International Journal of Engineering Research and Development
e-ISSN: 2278-067X, **p**-ISSN: 2278-800X, www.ijerd.com
Volume 6, Issue 7 (April 2013), PP.92-95

New Stiffness Matrices for Stability and Dynamic Analyses of Line Continuum

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Abstract:- The critical buckling load and the natural frequency of frame structures are usually determined to avoid failures due to instability and resonance. Classical analyses are intractable and many analysts resort to numerical method of analysis using the traditional 4x4 matrix stiffness systems, which also have great limitation. This work presents new 5x5 stiffness matrices for classical and effective stability and dynamic analyses of line continua. Energy variational principle was employed in developing the matrices, with five term Mclaurin's polynomial series as the shape function. A central deflection node was considered, making a total of five deformable nodes. The new 5x5 matrices were derived by minimizing the geometric work and kinetic energy of the line continuum of five term shape function. They were employed, as well as the traditional 4x4 matrices, in classical stability and free vibration analyses of four line continua and a portal frame. The results from the new 5x5 matrices were very close to exact results, with average percentage differences of 2.55% for stability and 0.14% for free vibration, whereas those from the traditional 4x4 matrices differed greatly from exact results, with average percentage differences of 23.73% for stability and 14.72% for free vibration. Thus, the newly developed stiffness matrices are suitable for stability and dynamic analyses of line continua and should be used by structural engineering analysts accordingly.

Keywords:- 5x5 stiffness system; matrices of geometry and inertia; line continuum; variational principle; deformable node; shape function; geometric work; classical analysis; kinetic energy

I. INTRODUCTION

Stability analysis of multi-storey-building frames is a necessary step in determining the critical buckling load so as to avoid subjecting the frame to loads beyond critical values. It is an established fact that most buildings collapse due to instability of the column elements rather than foundation failures, excessive deflection, or exceeding flexural and axial stresses on the frame elements. It is also well known that vibrations induce inertia stresses which augment the static stresses, resulting in very high stresses in the frame. Failure occurs when the stress capacities of the frame elements are exceeded. However, the most dangerous phenomenon is resonance, which occurs when the frame vibrates at its natural frequency. Thus, it is important to always verify the critical buckling load and the natural frequency of the frame. Classical analyses that treat each element as a whole are somewhat intractable because of the great number of elements usually involved in multi-storey frames. Therefore, many analysts resort to numerical method of analysis using the traditional 4 x 4 matrix stiffness systems such as that given by Yoo and Lee (2011) as expressed in equation for stability analysis and that given by Chopra (1995) as expressed in equation 2 for dynamic analysis. Unfortunately, such numerical approachesare also tedious (Melosh, 1963; Long, 1978, 1992, 2009; Cook et al., 1989; Huebner et al., 1995; Bathe, 1996; Zienkiewicz and Taylor, 2000) and frequently give results that differ greatly with exact classical results. Moreover, as observed by Ibearugbulem et al. (2013), the traditional 4 x 4 stiffness matrix and its load vector cannot classically analyse flexural line continua except using them numerically (more than one element in one analysis). This difficulty in using the traditional classical approach is evident in the works of Iyengar (1988), Chopra (1995), and Yoo and Lee (2011). Hence, there is need for a classical matrix approach that would be less cumbersome and at the same time give results that are close to exact results. Ibearugbulem et al. (2013) developed a 5 x 5 stiffness matrix system capable of classically analysing bending of line continua of different boundary conditions, as shown in equation 3. This work presents new stiffness matrices for classical and effective stability and dynamic analyses of line continua. These5 x 5 matrices of geometry and inertia are based on the same principle of introducing a central nodal deflection in the line continuum used by Ibearugbulem et al. (2013).

