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Modal Based Analysis and Evaluation of Voltage Stability of Bulk Power System

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Abstract:- This paper reports on Modal based analysis and its application in the evaluation of voltage stability of bulk power system. This method makes use of the power system Jacobian matrix to determine the eigenvalues necessary for the evaluation of the voltage stability of the power system. It identifies if the eigenvalues are positive or negative which is an indicator of the system stability. For a steady state power system, a positive eigenvalue shows that the system is stable while a negative eigenvalue indicates that the system is instable. The least eigenvalue indicates the proximity of the system to voltage instability. This method was used to determine the components of the system that contribute to instability through the use of the participating factors. The method was implemented on IEEE 14 bus system and it calculated the various eigenvalues with the least value used to calculate the participation factors that indicated the generator, branches and buses that will contribute most to the bulk system voltage instability.

Keywords:- Modal, voltage stability, eigenvalue, participation factors.

I. INTRODUCTION

Voltage stability issues are of major concern worldwide because of the significant number of black-outs that have occurred in recent times in which it was involved [1]. For many power systems, assessment of voltage stability and prediction of voltage instability or collapse have become the most important types of analysis performed as part of system planning, operational planning and real-time operation. It is important to have an analytical method to investigate voltage stability in the power system, particularly with a complex and large interconnected network. The work presented in this paper used modal analysis to evaluate the voltage stability of a bulk power system It involves the calculation of a small number of eigenvalues of the reduced Jacobian matrix obtained from the load flow solution. In the past, the electric utility industry has largely depended on conventional power-flow programs for static analysis of voltage stability. Stability is determined by computing the V-P and V-Q curves at selected load buses. Generally, such curves are generated by executing a large number of power flows using conventional models. While such procedure can be automated, they are time –consuming and do not readily provide information useful in gaining insight into causes of stability problems. In addition, these procedures focus on individual buses, that is, the stability characteristics are established by stressing each bus independently. This may unrealistically distort the stability condition of the system. Also, the buses selected for V-Q and V-P analysis must be chosen carefully, and a large number of such curves may be required to obtain complete information. In fact, it may not be possible to generate the V-Q curves completely due to power-flow divergence caused by problems elsewhere in the system.

A number of special techniques have been proposed in the literature for voltage stability analysis [2, 3, 4]. Generally, many of these have not found widespread practical application. A practical approaches based modal analysis has advantage that it gives voltage stability related information from a system wide perspective and clearly identifies areas that have potential problems. It has the added advantage that it provides information regarding the mechanism of instability. This is the principal reason for considering the modal analysis. The understanding of modal analysis starts with the formulation of power flow problem.

II. POWER FLOW PROBLEM

The power flow or load flow is widely used in power system analysis. It plays a major role in planning the future expansion of the power system as well as helping to run existing systems in the best possible way. The network load flow solution techniques are used for steady state and dynamic analysis programme [5, 6].

The solution of power flow predicts what the electrical state of the network will be when it is subjected to a specified loading condition. The result of the power flow is the voltage magnitude and the angle of each of the system nodes. These bus voltage magnitudes and angles are defined as the system state variables.

That is because they allow all other system quantities to be computed such as real and reactive power flows, current flows, voltage drops, power losses, etc … power flow solution is closely associated with voltage stability analysis. It is an essential tool for voltage stability evaluation. Much of the research on voltage stability deals with the power-flow computation method.
The power flow problem solves the complex matrix equation:

$$I = YV = \frac{S^{\bullet}}{V^{\bullet}} \qquad (1)$$

where
I = nodal current injection matrix
Y = system nodal admittance matrix
V = unknown complex node voltage vector
S = apparent power nodal injection vector representing specified load and generation of nodes
Where

$$S = P + jQ \qquad (2)$$

The Newton-Raphson method is the most general and reliable algorithm to solve the power-flow problem. It involves interactions based on successive linearization using the first term of Taylor expansion of the equation to be solved. From equation (1), we can write the equation for node k (bus k) as:

$$I_k = \sum_{m=1}^{n} Y_{km} V_m \qquad (3)$$

$$P_k + jQ_k = V_k I_k^{*} = V_k \sum_{m=1}^{n} Y_{km}^{*} V_m^{*} \qquad (4)$$

Where:
m = 1, 2, …, n
n = number of buses
V_k is the voltage of the k_{th} bus
Y_{km} is the element of the admittance bus
Equating the real and imaginary parts

$$P_k = R_e\left(V_k\left(\sum_{m=1}^{n} Y_{km}^{*} V_k^{*}\right)\right) \qquad (5)$$

$$Q_k = I_m\left(V_k\left(\sum_{m=1}^{n} Y_{km}^{*} V_k^{*}\right)\right) \qquad (6)$$

