section having re-entrant corners. In this process large reduction achieved even at high strain rates has made it one of the fastest growing metal working methods. Because of large force required in extrusion most metals are extruded hot when the deformation resistance of metal is low. However cold extrusion is also possible for many metals and has become an important commercial process. The reaction of the billet with the container and the die results in high compressive stresses that effectively reduce cracking of materials during primary breakdown from ingot. This is an important reason for increased commercial adoption of extrusion in the working of metals difficult to form such as stainless steel, nickel, nickel based alloys and other high temperature materials. More than ten years ago, researchers started to be attracted by three-dimensional problems in metal forming.

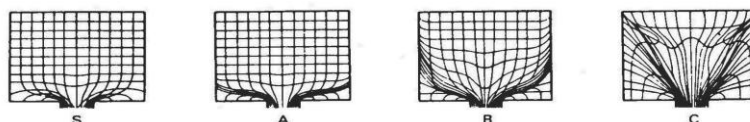


Fig. 1. Different types of metal flow in metal extrusion.

Typical flow patterns observed in extrusion are shown in Fig.1 [1].Flow pattern *S* is found in the absence of friction at the container and die interfaces, during extrusion of homogeneous materials. Flow pattern *A* is obtained in extrusion of homogeneous materials in the presence of friction at the die interface only. In the corner of the leading end of the billet, a separate metal zone (known as the *dead metal zone*) is formed between the die face and the container wall. Flow pattern *B* is obtained in homogeneous materials when there is friction at both die and container interfaces, resulting in an extended dead metal zone Flow pattern *C* is observed with billets having inhomogeneous material properties or with nonuniform temperature distribution in the billet; a more extended dead metal zone is formed and the material undergoes a more severe shear deformation at the container wall. Studies show that for die cone semi-angles under 45°, the dead metal zone does not form.

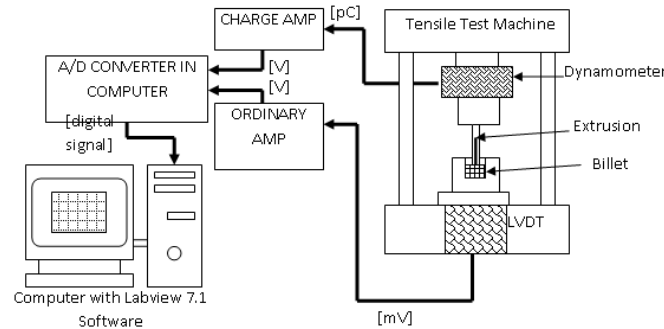
It has been established by various studies that the size and shape of the dead metal zone, and the pattern and homogeneity of flow lines in extrusion are directly related to the die cone angle  $I$ , to friction at the billet-container interface,

and to a lesser extent at the billet-die interface friction [10-13]. However, there are several other factors that can significantly affect extrusion metal flow. Unfortunately, these aspects have not been examined in detail in most of the published literature. Many studies use model materials (such as plasticine) and for metal flow study, lack of real material properties may contribute to inconsistencies with actual metal behavior. Numerical investigations are generally limited to the discussion of only one or two factors at a time. Also, enough experimental data is usually not available for satisfactory validation of numerical models, especially for metal flow pattern and dead metal zone analysis. Though shape complexity is a very important parameter in extrusion, no published work can be found on the relationship (if any) between complexity and metal flow. In a previous work by the authors [14] studied about the effect of various process parameters (such as extrusion ratio, ram speed, extrusion pressure, and billet material) on metal flow and DMZ in metal extrusion have been looked into & the author [15] studied about a detailed investigation of how the metal flow pattern and dead metal zone are affected by shape complexity in cold extrusion of solid profiles. The current paper reports some detailed investigation of how the metal flow pattern & DMZ are affected by varying the die cone angle  $\alpha$  (alpha) in cold extrusion of solid profiles.

## II. EXPERIMENTATION

An extrusion chamber assembly (consisting of container, bolster and ram) and three different compatible dies were designed and manufactured in collaboration with the die manufacturing facility of a commercial extrusion setup.

To emulate real world extrusion environment, these flat-face dies (90° die angle) were fabricated from H13 die steel using advanced machining techniques such as EDM, and were later subjected to standard heat treatment and surface hardening routines. Split billets of aluminum, lead, and Al-6063 were prepared to study metal flow patterns and dead metal zones. A 250-kN Instron universal testing machine, fitted with an extensometer (LVDT) and hooked up to a computer through a data logger, was used in the compression mode to perform and record the cold extrusion experiments as shown in figure 2.



**Fig 2:** Extrusion Data Gathering Schematic

### 2.1 Computer simulation

Extrusion runs on the three dies were simulated using the commercial finite element package ANSYS 7.0. To study the dynamic effects of changing ram speed, the software utilized was ANSYS-LSDYNA. Lead and aluminum being soft metals, further softened by annealing, billet material was modeled as elastic- perfectly plastic. As the container, die and billet surfaces were highly polished, and as a lubricant was also used, coefficient of friction was assumed to be negligible. Due to the symmetrical nature of the solid circular die & conical die, a 2D model was deemed to be sufficient, with an axis of symmetry along the line representing the split billet surface. The 2D 6-node triangular structural solid element PLANE2 (possessing plasticity, large deformation and large strain capabilities) was used in the axisymmetric mode to model the material geometry in ANSYS. As ANSYS-LSDYNA has a different set of element types, the explicit 2D structural solid element PLANE162 (capable of representing translations, velocities and accelerations) was used for explicit dynamic analysis. Meshing was done using 1210 elements, grid refinement being used near the die entrance area of the billet to assist smooth flow of elements in the dead metal zone and surrounding regions. Contacts were defined at the billet-container and billet-die interfaces, container and die being modeled as rigid materials. Load was applied (in the form of displacement in ANSYS, and as velocity in ANSYS-LSDYNA) at the free end of the billet to replicate the ram pressure. To represent the shape and geometry of the other two dies, 3D modeling had to be used. The 8-node 3D structural solid element SOLID164 was used for explicit dynamic analysis in ANSYS-LSDYNA. Model 1, having only one axis of symmetry, had to be modeled as shown in Fig 3.11. Model 2, possessing biaxial symmetry, is depicted in Fig.3.12.

