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Analysis of diffusion in a plane sheet and thin hollow cylinder for some boundary conditions

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Abstract—A general mathematical analysis for diffusion in a plane sheet and thin hollow cylinder is given. The analysis relates to the flow of mass or heat in the plate sheet and thin hollow cylinder, including set up transient and decay transient states. Under the condition where the diffusion coefficient is constant, the concentration profiles indicated in the general equations for some cases have been demonstrated and discussed Comparing the concentration profile of the set up transient and decay transient states for the plane sheet and thin hollow cylinder, the concentration profiles show the same behavior. However, diffusion in the thin hollow cylinder is faster than that in the plane sheet in reaching the steady state because of the geometry shape effect.

Keywords—Mathematical analysis; Diffusion; Concentration profile; Hollow cylinder; geometry shape effect

I. INTRODUCTION

Measurements of the diffusion coefficient and the permeation rate of hydrogen through a metal membrane have been widely investigated because of not only a sensitive electrochemical method developed by Devanathan and Stachurski [1] but also some mathematical solutions of the pertinent diffusion equation given by McBreen et al. [2], Kiuchi and McLellan [3], and Yen and Shih [4]. Critical hydrogen concentration induced cracking in a plate sheet [5] has been investigated. However, the critical hydrogen concentration of the metal pipe (hollow cylinder) has become an important factor [6-7]. Therefore, the need to understand the concentration profile in a hollow cylinder might be urgent. Several decades ago, Ash et al. [8] provided a means of measuring the diffusion coefficient D for a material in the form of a slab and hollow cylinder by the time lag method. Carslaw and Jaeger [9] also gave some solutions to the problem of heat conduction through a hollow cylinder shell with some initial and boundary conditions. Crank [10] applied the above mathematics to diffusion in hollow cylinder shells for some special case. However, a more general mathematical solution including steady, set up transient, and decay transient states of the concentration distribution and permeation rate of hydrogen in a hollow cylinder shell was investigated in our earlier study [11].

The object of this work is to present a mathematical analysis of diffusion in a plane sheet and thin hollow cylinder including set up transient and decay transient states for some boundary conditions. The concentration distribution profiles are presented for a plane sheet and thin hollow cylinder, respectively.

II. MATHEMATICAL ANALYSIS

2.1. Set up transient in a plane sheet

Consider the case of diffusion through a plane sheet or membrane of thickness L and a constant diffusion coefficient D, where the governing equation is described by Fick`s second law as follows

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} \tag{1}$$

2.1.1. Case A

Let concentration at x = 0 be maintained at C_0 , while at x = L be maintained at zero, as shown in Fig.1 (a), the following boundary conditions are fulfilled.

$$\begin{aligned} &t = 0,\ 0 < x < L, C = 0 \\ &t > 0,\ x = 0, C = C_0 \\ &t > 0,\ x = L, C = 0 \end{aligned} \tag{2}$$

Apply the Laplace transformation method to Eq. (1) with the initial and boundary conditions of Eq. (2) to obtain

$$\frac{C}{C_0} = (1 - X) - \frac{2}{n\pi}\sum_{n=1}^{\infty} Sin(n\pi X)\cdot \exp(-n^2\pi^2\tau) \qquad (3)$$

Where

$$X = \frac{x}{L} \quad \tau = \frac{Dt}{L^2}$$

2.1.2. Case B

Consider the plane sheet with zero concentration initially and the concentration at x = 0 is zero while at x = L it is maintained at C_0 , as shown in Fig.1 (b), then the following boundary conditions on the two surfaces, are

$$t = 0, \quad 0 < x < L, C = 0$$

$$t > 0, \quad x = 0, C = 0$$

$$t > 0, \quad x = L, C = C_0 \qquad (4)$$

Apply the Laplace transformation method to Eq. (1) with the initial and boundary conditions of Eq. (4) to obtain

$$\frac{C}{C_0} = \sum_{n=0}^{\infty}\{erfc[\frac{(2n+1) - X}{2\sqrt{\tau}}] - erfc[\frac{(2n+1) + X}{2\sqrt{\tau}}]\} \qquad (5)$$

Where

$$X = \frac{x}{L} \quad \tau = \frac{Dt}{L^2}$$

2.1.3. Case C

Consider the plane sheet with zero concentration initially and the boundary conditions on the two surfaces at constant concentration , as shown in Fig.1 (c), are

$$t = 0, \quad 0 < x < L, C = 0$$

$$t > 0, \quad x = 0, C = C_0$$

$$t > 0, \quad x = L, C = C_0 \qquad (6)$$

Apply the Laplace transformation method to Eq. (1) with the initial and boundary conditions of Eq. (6) to obtain

$$\frac{C}{C_0} = 1 + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{Cos(n\pi) - 1}{n}\cdot Sin(n\pi X)\cdot \exp(-n^2\pi^2\tau) \qquad (7)$$