Where
P_k is the real power
Q_k is the reactive power
With the following notation:

$$V_k = |V_k| e^{j\theta_k}, V_m = |V_m| e^{j\theta_m}, Y_{km} = |Y_{km}| e^{j\theta_{km}} \qquad (7)$$

Where
$|V_k|$ is the magnitude of the voltage
δ_k is the angle of the voltage
δ_{km} is the load angle
Substituting for V_k, V_m, and Y_{km} in Equation (4)

$$P_k + jQ_k = |V_k| e^{j\delta_k} \sum_{m=1}^{n} |V_k| e^{-j\delta_k} |Y_{km}| e^{-j\theta_{km}}$$

Or

$$P_K + jQ_K = |V_k| \sum_{m=1}^{n} |V_k| |Y_{km}| e^{j(\delta_k - \delta_m - \delta_{km})}$$

Or

$$P_k + jQ_k = |V_k| \sum_{m=1}^{n} |V_k||Y_{km}| \angle(\delta_k - \delta_k - \theta_{km})$$

Or

$$P_k + jQ_K = |V_k| \sum_{m=1}^{n} |V_k||Y_{km}| (\cos(\delta_k - \delta_m - \theta_{km})$$
$$+ j\sin(\delta_k - \delta_m - \theta_{km}))$$

Separating the real and imaginary parts of above equation to get real and reactive powers,

$$P_k = |V_k| \sum_{m=1}^{n} |V_k||Y_{km}| \cos(\delta_k - \delta_m - \theta_{km}) \quad (8)$$

$$Q_k = |V_k| \sum_{m=1}^{n} |V_k||Y_{km}| \sin(\delta_k - \delta_m - \theta_{km}) \quad (9)$$

The mismatch power at bus k is given by:

$$\Delta P_k = p_k^{sch} - p_k \quad (10)$$

$$\Delta Q_k = Q_k^{sch} - Q_k \quad (11)$$

The P_k and Q_k are calculated from Equation (8) and (9)

The Newton – Raphson method solves the partitioned matrix equation:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = J \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} \quad (12)$$

Where,

ΔP and ΔQ = mismatch active and reactive power vectors.

ΔV and ΔΘ = unknown voltage magnitude and angle correction vectors.

J = Jacobian matrix of partial derivative terms

III. MODAL ANALYSIS

The Modal analysis mainly depends on the power-flow Jacobian matrix of equation (12). Gao, Morison and Kundur [8] proposed this method in 1992. It can predict voltage collapse in complex power system networks. It involves mainly the computing of the smallest eigenvalues and associated eigenvectors of the reduced Jacobian matrix obtained from the load flow solution. The eigenvalues are associated with a mode of voltage and reactive power variation which can provide a relative measure of proximity to voltage instability. Then, the participation factor can be used effectively to find out the weakest nodes or buses in the system. The analysis is expressed as follows:

Equation (12) can be rewritten as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} \quad (13)$$

By letting $\Delta P = 0$ in Equation (13)

$$\Delta P = 0 = J_{11}\Delta\theta + J_{12}\Delta V,\ \Delta\theta = -J_{11}^{-1}J_{12}\Delta V \quad (14)$$ and

$$\Delta Q = J_{21}\Delta\theta + J_{22}\Delta V \quad (15)$$

Substituting Equation (14) in Equation (15):

$$\Delta Q = J_R \Delta V \quad (16)$$

Where

$$J_R = \lfloor J_{22} - J_{21}J_{11}^{-1}J_{12} \rfloor$$

J_R is the reduced Jacobian matrix of the system.