## III. MATERIALS PROPERTIES

Details	Container	Die	Material
Young's modulus	200GPa	210Gpa	70Gpa
Poison's ratio	0.3	0.32	0.34
Density	7850kg/m <sup>3</sup>	7800kg/m <sup>3</sup>	2800kg/m <sup>3</sup>
Yield stress	350Mpa	550Mpa	140Mpa
Tangent Modulus	5000Mpa	1388Mpa	1000Mpa

### 3.1 Profile & dimension of dies used in the study :

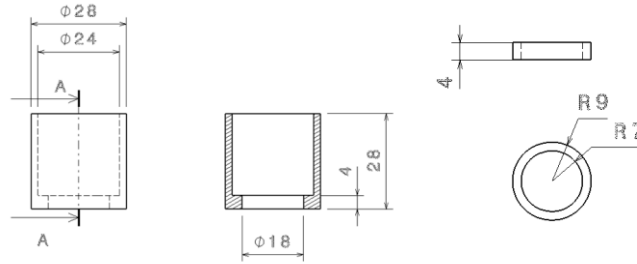


Fig.3.1 : 2 Dimensional representation of the problem

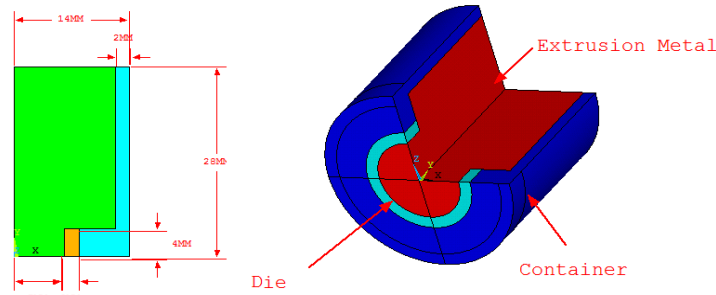


Fig.3.2: Axisymmetric & three dimensional view

Initially the analysis is carried out with cylindrical die and geometrical dimensions and views are as follows. Catia software is used to represent the problem in two dimensional space and later 3 dimensional space for clear dimensioning of the problem. The figure3.1 shows container and die dimensions. The outer dimension of the container is 28mm and internal diameter of the die is 14mm with 2mm thickness. Total length of the container is 28mm. The figure3.2 shows 2 dimensional and 3 dimensional views of the problem modelled using Ansys software. The dimensions of the problem are also represented. Here the die shape will be changed to find the angular effect of the die on dead metal zone formation. DMZ is unwanted behaviour for the smoother and finer extrusion parts. Extrusion metal, Container and die geometries are split to show the inner view of the geometrical model. Ansys mixed approach is used to built the above geometry. Even though models are built using Catia, the catia models are used for clear geometrical representation and ansys models are useful for analysis. Ansys geometry can be easily split using workplane options. Generally map mesh gives better accuracy compared to the free mesh for the imported geometries. Hypermesh can be used to split the catia models. But it takes time and for smaller changes again the process should be repeated which is not the case with ansys geometries.

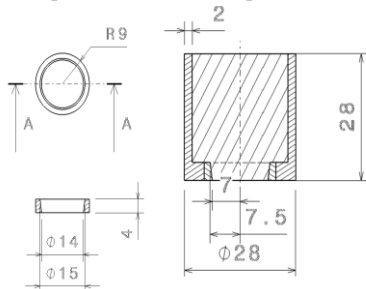


Fig3.3: Model 2(7° taper)

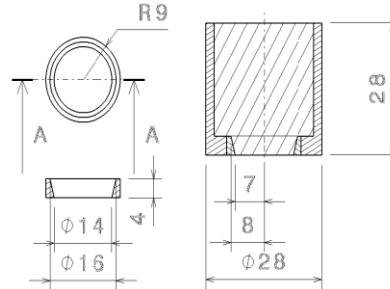


Fig3.4: Model 3(14° taper)

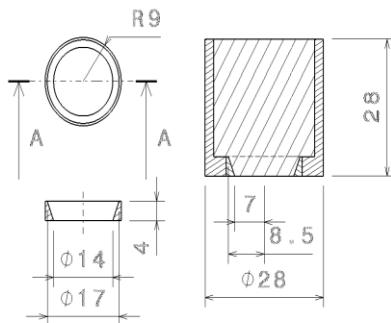


Fig3.5: Model 4 (21° taper)

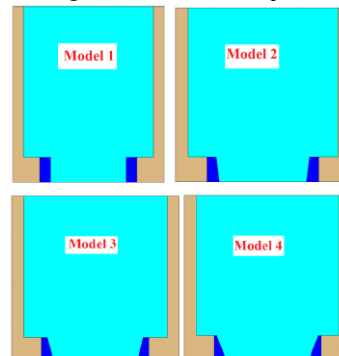
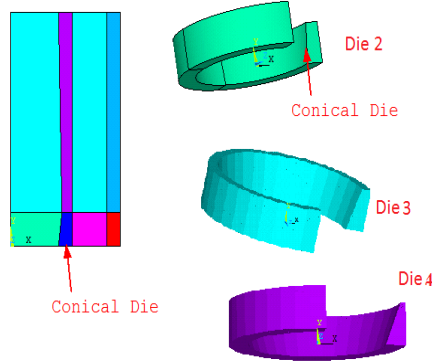


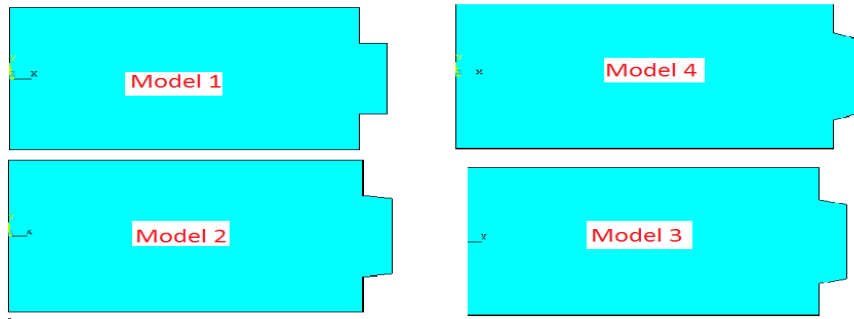
Fig3.6: Catia Sectional Models

The figure 3.3, 3.4 & 3.5 shows taper of dies using Catia drafting. The taper for the die is provided for  $7^{\circ}, 14^{\circ}, 21^{\circ}$  to check the dead metal zone in the problem. All other dimensions are similar. Catia drafting gives better dimensional details compared to the ansys dimensioning. Also ansys dimensioning is based on manual entry since ansys is analysis software. After the models are built in three dimensional space, drafting module is used to represent in two dimensional space. Sectioning is possible with modelling softwares. The figure 3.6 shows varying taper from model 1 to model 4. The taper angle is increasing with the models. All the members of assembly (container, die and material) are represented with different colours. The models. All the members of assembly (container, die and material) are represented with different colours. The figure 3.7 shows expanded plots of axisymmetric problems. Only conical dies are expanded to show in three dimensional view. Increasing taper can be observed in the problem. The conical dies are formed with keypoints generation and splitting the geometry. Ansys modelling helps in building the better meshed structures.