Where

$$X = \frac{x}{L} \quad \tau = \frac{Dt}{L^2}$$

2.1.4. Case D

Consider the plane sheet with zero concentration initially, and the boundary conditions at the surface x = 0 is impermeable, i.e., the concentration gradient is zero and on the other surface x = L, the concentration is constant at C_0, as shown in Fig.1 (d), are

$$t = 0,\quad 0 < x < L, C = 0$$
$$t > 0,\quad x = 0, \frac{\partial C}{\partial x} = 0$$
$$t > 0,\quad x = L, C = C_0 \qquad (8)$$

Apply the Laplace transformation method to Eq. (1) with the initial and boundary conditions of Eq. (8) to obtain

$$\frac{C}{C_0} = \sum_{n=0}^{\infty} (-1)^n \{erfc[\frac{(2n+1)+X}{2\sqrt{\tau}}] + erfc[\frac{(2n+1)-X}{2\sqrt{\tau}}]\} \qquad (9)$$

Where

$$X = \frac{x}{L} \quad \tau = \frac{Dt}{L^2}$$

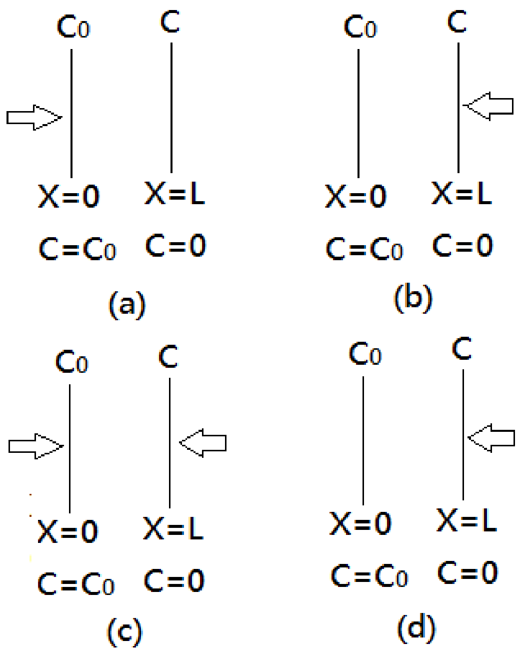

Fig. 1. Schematic representation of diffusion in plate sheet for cases: (a) A (b) B (c) C (d) D.

2.2. Set up transient in a hollow cylinder

Consider a long circular cylinder in which diffusion is radial everywhere. Concentration is then a function of radius r and time t only, and the diffusion equation becomes

$$\frac{\partial C}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial C}{\partial r}\right) \qquad (10)$$

2.2.1. Case A

Consider the hollow cylinder with zero concentration initially, and the concentration at r = a is zero while at r = b it is maintained at Co , as shown in Fig.2 (a), then the following boundary conditions on the two surfaces are

$$t \le 0, a < r < b, C = 0$$

$$t > 0, \quad r = a, C = 0$$

$$t > 0, \quad r = b, C = C_0 \tag{11}$$

The solution to this case has been derived and discussed in a previous paper [11] as

$$\frac{C}{C_0} = \frac{\ln r_A^*}{\ln k} - \pi \sum_{n=1}^{\infty} \frac{J_0(a\alpha_n) J_0(ka\alpha_n) U_0(a\alpha_n r_A^*)}{J_0^2(a\alpha_n) - J_0^2(ka\alpha_n)} \exp\left[-(a\alpha_n)^2 \tau\right] \tag{12}$$

Where

$$U_0(a\alpha_n r_A^*) = J_0(a\alpha_n r_A^*) Y_0(a\alpha_n) - Y_0(a\alpha_n r_A^*) J_0(a\alpha_n)$$

and

$$r_A^* = \frac{r}{a} \quad , \quad k = \frac{b}{a} \quad , \quad \tau = \frac{Dt}{a^2}$$

2.2.2. Case B

Consider the hollow cylinder with zero concentration initially, and the concentration at r = a is Co while at r = b it is maintained at zero, as shown in Fig.2 (b), then the following boundary conditions on the two surfaces are

$$t \le 0, a < r < b, C = 0$$

$$t > 0, \quad r = a, C = C_0$$

$$t > 0, \quad r = b, C = 0 \tag{13}$$

The solution to this case has been derived and discussed in a previous paper [11] as

$$\frac{C}{C_0} = -\frac{\ln r_B^*}{\ln k} + \pi \sum_{n=1}^{\infty} \frac{J_0^2(ka\alpha_n) U_0(ka\alpha_n r_B^*)}{J_0^2(a\alpha_n) - J_0^2(ka\alpha_n)} \exp\left[-(a\alpha_n)^2 t\right] \tag{14}$$

Where

$$U_0(ka\alpha_n r_B^*) = J_0(ka\alpha_n r_B^*) Y_0(a\alpha_n) - Y_0(ka\alpha_n r_B^*) J_0(a\alpha_n)$$

And

$$r_B^* = \frac{r}{b} \quad , \quad k = \frac{b}{a} \quad , \quad \tau = \frac{Dt}{a^2}$$