$$Kg = \frac{P}{L}\begin{bmatrix} 1.2 & 0.1L & -1.2 & 0.1000L \\ 0.1L & 0.1333L^2 & -0.1L & -3.00L^2 \\ -1.2 & -0.1L & 1.2 & -0.1L \\ 0.1L & -3L^2 & -0.1L & 0.1333L^2 \end{bmatrix} \qquad (1)$$

$$Km = \frac{\omega^2 mL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 55 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \quad (2)$$

$$Ks = \begin{bmatrix} \frac{316EI}{5L^3} & \frac{94EI}{5L^2} & \frac{-512EI}{5L^3} & \frac{196EI}{5L^3} & \frac{-34EI}{5L^2} \\ \frac{94EI}{5L^2} & \frac{36EI}{5L} & \frac{-128EI}{5L^2} & \frac{34EI}{5L^2} & \frac{-6EI}{5L} \\ \frac{-512EI}{5L^3} & \frac{-128EI}{5L^2} & \frac{1024EI}{5L^3} & \frac{-512EI}{5L^3} & \frac{128EI}{5L^2} \\ \frac{196EI}{5L^3} & \frac{34EI}{5L^2} & \frac{-512EI}{5L^3} & \frac{316EI}{5L^3} & \frac{-94EI}{5L^2} \\ \frac{-34EI}{5L^2} & \frac{-6EI}{5L} & \frac{128EI}{5L^2} & \frac{-94EI}{5L^2} & \frac{36EI}{5L} \end{bmatrix} \quad (3)$$

II. NEW 5 X 5 MATRICES FOR STABILITY AND DYNAMIC ANALYSES

The shape function given by Ibearugbulem et al. (2013) is as shown in equation (4).

$$W(x) = a_0 + a_1X + a_2X^2 + a_3X^3 + a_4X^4 \quad (4)$$

Stability and vibration energy fuctionals given by El-Naschie(1990) are as shown in equations (5) and (6) respectively.

$$\text{Ug} = -\frac{P}{2}\int_0^L \frac{dw}{dx}dx \quad (5)$$

$$\text{Um} = -\frac{m\omega^2}{2}\int_0^L w^2\,dx \quad (6)$$

Substituting equation (4) in equations (5) and (6) and minimizingthem in variational principleresults in equations (7) and (8) respectively.

$$\frac{\text{dUg}}{d\Delta} = [Kg].[\Delta] \quad (7)$$

$$\frac{\text{dUm}}{d\Delta} = [Km].[\Delta] \quad (8)$$

WhereΔ *is the nodal deformation vector expressed in equation* (9).

$$[\Delta]^T = [w_1 \quad \theta_1 \quad w_2 \quad w_3 \quad \theta_3] \quad (9)$$

Kg and Km are the required new 5x5 stiffness matrices for stability and dynamic (vibration) analyses of line continua as expressed in equations (10) and (11) respectively.

$$\text{Kg} = \begin{vmatrix} \frac{2.419048P}{L} & 0.138095P & \frac{-2.4381P}{L} & \frac{0.019048P}{L} & 0.061905P \\ 0.138095P & 0.07619PL & -0.07619P & -0.0619P & 0.02381PL \\ \frac{-2.4381P}{L} & -0.07619P & \frac{4.87619P}{L} & \frac{-2.4381P}{L} & 0.07619P \\ \frac{0.019048P}{L} & -0.0619P & \frac{-2.4381P}{L} & \frac{2.419048P}{L} & -0.1381P \\ 0.061905P & 0.02381PL & 0.07619P & -0.1381P & 0.07619PL \end{vmatrix} \quad (10)$$

$$\text{Km} = \omega^2 mL \begin{vmatrix} 0.206349 & 0.015873L & 0.063492 & -0.03651 & 0.005556L \\ 0.015873L & 0.001587L^2 & 0.006349L & -0.00556L & 0.000794L^2 \\ 0.063492 & 0.006349L & 0.406349 & 0.063492 & -0.00635L \\ -0.03651 & -0.00556L & 0.063492 & 0.206349 & -0.01587L \\ 0.005556L & 0.000794L^2 & -0.00635L & -0.01587L & 0.001587L^2 \end{vmatrix} \quad (11)$$

III. CLASSICAL APPLICATION TO STABILITY AND FREE VIBRATION ANALYSES

1. Four line continua with the following boundary conditions were analysed for critical buckling loads and fundamental natural frequencies using the new 5 x 5 and the traditional 4 x 4 stiffness systems:
   i. P – R line continuum: one end is pinned and the other end is on roller.

ii. C – C line continuum: both ends are clamped.
iii. C – R line continuum: one end is clamped and the other end is on roller.
iv. C – F line continuum: one end is clamped and the other end is free.