**Fig3.7 :** Conical die

**3.2 CFD analysis models :**

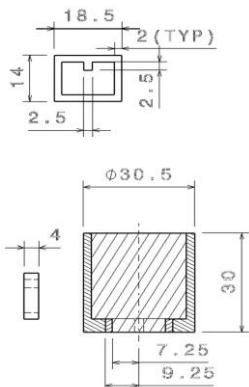


**Fig3.8:** CFD models

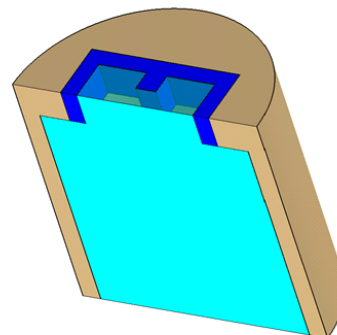
Only extrusion geometry is required in the CFD analysis. Hydrodynamic boundary conditions are applied to obtain the solution for the problem. Inlet, outlet and wall boundary conditions are applied for the CFD problems. CFD simulation helps in representing fluid conditions. Here viscous models can be considered for proper analysis. All modes in fluid domain represented as shown in figures.

**3.3 : 3D MODELS :**

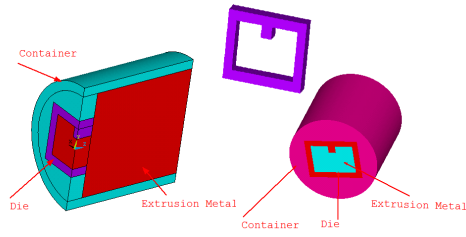
**MODEL 1**



**Fig3.9:** Model 1 : Rectangular cavity



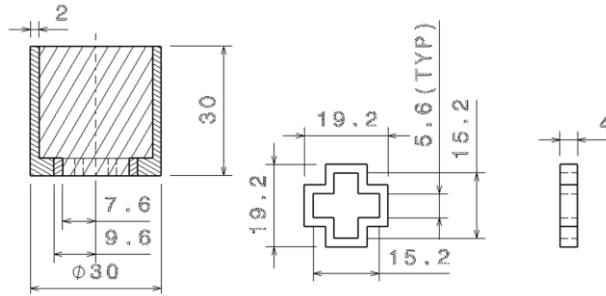
**Fig : 3.10 :** Three dimensional representation



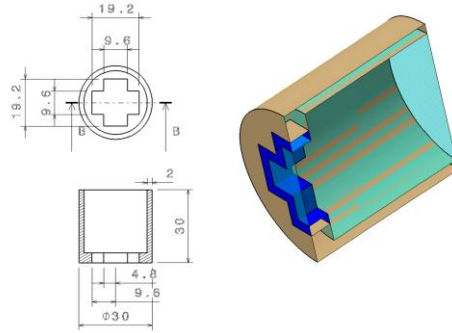
**Fig 3.11:** Axis symmetry Model 1 Geometry

The figure 3.11 shows geometry considered for the analysis. The effect of die shape on extrusion process need to be studied. So the above geometry is considered. All the dimensions are represented as shown in figures.

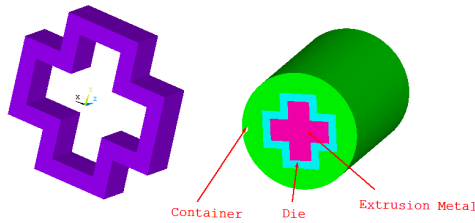
**MODEL 2**



**Fig3.12 :** Model 2 - Die shape

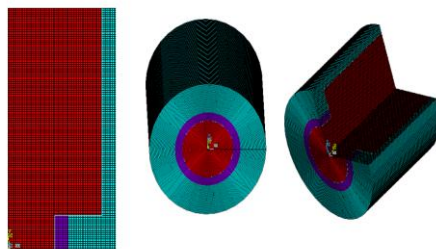


**Fig 3.13:** Cad Models for model 2



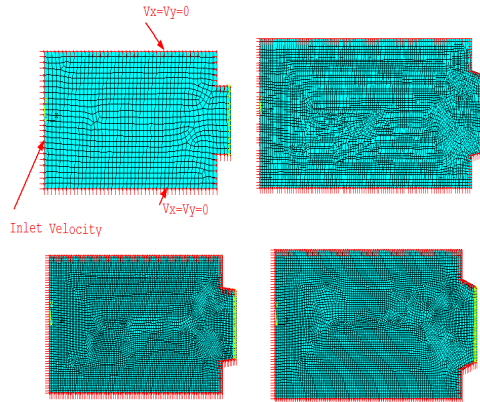
**Fig3.14:** Die shape 2

**3.4 MESHED PLOTS**



**Fig3.15 :** 2 dimensional map mesh

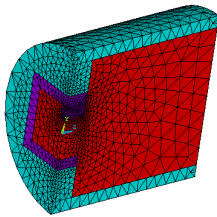
The geometry is meshed with 4 noded quad element Plane182. Different materials are applied for die, container and billet while meshing. 7448 elements and 7768 nodes are used to mesh the problem. 0.25 mm size is used for meshing. The structure is map meshed after splitting the geometry using workplane. Generally map mesh gives better results compared to the free mesh. Plane182 element is suitable for axisymmetric problems along with nonlinearity of the materials. Using mesh attributes, different material properties can be assigned before meshing. Meshed plot with boundary conditions : For CFD analysis, only extruded geometry is meshed.  $V_x$  and  $V_y$  are equal to zero on the solid boundaries as per the hydrodynamic theory. Inlet velocity of 1m/sec is applied along the line geometry as shown in the figure.