Equation (16) can be written as

$$\Delta V = J_R^{-1}\Delta Q \quad (17)$$

The matrix J_R represents the linearized relationship between the incremental changes in bus voltage (ΔV) and bus reactive power injection (ΔQ). It's well known that, the system voltage is affected by both real and reactive power variations. In order to focus the study of the reactive demand and supply problem of the system

as well as minimize computational effort by reducing dimensions of the Jacobian matrix J the real power (ΔP = o) and angle part from the system in Equation (13) are eliminated.
The eigenvalues and eigenvectors of the reduced order Jacobian matrix J_R are used for the voltage stability characteristics analysis. Voltage instability can be detected by identifying modes of the eigenvalues matrix J_R. The magnitude of the eigenvalues provides a relative measure of proximity to instability. The eigenvectors on the other hand present information related to the mechanism of loss of voltage stability.
Modal analysis of J_R results in the following.

$$J_R = \lambda \Phi \xi \quad (18)$$

where Φ = right eigenvector matrix of J_R

ξ = left eigenvector matrix of J_R

λ = diagonal eigenvalue matrix of J_R

Equation (18) can be written as:

$$J_R^{-1} = \Phi \lambda^{-1} \xi \quad (19)$$

In general it can be said that, a system is voltage stable if the eigenvalues of J_R are all positive. This is different from dynamic systems where eigenvalues with negative real parts are stable. The relationship between system voltage stability and eigenvalues of the J_R matrix is best understood by relating the eigenvalues with the V-Q. sensitivities of each bus (which must be positive for stability). J_R can be taken as a symmetric matrix and therefore the eigenvalues of J_R are close to being purely real. If all the eigenvalues are positive, J_R is positive definite and the V-Q sensitivities are also positive, indicating that the system is voltage stable.
The system is considered voltage unstable if at least one of the eigenvalues is negative. A zero eigenvalue of J_R means that the system is on the verge of voltage instability. Furthermore, small eigenvalue of J_R determine the proximity of the system to being voltage unstable [9].

There is no need to evaluate all the eigenvalues of J_R of a large power system because it is known that once the minimum eigenvalues becomes zeros the system Jacobian matrix becomes singular and voltage instability occurs. So the eigenvalues of importance are the critical eigenvalues of the reduced Jacobian matrix J_R. Thus, the smallest eigenvalues of J_R are taken to be the least stable modes of the system. The rest of the eigenvalues are neglected because they are considered to be strong enough modes. Once the minimum eigenvalues and the corresponding left and right eigenvectors have been calculated the participation factor can be used to identify the weakest node or bus in the system. An algorithm for the modal method analysis used in this study is shown in figure1.

The appropriate definition and determination as to which node or load bus participates in the selected modes become very important. This necessitates a tool, called the participation factor, for identifying the weakest nodes or load buses that are making significant contribution to the selected modes [10].

$$\Delta V = \sum_i \frac{\Phi_i \xi_i}{\lambda_i} \Delta Q \quad (20)$$

where λ_i is the íth eigenvalue, Φ_i is the íth column right eigenvector and ξ_i is the íth row left eigenvector of matrix
J_R. Each eigenvalue λ_i and corresponding right and left eigenvectors Φ_i and ξ_i , defined the íth mode of the system.

IV. IDENTIFICATION OF THE WEAK LOAD BUSES

The minimum eigenvalues, which become close to instability, need to be observed more closely. parts are stable. The relationship between system voltage stability and eigenvalues of the J_R matrix is best understood by relating the eigenvalues with the V-Q. sensitivities of each bus (which must be positive for stability). J_R can be taken as a symmetric matrix and therefore the eigenvalues of J_R are close to being purely real. If all the eigenvalues are positive, J_R is positive definite and the V-Q sensitivities

V. SAMPLE SYSTEM EVALUATION AND RESULTS ANALYSIS

The modal analysis method has been successfully applied to IEEE 14 bus power system shown in Figure 2, while Tables 1 and 2 are the line and load parameters respectively. A power flow program based on Matlab is developed to:

1. Calculate the power flow solution
2. Analyze the voltage stability based on modal analysis
3. Generate the Q-V sensitivities and Participation factors

The voltage profile of the buses is presented from the load flow simulation as shown in Figure 3. It can be seen that all the bus voltages are within the acceptable level (± 5%). The lowest voltage compared to the other buses can be noticed in bus number 3

Since there are 14 buses among which there is one swing bus and 4 PV buses, then the total number of eigenvalues of the reduced Jacobian matrix J_R is expected to be 9 as shown in Table 3.