2.2.3. Case C

Consider the hollow cylinder with zero concentration initially, and the boundary conditions on the two surfaces are constant concentration Co , as shown in Fig.2 (c), are

$$t \le 0, a < r < b, C = 0$$

$$t > 0, \quad r = a, C = C_0$$

$$t > 0, \quad r = b, C = C_0 \tag{15}$$

The solution to this case has been derived and discussed in a previous paper [11] as

$$\frac{C}{C_0} = 1 - \pi \sum_{n=1}^{\infty} \frac{J_0(ka\alpha_n) U_0(a\alpha_n r_C^*)}{J_0(a\alpha_n) + J_0(ka\alpha_n)} \exp[-(a\alpha_n)^2 \tau] \tag{16}$$

where

$$U_0(a\alpha_n r_C^*) = J_0(a\alpha_n r_C^*) Y_0(a\alpha_n) - J_0(a\alpha_n) Y_0(a\alpha_n r_C^*)$$

and

, ,

$$r_C^* = \frac{r}{a} \quad K = \frac{b}{a} \quad \tau = \frac{Dt}{a^2}$$

2.2.4. Case D

Consider the hollow cylinder with zero concentration initially, and the boundary conditions at the surface r = a is impermeable, i.e., the concentration gradient is zero, and the other surface r = b is at constant concentration Co, as shown in Fig.2 (d), are

$$t \le 0, \quad a < r < b, C = 0$$

$$t > 0, \quad r = a, \frac{\partial C}{\partial r} = 0$$

$$t > 0, \quad r = b, C = C_0 \tag{17}$$

This is similar to the solution of the temperature distribution solved by Carslaw and Jaegar [9], the solution to this case has been derived as

$$\frac{C}{C_0} = 1 - \pi \sum_{n=1}^{\infty} \frac{J_1(a\alpha_n) J_0(ka\alpha_n) [J_0(a\alpha_n r_D^*) Y_1(a\alpha_n) - J_1(a\alpha_n) Y_0(a\alpha_n r_D^*)]}{J_0^2(a\alpha_n) - J_0^2(ka\alpha_n)} \exp[-(a\alpha_n)^2 \tau] \tag{18}$$

Where

$$r_D^* = \frac{r}{a} \quad K = \frac{b}{a} \quad \tau = \frac{Dt}{a^2}$$

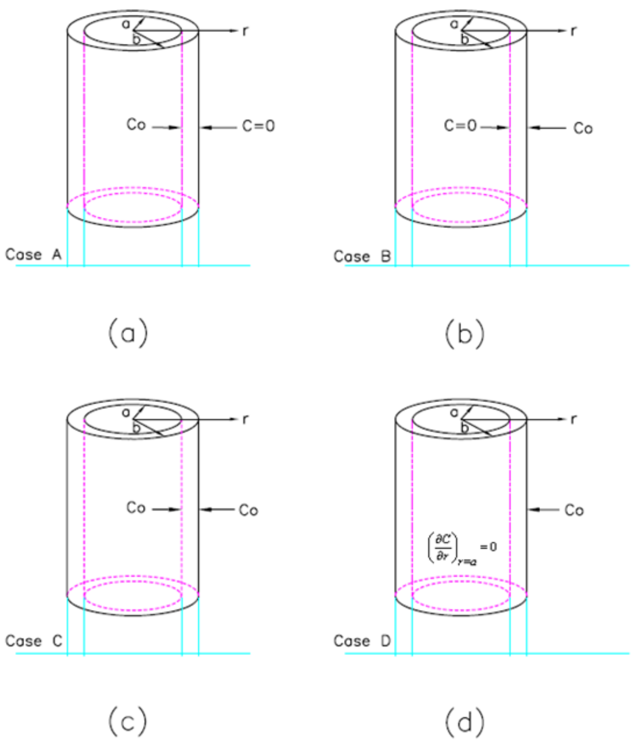

Fig. 2. Schematic representation of diffusion in hollow cylinder for cases: (a) A (b) B (c) C (d) D.

2.3. Decay transient state in a plate sheet

2.3.1. Case A

Consider the steady state for case A shown in Fig.1 (a) has been approached, and the concentration of steady state C/Co = 1-x/L as the initial condition. The boundary condition is

$$t \le 0,\quad 0 < x < L, C/C_0 = 1 - x/L$$

$$t > 0,\quad x = 0, C = 0$$

$$t > 0,\quad x = L, C = 0 \qquad (19)$$

Use the method of separation of variables to Eq. (1) with the initial and boundary conditions of Eq. (19) to obtain

$$\frac{C}{C_0} = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \cdot Sin(n\pi X) \cdot \exp(-n^2 \pi^2 \tau) \qquad (20)$$

Where

$$X = \frac{x}{L} \quad \tau = \frac{Dt}{L^2}$$

2.3.2. Case B

Consider the steady state for case B shown in Fig.1 (b) has been approached, and the concentration of steady state C/Co = x/L as the initial condition. The boundary condition is

$$t \le 0,\quad 0 < x < L, C/C_0 = x/L$$

$$t > 0,\quad x = 0, C = 0$$

$$t > 0,\quad x = L, C = 0 \qquad (21)$$

Use the method of separation of variables to Eq. (1) with the initial and boundary conditions of Eq. (21) to obtain

$$\frac{C}{C_0} = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \cdot Sin(n\pi X) \cdot \exp(-n^2 \pi^2 \tau) \qquad (22)$$