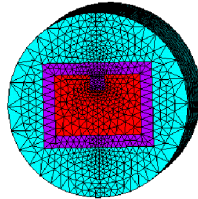
**Fig3.16 :** CFD boundary conditions

The geometry is meshed with 2 dimensional Fluid141 element. 1171 elements and 1246 nodes are used for the mesh. Fluid141 is a fluid element based flotrán logic for solving the flow problems. Only one element is available in Flotrán for during flow and heat transfer problem. The element is defined by flow variable degree of freedom along with temperature. Fine mesh is used to capture the flow pattern in the problem. But flow problem requires extensive computational time due to the requirement of many conditional variables. Atmospheric condition is assumed at the exit conditions and container walls flow velocity is assumed zero.

A Three dimensional meshed Models :



**Fig: 3.17:** Meshed Model 1



**Fig3.18:** Meshed model 2

The figure shows meshed models for model1 and model2. Solid185 element is used for meshing. Due to complication in the shape, tetra mesh is used for meshing the objects. Also complete contact is assumed between wall boundary and extrusion metal boundary.

#### IV. RESULTS

The analysis has been done to find dead band zone during metal extrusion process. Initially the analysis is carried out with cylindrical die and later analysis is carried out with tapered or conical dies. The results are as presented below.

The figure4.1 shows displacement pattern in the extrusion process. The status bar indicates the varying displacements in the problem. DMZ(Dead Metal Zone) area is also represented in the problem. The blue region is the slow moving region by which pattern of displacement can be observed. The relative movement of the metal vary slow in this region. This region should be reduced to prevent dead metal zone formation in the extrusion process. The left side figure shows only extruded metal in the axisymmetric domain and the right side picture shows deformation plot of the complete assembly. Contact elements are defined across the interface of metal, container interface and metal die interface. Targe169 and Contac172 elements are used for contact definition.

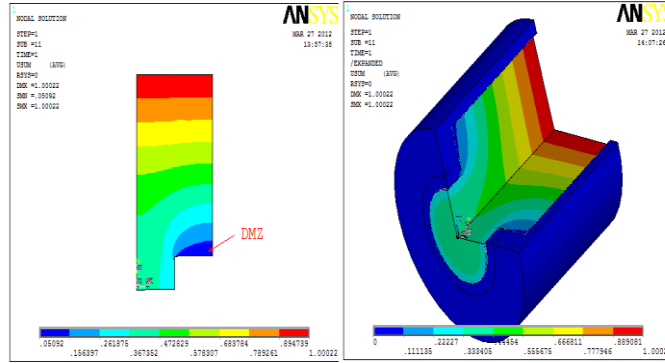


Fig4.1 : Displacement in the structure

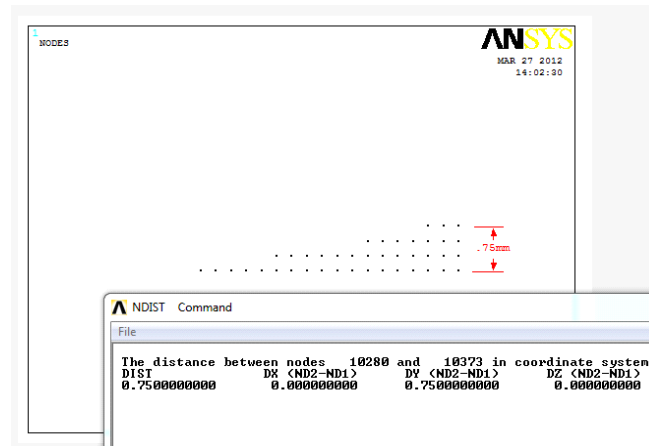


Fig4.2: DMZ value

The figure4.2 shows DMZ of 0.75mm in the problem. So the DMZ is spread for a height of 0.75mm.

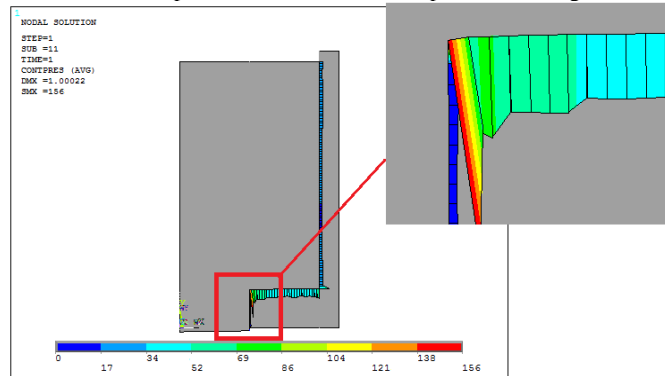


Fig4.3 : Contact pressure plot

Contact pressure is shown above figure4.3. Higher contact pressure indicates zone of dead metal zone as the billet is attached either to the container or die surface. So minimum contact pressure is desirable for smooth flow of billet metal for extrusion process.

The displacement plot represents variation of displacement along the contact area between extrusion metal and the container inner geometry. The plot shows reducing displacement values from top to bottom. So the movement of metal is very slow at the bottom of the container. So die shape can be predicted to reduce the DMZ in the problem. So the simulation helps in predicting the optimum shape of the container for reducing the DMZ regions.

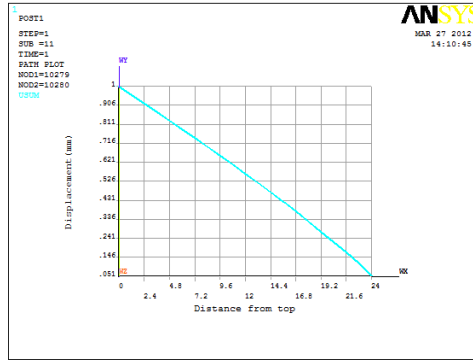


Fig4.4 : Displacement plot

CONE (7°) RESULTS :

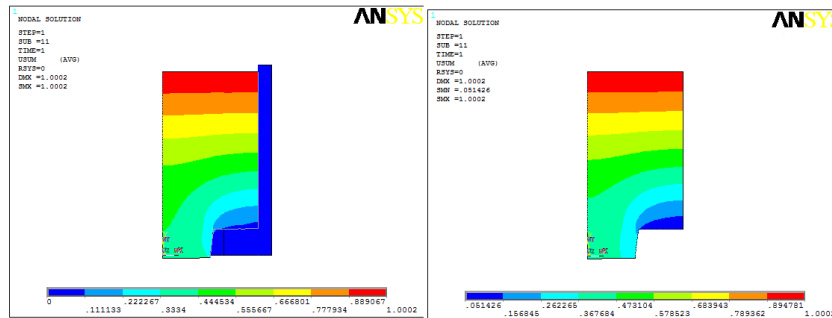


Fig4.5: Overall displacement plot (7° die angle)

Blue region indicates DMZ region. The DMZ region is reducing compared to the cylindrical case by visual observation. The value of DMZ is represented in the next picture. Right side picture shows deformation only in the billet metal.