Table 1 Line data for IEEE 14 Bus system

(Length 1km)

	ID	Name	R(1)	X(1)	C(1)	B(0)
			Ohm/…	Ohm/…	uF/…	Us/…
1.	421	Branch-3	2.237193	9.425351	2.92837	919.975
2.	424	Branch-2	2.572368	10.61893	3.289402	1033.396
3.	427	Branch-1	0.922681	2.817083	3.53009	1109.01
4.	430	Branch-7	0.635593	2.004857	0.855779	268.851
5.	433	Branch-6	3.190346	8.142738	2.313279	726.738
6.	436	Branch-5	2.711389	8.278426	2.273164	714.136
7.	439	Branch-4	2.766617	8.394595	2.50048	785.549
8.	442	Branch-13	0.125976	0.248086	0	0
9.	445	Branch-12	0.234069	0.487164	0	0
10.	448	Branch-11	0.180879	0.378785	0	0
11.	451	Branch-19	0.42072	0.380651	0	0
12.	454	Branch-18	0.156256	0.365778	0	0
13.	457	Branch-17	0.242068	0.514911	0	0
14.	460	Branch-16	0.060578	0.160921	0	0
15.	463	Branch-15	0	0.209503	0	0
16.	466	Branch-20	0.325519	0.662769	0	0

Table 2 Load Distribution for IEEE 14 Bus System

	ID	**Name**	**LF Type**	**P MW**	**Q MVAR**
1.	686	Load 2	PQ	21.7	12.7
2.	687	Load 6	PQ	11.2	7.5
3.	688	Load 5	PQ	7.6	1.6
4.	689	Load 4	PQ	47.8	4
5.	690	Load 3	PQ	94.2	19
6.	691	Load 11	PQ	3.5	1.8
7.	692	Load 10	PQ	9	5.8
8.	693	Load 9	PQ	29.5	16.6
9.	694	Load 7	PQ	0	0
10.	695	Load 14	PQ	14.9	5
11.	696	Load 13	PQ	13.5	5.8
12.	697	Load 12	PQ	6.1	1.6

Table 3 IEEE 14 Bus System Eigenvalues

	1	2	3	4
Eigenvalue	2.0792	5.3677	7.5987	9.4942
#	5	6	7	8
Eigenvalue	16.0985	18.8474	19.3553	38.5332
#	9			
Eigenvalue	64.9284			