Where

$$X = \frac{x}{L} \quad \tau = \frac{Dt}{L^2}$$

2.3.3. Cases C

Consider the steady state for cases C and D shown in Fig.1 (c)-(d) have been approached, and the concentration of steady state as the initial conditions. The boundary condition is

$$t \le 0,\quad 0 < x < L, C = C_0$$

$$t > 0,\quad x = 0, C = 0$$

$$t > 0,\quad x = L, C = 0 \qquad (23)$$

Use the method of separation of variables to Eq. (1) with the initial and boundary conditions of Eq. (23) to obtain

$$\frac{C}{C_0} = \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} \cdot Sin[(2n-1)\pi X] \cdot \exp[-(2n-1)^2 \pi^2 \tau] \qquad (24)$$

Where

$$X = \frac{x}{L} \quad \tau = \frac{Dt}{L^2}$$

2.4. Decay transient state in a hollow cylinder

2.4.1. Case A

Consider the steady state for case A shown in Fig.2 (a) has been approached, and the concentration of steady state as the initial condition. The boundary condition is

$$t \le 0, a < r < b, C = C_0 \frac{\ln(r/a)}{\ln(b/a)}$$

$$t > 0, \quad r = a, C = 0$$

$$t > 0, \quad r = b, C = 0 \qquad (25)$$

The solution to this case has been derived and discussed in a previous paper [11] as

$$\frac{C}{C_0} = \pi \sum_{n=1}^{\infty} \frac{J_0^2(a\alpha_n) U_0(a\alpha_n r_A^{**})}{J_0^2(a\alpha_n) - J_0^2(ka\alpha_n)} \exp\left[-(a\alpha_n)^2 \tau\right] \qquad (26)$$

Where

$$U_0(a\alpha_n r_A^{**}) = J_0(a\alpha_n r_A^{**}) Y_0(ka\alpha_n) - Y_0(a\alpha_n r_A^{**}) J_0(ka\alpha_n)$$

$$r_A^{**} = \frac{r}{a} \quad k = \frac{b}{a} \quad \tau = \frac{Dt}{a^2}$$

2.4.2. Case B

Consider the steady state for case B shown in Fig.2 (b) has been approached, and the concentration of steady state as the initial condition. The boundary condition is

$$t \le 0, a < r < b, C = C_0 \frac{\ln(b/r)}{\ln(b/a)}$$

$$t > 0, \quad r = a, C = 0$$

$$t > 0, \quad r = b, C = 0 \qquad (27)$$

The solution to this case has been derived and discussed in a previous paper [11] as

$$\frac{C}{C_0} = -\pi \sum_{n=1}^{\infty} \frac{J_0(a\alpha_n) J_0(ka\alpha_n) U_0(ka\alpha_n r_B^{**})}{J_0^2(a\alpha_n) - J_0^2(ka\alpha_n)} \exp\left[-(a\alpha_n)^2 \tau\right] \qquad (28)$$

Where

$$U_0(ka\alpha_n r_B^{**}) = J_0(ka\alpha_n r_B^{**}) Y_0(ka\alpha_n) - Y_0(ka\alpha_n r_B^{**}) J_0(ka\alpha_n)$$

And

$$r_B^{**} = \frac{r}{b} \quad k = \frac{b}{a} \quad \tau = \frac{Dt}{a^2}$$

2.4.3. Case C

Consider the steady state for cases C and D shown in Fig.2 (c)-(d) have been approached, and the concentration of steady state C=Co as the initial conditions. The boundary condition is

$$t \le 0, a < r < b, C = C_0$$

$$t > 0, \quad r = a, C = 0$$

$$t > 0, \quad r = b, C = 0$$

(29)

This is similar to the solution of the temperature distribution solved by Carslaw and Jaegar [12], the solution to this case has been derived as

$$\frac{C}{C_0} = \pi \sum_{n=1}^{\infty} \frac{J_0(a\alpha_n) U_0(a\alpha_n r_C^{**})}{J_0(a\alpha_n) + J_0(ka\alpha_n)} \exp\left[-(a\alpha_n)^2 \tau\right] \quad (30)$$

Where

$$U_0(a\alpha_n r_C^{**}) = J_0(a\alpha_n r_C^{**}) Y_0(ka\alpha_n) - Y_0(a\alpha_n r_C^{**}) J_0(ka\alpha_n)$$

And

$$r_C^{**} = \frac{r}{a} \quad K = \frac{b}{a} \quad \tau = \frac{Dt}{a^2}$$

III. RESULTS AND DISCUSSION

3.1. Set up the transient in a plane sheet

For cases A, B, C and D, the concentration distribution profiles of diffusion in the plane sheet have been plotted according to Eqs. (3), (5), (7), and (9), as shown in Fig. 3 (a) - (d), respectively. For cases A and B, the concentration profiles are transferred from nonlinear to linear as is increased. However, the concentration profiles are concave for case A but convex for case B, respectively.