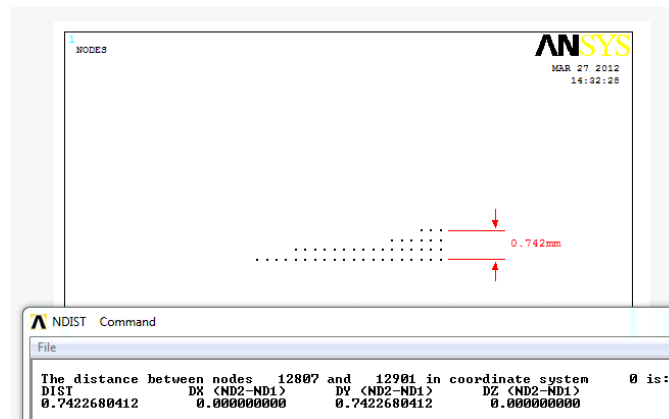


Fig.4.6 : DMZ value

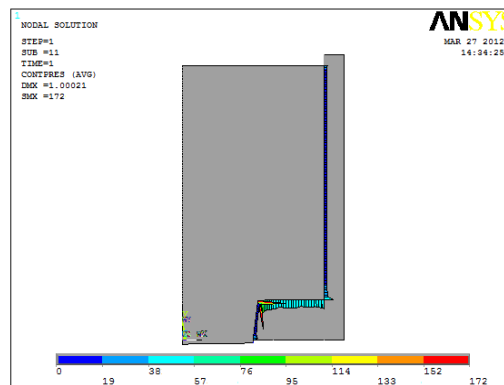


Fig4.7 : Contact pressure (7° die angle)



The figure4.7 shows DMZ region and contact pressure in the problem. Maximum contact pressure is taking place at the edge of die. The drop in DMZ value can be observed compared to the previous case. CONE2(14<sup>0</sup>) RESULTS :

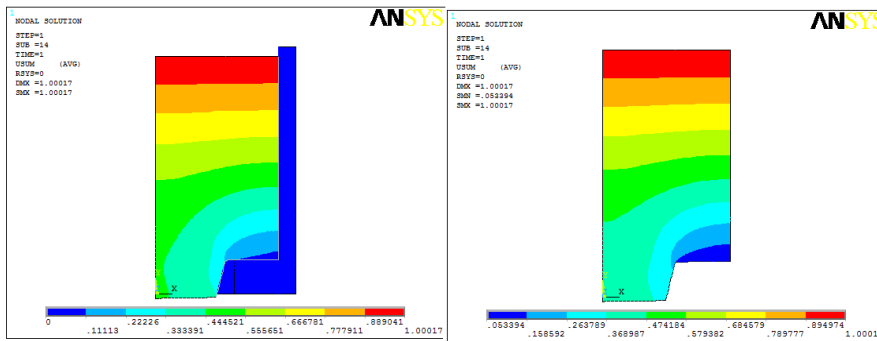


Fig4.8: Overall displacement plot (14<sup>0</sup> die angle)

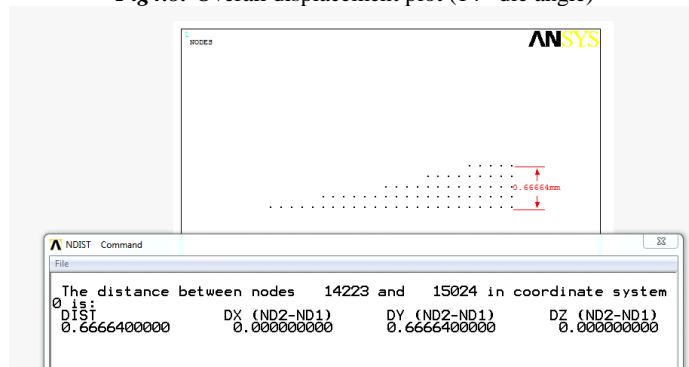


Fig4.9 : DMZ value

The figure shows further drop in DMZ value compared to the previous two cases. So taper of die also plays role in reducing the DMZ values. The values are queried by local refinement of the mesh to capture it to decimal values. CONE 3 RESULTS :

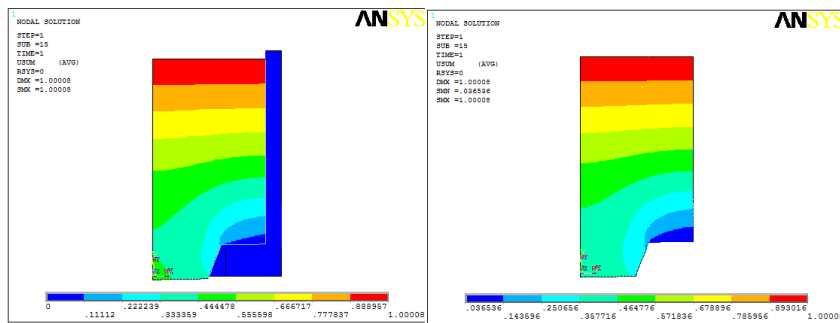


Fig : 4.10 : Overall displacement plot (21<sup>0</sup> die angle)