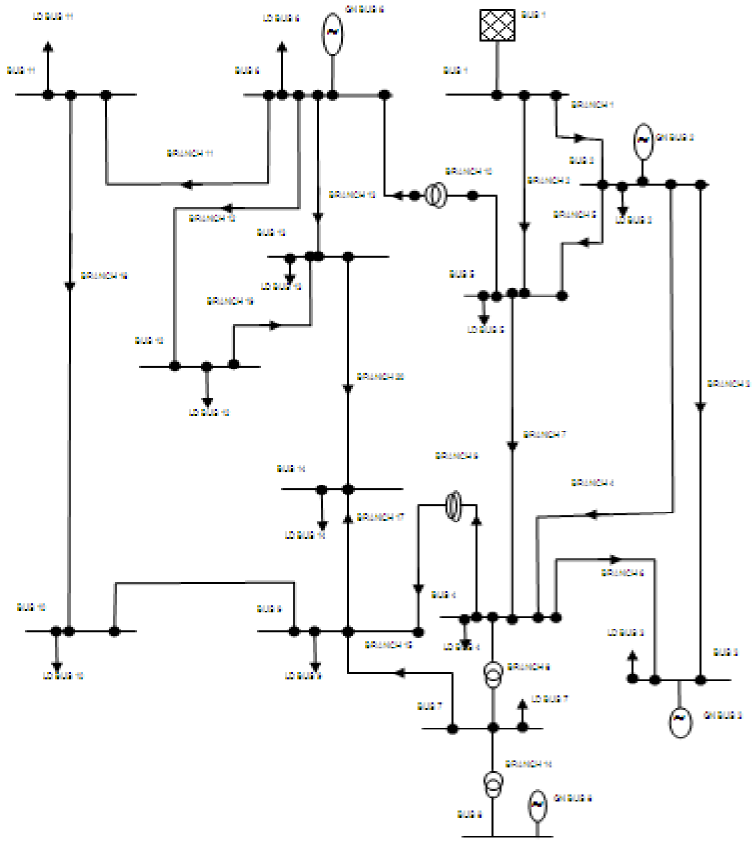

Figure 1.0 Line diagram of the IEEE 14 Bus interconnected network

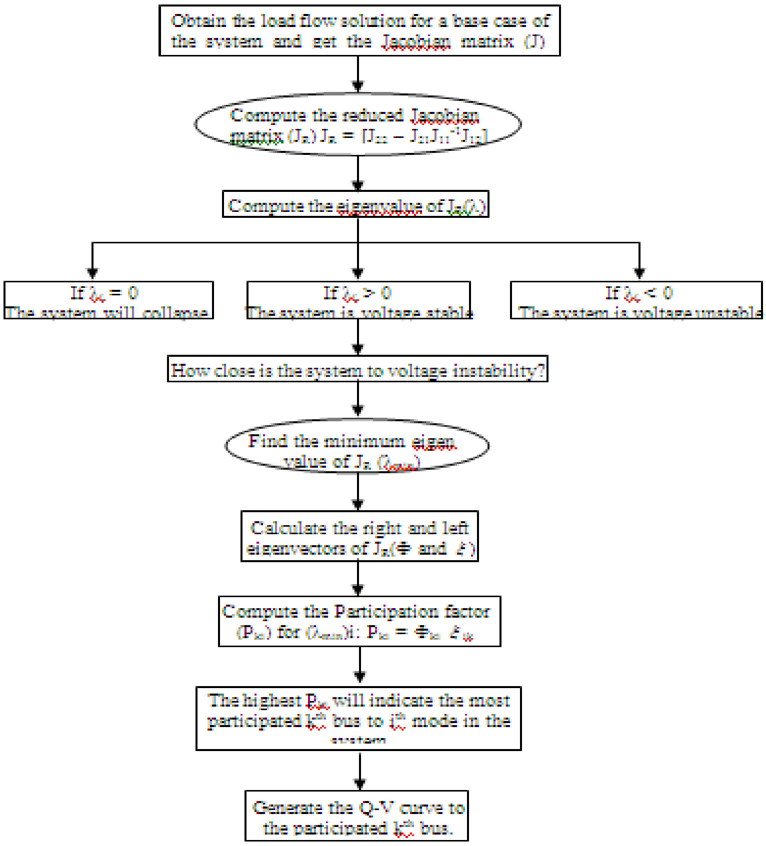

Figure 2.0 Algorithm for the voltage stability analysis

VI. CONCLUSION

The results of the analysis of the 14 Bus system are shown in the bar charts below.

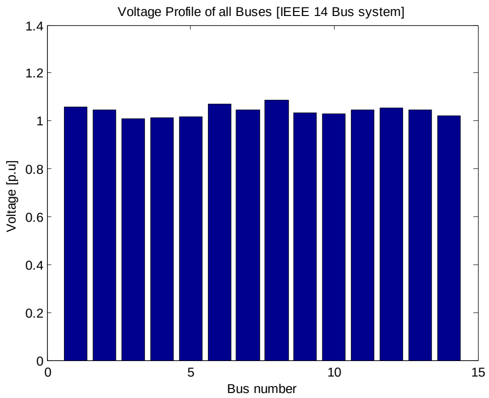

Figure 3.0 Voltage profiles of all Buses of the IEEE 14 Bus system

From Table 3, it can be notices that the minimum eigenvalue λ = 2.0792 is the most critical mode. The Q-V Sensitivity is calculated to determine the most critical bus. The simulation showed that bus 14 is the most critical with sensitivity of 0.222725 as shown in Figure 4.