For case C, the concentration profiles are symmetrical about the central plane as τ is increased. However, the concentration profiles are unsymmetrical in less τ , as shown in Fig. 3(c). For case D, the concentration profiles are transferred from nonlinear to linear as τ is increased, as shown in Fig. 3(d).

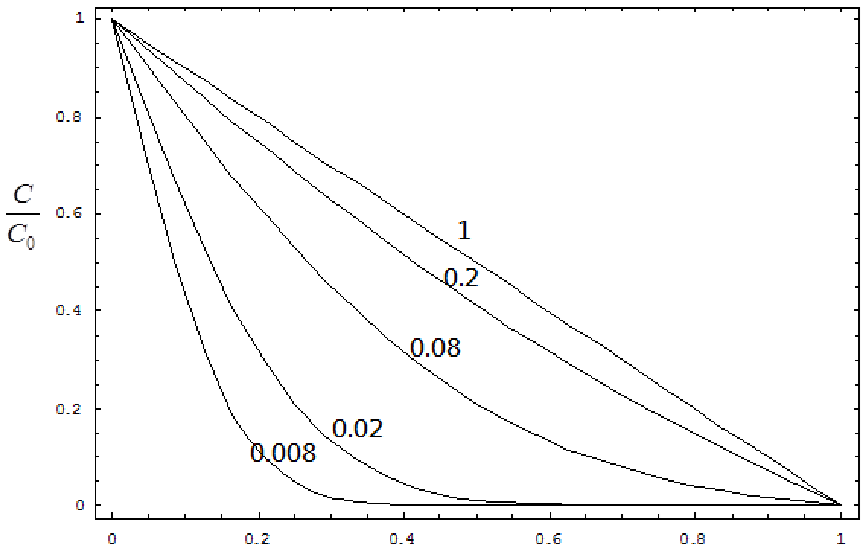

Fig.3 (a)

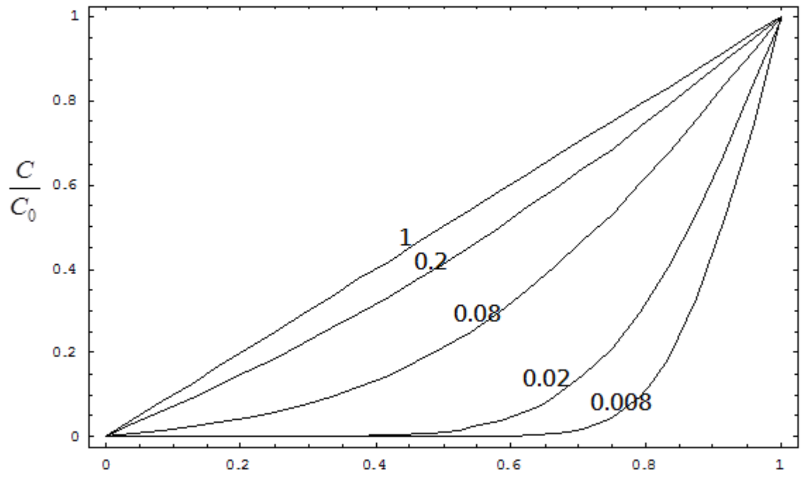

Fig.3 (b)

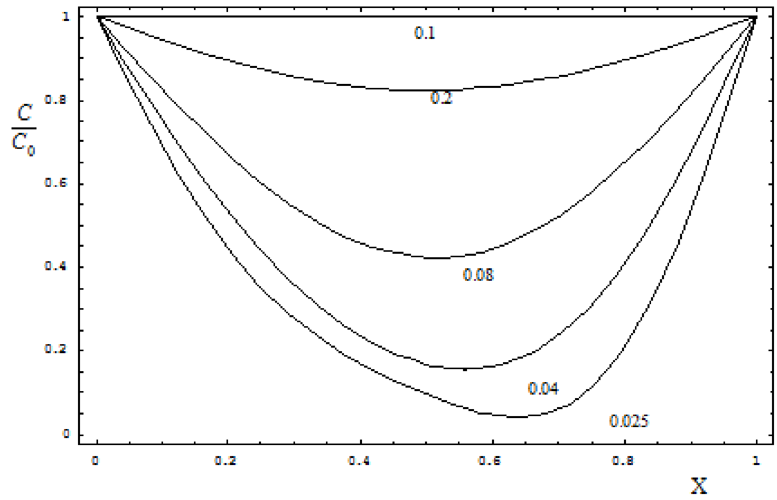

Fig.3 (c)

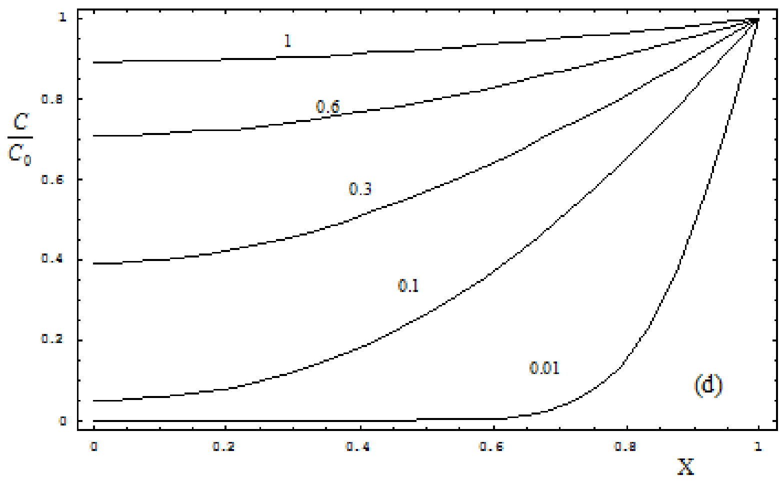

Fig.3 (d)

Fig. 3. Transient state concentration distribution through plate sheet for cases: (a) A (b) B (c) C (d) D. Numbers on curves are values of τ.