2. The portal frame shown in figure 1 was also analysed for critical buckling loads and fundamental natural frequencies using the new 5 x 5 and the traditional 4 x 4 stiffness systems.

IV. RESULTS AND DISCUSSIONS

The results of the stability and free vibration analyses of line continua of four different boundary conditions are presented in tables 1 and 2, together with results from classical exact solutions.The results of the stability and free vibration analyses of the portal frame are presented in table 3. It can be seen from tables 1 and 2 that the results from the traditional 4 x 4 stiffness system differ very much from the exact results, except for C – F line continuum, with average percentage differences of 23.73% for stability and 14.72% for free vibration.The highest differencesare 48.59% for stability and 32.94% for free vibrationwith C – R line continuum. It was not possible to analyse C – C line continuum with the traditional 4 x 4 stiffness system. These results clearly show that the traditional 4 x 4 stiffness system is not suitable for multi storey frame analysis.

On the other hand, the results for the new 5 x 5 stiffness systemare very close to the exact results, the highest percentage differences being 6.46% for stability and 0.36 % for free vibration, with average percentage differences of 2.55% for stability and 0.14 % for free vibration.Thus, the new 5 x 5 stiffness system would be suitable for multi storey frame analysis.

It can be seen from table 3that the resultsobtained using the traditional 4 x 4 stiffness system differvery much from those obtained using the new 5 x 5 stiffness system for classical analysis of portal frame, the percentage differences being 73.48% for stability and 50.10% for free vibration. This wide difference could be due to the fact that, as previously observed, the traditional 4 x 4 stiffness system is not suitable for classical analysis but could be good for use in numerical analysis.

It can easily be concluded that the newly developed stiffness matrices are suitable for stability and dynamic analyses of line continua and should be used by structural engineering analysts accordingly.

Table 1: Critical buckling load of the continuum, Pcr (EI/L^2) from classical analysis

Continua Boundary Conditions	Exact Result (Pcr)	Result from 4 X 4 stiffness system (Pcr)	Percentage Difference With Exact Result	Result from 5 X 5 stiffness system (Pcr)	Percentage Difference With Exact Result
P - R BEAM	9.87	12.02	21.78318136	9.88	0.10
C - C BEAM	39.45	Impossible	--	42	6.46
C -R BEAM	20.19	30	48.5884101	20.92	3.62
C - F BEAM	2.47	2.49	0.809716599	2.47	0.00
Average % difference			**23.72710269**		**2.55**

Table2: Fundamental natural frequency, $\omega\left(\sqrt{\frac{EI}{mL^4}}\right)$ from classical analysis

Continua Boundary Conditions	Exact Result (ω)	Result from 4 X 4 stiffness system (ω)	Percentage Difference With Exact Result	Result from 5 X 5 stiffness system (ω)	Percentage Difference With Exact Result
P - R BEAM	9.87	10.95	10.94224924	9.87	0.00
C - C BEAM	22.37	Impossible	--	22.45	0.36
C -R BEAM	15.42	20.5	32.94422827	15.45	0.19
C - F BEAM	3.52	3.53	0.284090909	3.52	0.00
AVERAGE % DIFFERENCE			**14.72352281**	(EI)	**0.14**

Table 3: Result for Portal frame Analysis

Analysis	5 x 5 Stiffnes system	4 x 4 Stiffness system	Percentage difference with 5 x 5 Stiffness system
Critical buckling load, Pcr $\left(\frac{EI}{L^2}\right)$	2.704	0.7172	73.48
Fundamental natural frequency, ω $\left(\sqrt{\frac{EI}{mL^4}}\right)$	4.8555	2.4228	50.10

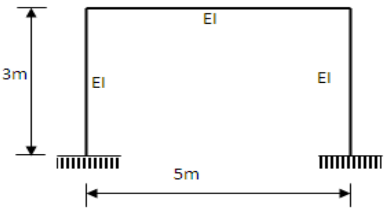

Figure 1: Portal frame for stability and free vibration analysis

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