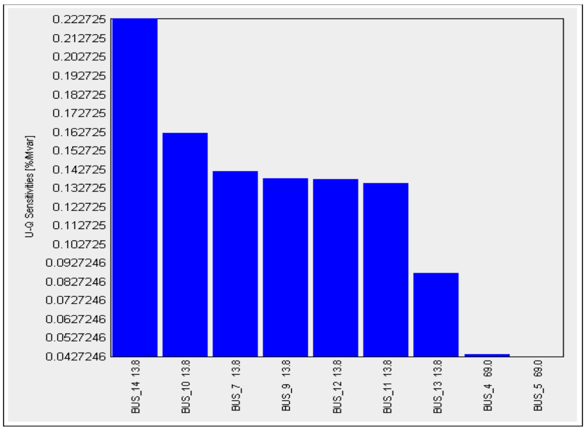

Figure 4.0 Q-V Sensitivity of IEEE 14 Bus system

The bus participation factor was calculated; the result showed that bus 14 had the highest bus participation factor of 0.232557 at the least eigenvalue λ = 2.0792 as shown in Fig. 5.

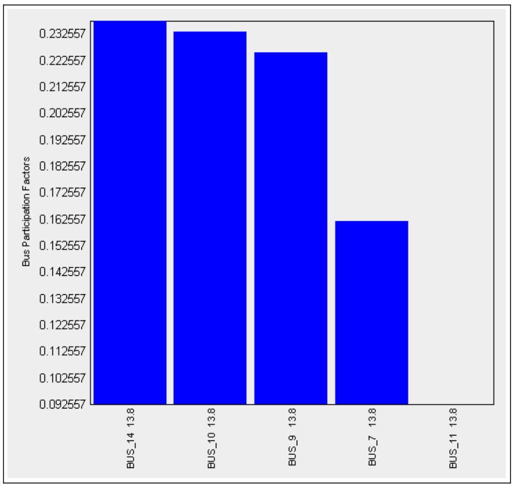

Figure 5.0 Highest contribution to voltage collapse

The branch participation factors was calculated, the result showed that branch 10 has the highest branch participation factor of 0.968457 at the least eigenvalue λ = 2.0792 as shown in Figure 6. It indicates the highest contribution of this branch to the voltage collapse.
The generator participation factors was calculated, the result showed that Gnbus 6 has the highest generator participation factor of 0.952139 at the least eigenvalue λ = 2.0792 as shown in Figure 7, it indicates the highest contribution of this generator to the voltage collapse.

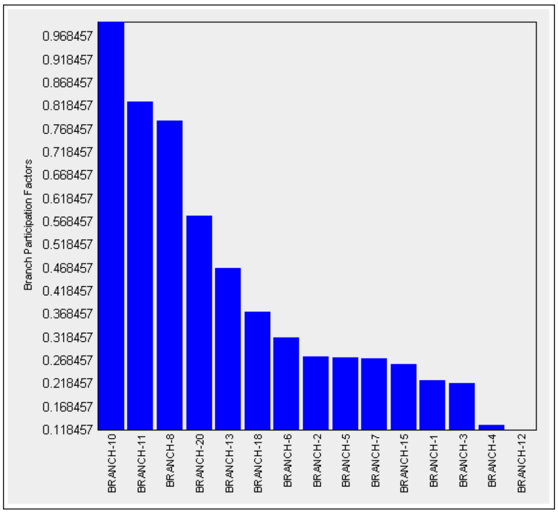

Figure 6 The branch Participation Factors for Least Eigenvalue λ = 2.0792

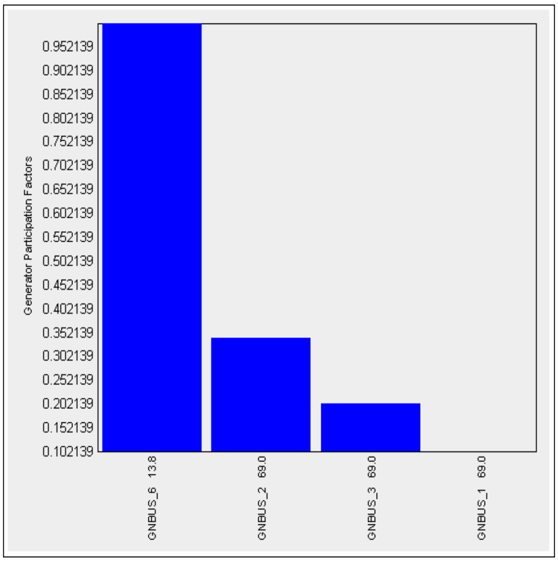

Figure 7 Generator Participation Factors at least Eigenvalue of λ = 2.0792

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