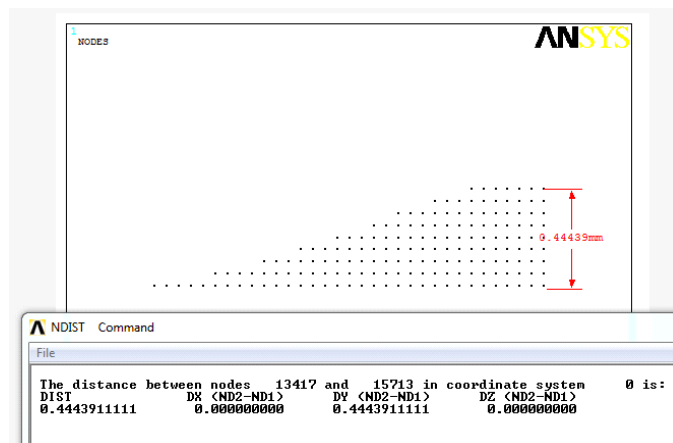


Fig: 4.11 : DMZ value

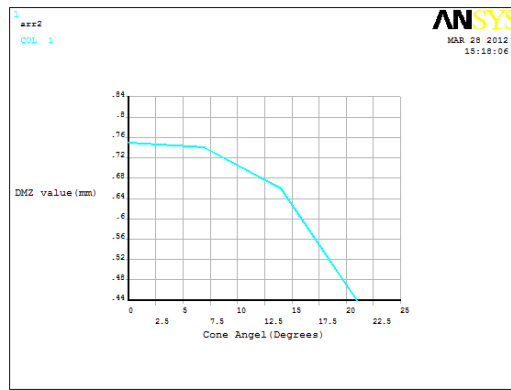
The above picture 4.11 shows DMZ value as queried in the Ansys software. Maximum value of 0.44435mm can be observed in the problem. For better results mesh is refined in the possible DMZ region. This value is smaller than cylindrical, 7° cone and 14° cone angle dies. So analysis software with the help of finite element analysis helps in estimating the DMZ regions. It gives a chance to the die designer for selection and modified designs for forging dies.

**4.1 TABULAR RESULTS :**

Die Details	DMZ value(mm)
Cylindrical	0.75
Cone(7°)	0.74
Cone(14°)	0.66
Cone(21°)	0.44

**Table 4.1 :** Tabular results for DMZ with different cone angles of die

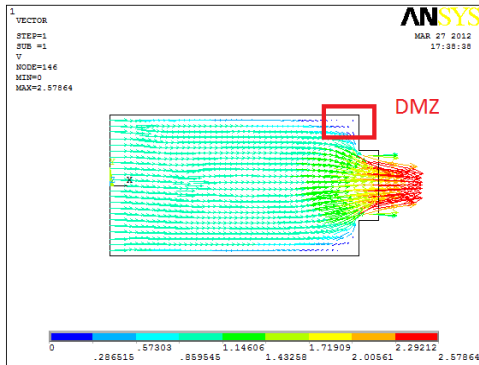
The table 4.1 results show reduction in DMZ values with increased cone angles. Almost a nonlinear relation can be observed between the cone angle and DMZ value.



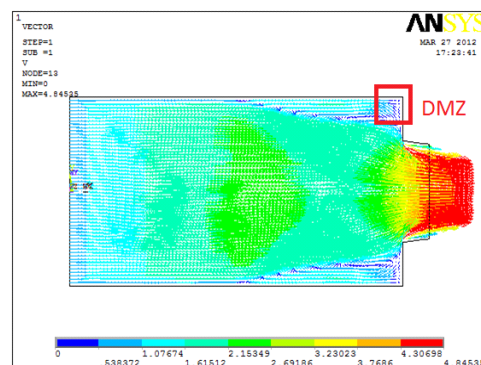
**Fig 4.12 :** Effect of Cone angle on Dead Metal zone

The effect of cone angle on Dead Metal zone is represented in the graph. The graph shows reducing DMZ with increased cone angle. So die cone angle has considerable effect in reducing the DMZ in the extrusion process. So higher cone angle is desirable for dies in the extrusion process. Almost a parabolic relation can be observed from the graph.

**4.2 CFD RESULTS:**



**Fig 4.13 :** Metal Flow and DMZ for cylindrical Die



**Fig 4.14 :** Metal Flow and DMZ for Conical die (7°)

CFD simulation shows the movement in vector form. The minimum flow region is represented by DMZ. CFD helps to simulate the flow patterns under extrusion and one can easily estimate and design the dies for optimum flows. DMZ area reduction can be observed in the Flotran simulation compared to the cylindrical dies. Due to cone angle, flow velocity is increasing. The increase in velocity is also an indication of movement of the extrusion material. Also turbulence can be observed in the DMZ region.

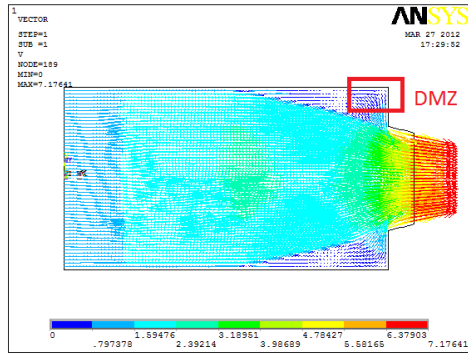


Fig4.15 : Metal Flow and DMZ for Conical die(14<sup>0</sup>)

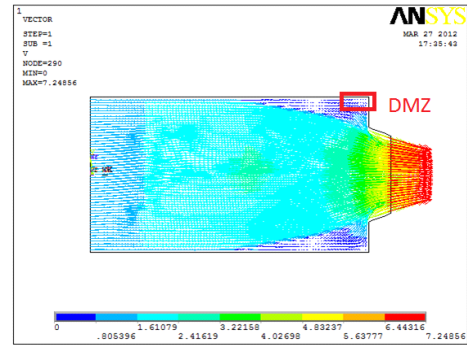


Fig4.16 : Metal Flow and DMZ for Conical die(21<sup>0</sup>)

DMZ area reduction can be observed in the Flotran simulation compared to the cylindrical dies. Due to cone angle, flow velocity is also increasing compared to the previous cases. Vector indication also shows slighter movement of the fluid in the DMZ regions. For the same configurations, velocity change can be considered for effect of die angle on DMZ formation.

Further increase of flow can be observed compared to the previous cases indicating reduction in DMZ regions. But change in velocity is marginal compared to the previous case. So optimization of die angle is also possible with computer simulations. The analysis is done for similar conditions except the die angle.