3.2. Set up transient in a hollow cylinder

For cases A, B, C and D, the concentration distribution profiles of diffusion in hollow cylinder have been plotted according to Eqs. (12), (14), (16), and (18), as shown in Fig. 4 (a) - (d), respectively. For cases A and B, the concentration profiles are transferred from nonlinear to linear as τ is increased. However, the concentration profiles are concave for case A but convex for case B, respectively.

For case C, the concentration profiles are symmetrical about the central plane during the diffusion periods, as shown in Fig. 4(c). For case D, the concentration profiles are transferred from nonlinear to linear as τ is increased, as shown in Fig. 4(d).

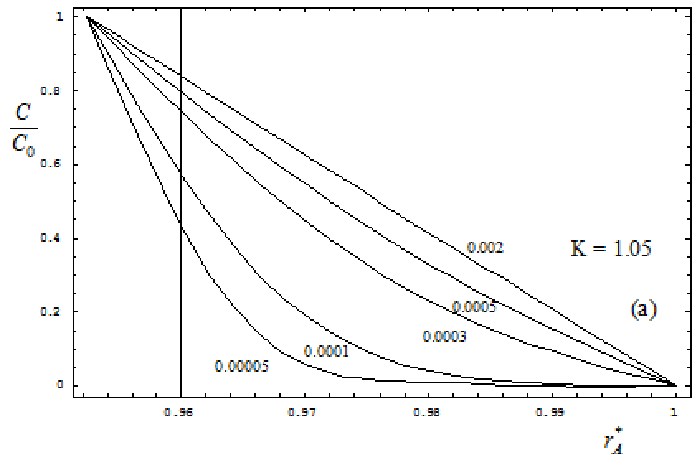

Fig.4 (a)

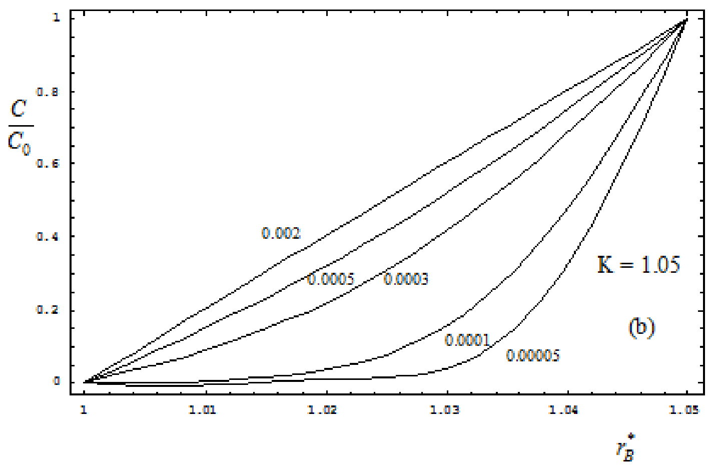

Fig.4 (b)

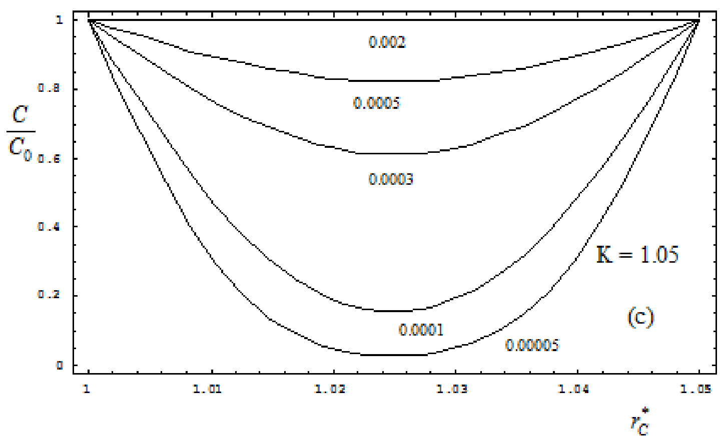

Fig.4 (c)