**4.3 THREE DIMENSIONAL DIE RESULTS :**

**MODEL 1 RESULTS :**

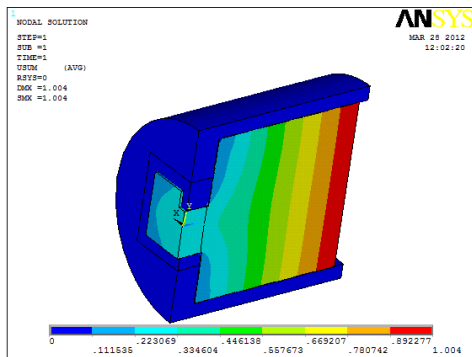


Fig4.17: Metal Flow and DMZ in the Die 2

The figure4.17 shows flow of extruded metal with the change in die shape. The geometry is considered to find the effect of die shape of formation of Dead metal zone. The simulation pictures shows variation of movement of extruded metal along the die path.

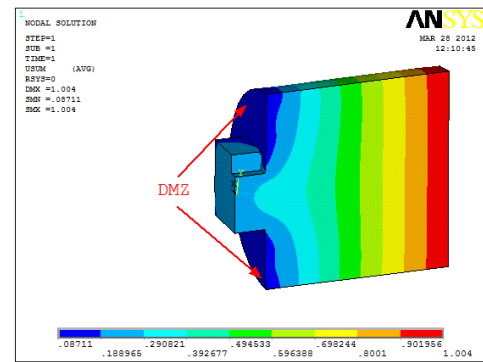


Fig4.18: DMZ location the Die 2

The figure4.18 shows dead metal formation close to the die rectangular shape. The projection provided at the center of extrusion part is also increasing the Dead metal zone as shown by colour representation.

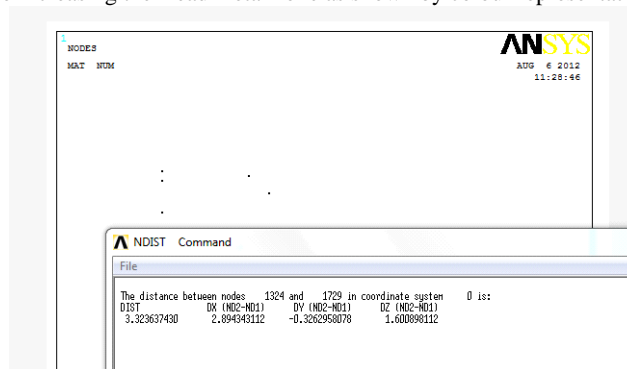


Fig4.19: DMZ value

Figure4.19 shows the DMZ value. Since flow direction is considered in 'Z' axis, upto a value of 1.6mm the Dead metal zone is spread. By query option this value can be obtained in Ansys.

**MODEL 3 RESULTS :**

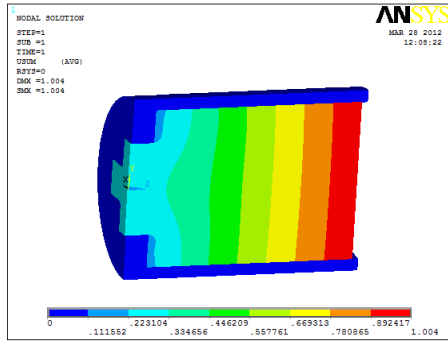


Fig4.20: Metal Flow and DMZ in the Die 2

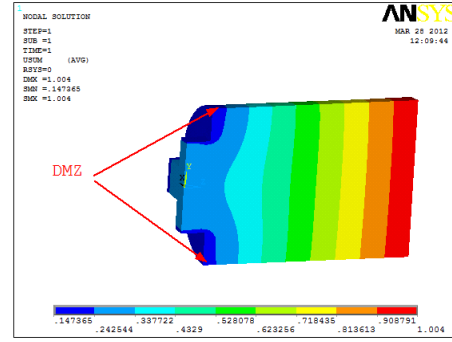


Fig4.21: DMZ spread in the Die configuration 3

The figure4.20 shows reduced DMZ by change of shape of the die. The simulation shows reduced blue region or less flow region is reduced compared to the previous design. So this die shape is compared to the previous die shape. So die shape also plays important role in reducing the DMZ.

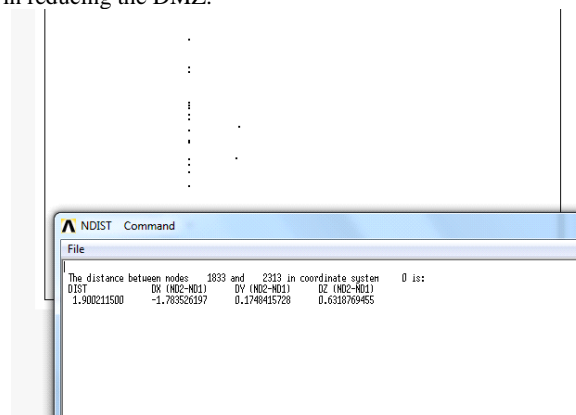


Fig. 4.22 : DMZ value for 3 Dimensional Model 2

The results shows lesser DMZ in the model 2 compared to Model 1. Figure 4.22 shows reduced DMZ value of 0.6318mm compared to the first model of 1.6mm. So shape of die has influence on flow characters and better die shape minimizes the DMZ.

## V. DISCUSSION

Extrusion is an important manufacturing process in Engineering Industry. Die plays important role in forming the shape. Certain drawbacks exists in the extrusion process. Due to the friction between the members and curvature the dead metal zone will be formed in the extrusion process which will reduce the speed of extrusion process and also quality of the extruded parts. So a Finite element analysis is carried out to analyse the nature of Dead Metal Zone(DMZ) formation. Due to the advances in finite element analysis techniques, through simulation, the process can be improved without being prototype built up and testing. Initially the effect of cone angle of the die is considered to find effect on DMZ formation. The cone angle is varied from zero degrees to 21° to find extent of DMZ formation. By considering finer mesh, the particle movement can be queried by ansys query option for results or nodes of negligible movement can be captured. The results shows drop in DMZ formation with cone angle. Also cone angle can be optimised by finite element simulation as much variation is not observed between 14° and 21° for given set of dimensions and material properties. An axisymmetric analysis is considered for analysing the regular cylindrical or conical geometries. Further CFD analysis is considered to find the DMZ formation. Here the change in velocity shows the effect of cone angle on dead metal formation. CFD is the best tool to find possible DMZ Regions with highly complicated shapes where structural analysis is not possible to apply. Also effect of viscosity, density and thermal effects can be considered in CFD. CFD solvers gives high pressure zones, high and low velocity zones by which design of die or container can be improved. Ansys/Flotran solver based on Fortran code is used for analysis. Fluid141 element with its 2 dimensional capabilities for flow simulation is used for meshing. At exit atmospheric conditions are considered for analysis. The analysis is done at similar conditions for all the models (Inlet, outlet and wall boundary conditions) except the cone geometry change. Further three dimensional analysis is considered to study irregular die shapes. Solid185 element with large deformation effects are considered for analysis. The results shows even die shape has effect on dead metal zone formation.