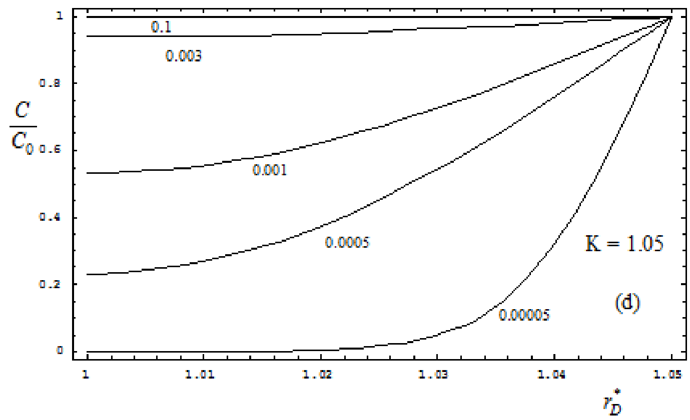

Fig.4 (d)

Fig. 4. Transient state concentration distribution through cylinder wall for cases: (a) A (b) B (c) C (d) D. Numbers on curves are values of τ

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3.3. Decay transient in a plane sheet

For cases A, B and C, the concentration distribution of C/Co calculated by Eqs. (20), (22), and (24) are plotted against X_A ,X_B and X_C at various τ, as shown in Figs. 5(a)-(c), respectively. The concentration profile decreases with increasing τ . The concentration profiles are unsymmetrical for cases A and B. However, case C is symmetrical. The maximum concentration is moving from the inner to the center for case A and from the outer to the center for case B, and then becoming symmetrical as τ increases, as shown in Figs. 5 (a) and (b), respectively.

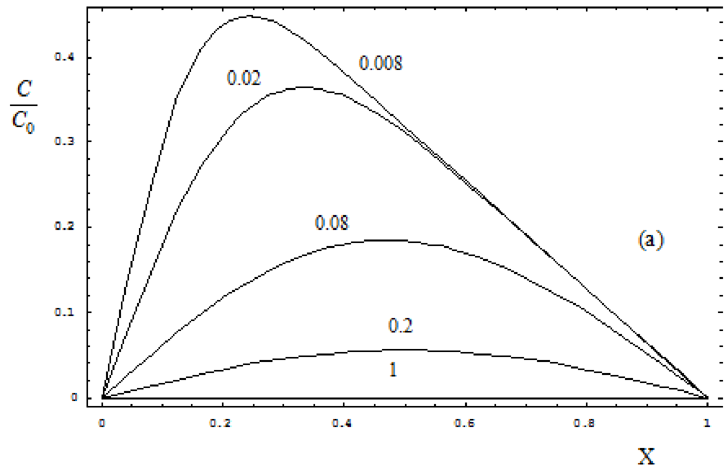

Fig.5 (a)

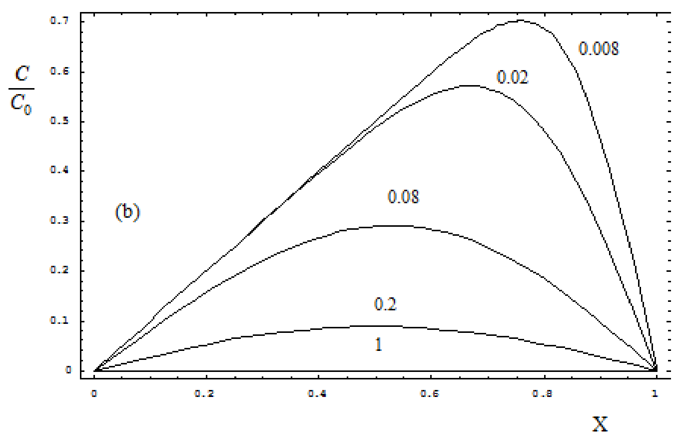

Fig.5 (b)

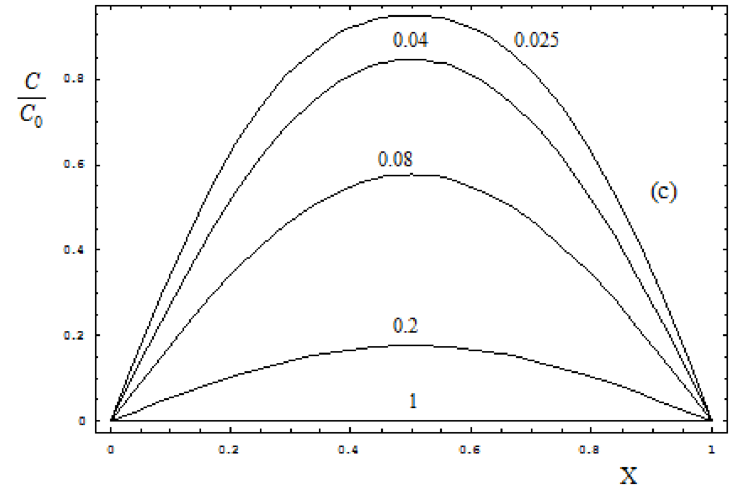

Fig.5 (c)

Fig. 5. Decay transient state concentration distribution through plate sheet for cases: (a) A (b) B (c) C. Numbers on curves are values of τ .