## VI. CONCLUSIONS

Dead Metal Zone is the critical topic in quality of the extruded parts. This should be minimised for better products. Also it increases the load requirements for forging operation. Die, container and the extruded metal are the major components of extrusion process.

Analysis has been carried out to find the effect of cone angle and die shape on dead metal zone formation. The problem is analysed using Ansys as well as Ansys-Flotran to have visual idealization of DMZ formation. Analysis has been

carried out in both structural and fluid domain. Also analysis is carried out both in axisymmetric, two dimensional and three dimensional domains.

Initially models are built in catia and dimensions are represented for better visualization. Since ansys is analysis software, dimensional representation is difficult with ansys. So catia a three dimensional modeling software is used for better dimensional representation and views. But the actual analysis models are built in Ansys for better finite element connectivity which is difficult with catia models. Also mesh control is better with ansys models. Axisymmetric and flotran models are analysed in two dimensional space. For irregular die shapes, the analysis is carried out in three dimensional space. Contact models are considered in the axisymmetric domain. But complete contact is considered in the three dimensional models due to convergence problems with three dimensional problems.

From the results it can be observed that cone angle plays important role in DMZ formation. Increased cone angle reduces the DMZ formation and improves the efficiency of forging process. Almost a parabolic relation can be observed with die angle. Computation fluid dynamic analysis is also carried to study the flow simulation of extruded metal. This simulation helps in optimizing the die angle. The simulation results can be interpreted with increase in velocity with reduced DMZ zone. In the beginning the velocity increase is more with cone angle. But later between  $14^{\circ}$  and  $21^{\circ}$  the velocity increase is marginal which indicates almost optimized cone angle. Also CFD studies helps in the finding any turbulence in the process. Also shape of die also plays important role in DMZ formation. Various methods has explored to find the DMZ formation in the forging dies. Ansys axisymmetric, CFD and three dimensional models are built to find the DMZ Region. The accuracy of prediction can be improved by fine meshing the geometries.

## VII. FURTHER SCOPE

- Analysis can be carried out with temperature effect on extruded metal
- Material nonlinearity of die can be considered in the analysis
- Composite materials for dies can be analysed.
- Viscosity effects can be studied with converged extruded metal
- LsDyna solver can be used for the analysis(Explicit solvers)

## REFERENCES

- [1]. V. Nagpal, T. Altan, Analysis of the three-dimensional metal flow in extrusion of shapes. With the use of dual stream function, in: Proceedings of the Third North American Metal Research Conference, Pittsburgh, PA, 1975, pp. 26–40.
- [2]. B.B. Basily, D.H. Sansome, Some theoretical considerations for the direct drawing of section rod from round bar, Int. J. Mech. Sci. 18 (1979) 201–209.
- [3]. D.Y. Yang, C.H. Lee, Analysis of three-dimensional extrusion of sections through curved dies by conformal transformations, Int. J. Mech. Sci. 20 (1978) 541–552.
- [4]. W. Johnson, H. Kudo, The Mechanics of Metal Extrusion.(1963)
- [5]. Hill, Analysis metal working process, mechanics of physical solids flow rule(1963)
- [6]. R. Prakash, O.H. Khan, An analysis of plastic flow through polygonal converging dies with generalised boundaries of the zone of plastic deformation, Int. J. Mach. Tool Des. Res. 19 (1979)1–9.
- [7]. F. Gatto, A. Giarda, The characteristics of three-dimensional analysis of plastic deformation according to the SERR method, Int. J. Mech. Sci. 23 (1981) 129–148
- [8]. P.K. Kar, N.S. Das, Upper bound analysis of extrusion of I-section bars from square:rectangular billets through square dies, Int. J. Mech. Sci. 39 (8) (1997) 925–934.
- [9]. P.K. Kar, R.K. Sahoo, Application of the SERR technique to the analysis of extrusion of sections from round billets, J. Inst. Engrs. (India) 78 (1997) 15
- [10]. S.K. Sahoo, P.K. Kar , K.C. Singh a. A numerical application of the upper-bound technique for round-to-hexagon extrusion through linearly converging dies,Int. J. Mat. Pro. Tec. 91 (1999) 105–110
- [11]. Narayanasamy R, Ponalagusamy R, Venkatesan R, Srinivasan P (2006) An upper bound solution to extrusion of circular billet to circular shape through cosine die. Mater Des 27(5):411– 415. doi:10.1016/j.matdes.2004.11.026
- [12]. J.S. Gunasekera, S. Hosino, Analysis of extrusion or drawing of polygonal sections through straightly converging dies, J. Engng. Ind. Trans. ASME 104 (1982) 38–43.
- [13]. C.B. Boer, W.R. Schneider, B. Avitzur, An upper bound approach for the direct drawing of square section rod from round bar, in: Proceedings of the 20th International Machine Tool Design and Research Conference, Birmingham, 1980, pp. 149–155
- [14]. D.Y. Yang, M.U. Kim, C.H. Lee, A new approach for generalized three-dimensional extrusion of sections from round billets by conformal transformation, IUTAM Symp. Metal Forming Plasticity, Germany, 1979, pp. 204–221.
- [15]. S.Z. Qamar\*FEM study of extrusion complexity & DMZ Mechanical and Industrial Engineering Department, Box 33, Sultan Qaboos University, Al Khoudh 123, Oman
- [16]. B. Avitzur, Metal forming process and analysis, chapter 10 (1977)
- [17]. Geoffrey W. Rowe, Introduction to the principle of metal working, Chapter-8
- [18]. V. Gopinathan, Plasticity Theory and its Application in Metal Forming, Chapter 7  
[www.google.com](http://www.google.com), [www.matweb.com](http://www.matweb.com)