3.4. Decay transient in a hollow cylinder

For cases A, B and C, the concentration distribution of C/Co calculated by Eqs. (26), (28) and (30) are plotted against r_A^{**}, r_B^{**} and r_C^{**} at various τ for K = 1.05, as shown in Figs. 6(a)-(c), respectively. The concentration profile was decreased with increasing τ. The concentration profiles are unsymmetrical for cases A and B. However, case C is symmetrical. The maximum concentration is moving from the outer to the center for case A and from the inner to the center for case B, and then becoming symmetrical as τ increases, as shown in Figs. 6 (a) and (b), respectively.

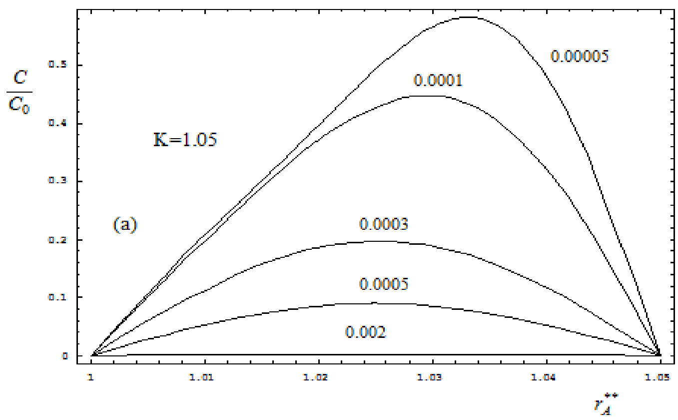

Fig.6 (a)

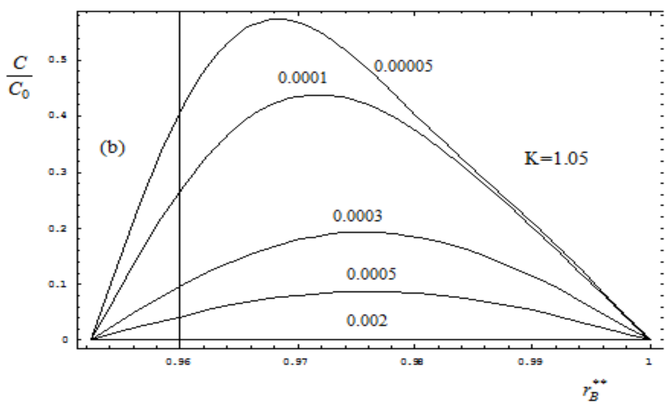

Fig.6 (b)

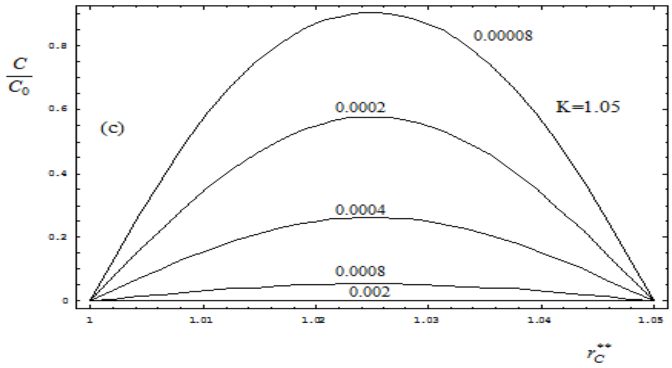

Fig.6 (c)

Fig. 6. Decay transient state concentration distribution through cylinder wall for cases: (a) A (b) B (c) C. Numbers on curves are values of τ .

3.5. Comparison of Set up transient in a plane sheet and hollow cylinder

The concentration profiles for the plane sheet and thin hollow cylinder are shown in Figs. 3(a)-(d) and 4(a)-(d), respectively. The concentration profiles show the same behavior. However, the diffusion in hollow cylinder is faster than the plane sheet to reach the steady state due to the geometry shape effect. The concentration profiles of case C are unsymmetrical in less τ for plane sheet. However, the concentration profiles of the hollow cylinder are symmetrical about the central plane during the diffusion periods.

3.6. Comparison of Decay transient in a plane sheet and hollow cylinder

The concentration profiles for the plane sheet and thin hollow cylinder are shown in Figs. 5(a)-(c) and 6(a)-(c), respectively. Comparing the decreasing rate of the concentration profile, the hollow cylinder is faster than the plane sheet to reach the steady state due to the geometry shape effect. The concentration profile of case C is unsymmetrical in less τ for the plane sheet. However, the concentration profiles of the hollow cylinder are symmetrical about the central plane.

IV. CONCLUSIONS

The mathematical solutions of diffusion in a plane sheet and thin hollow cylinder (k = 1.05) for the set up transient and decay transient states are given in Eqs. (3), (5), (7), (9), (12), (14), (16), (18), (20), (22), (24), (26), (28), and (30), respectively. Through the mathematical analysis and figure plotting, a few conclusions are drawn:

1. In the transient state, the concentration profiles show the same behavior for the plane sheet and thin hollow cylinder. However, diffusion in the hollow cylinder is faster than that of the plane sheet to reach the steady state due to the geometry shape effect.
2. In the decay transient state, the concentration profiles show the same behavior for the plane sheet and thin hollow cyl inder. Comparing the decreasing rate of the concentration profile, the hollow cylinder is faster than the plane sheet to reach the steady state due to the geometry shape effect.

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