International Journal of Engineering Research and Development
e-ISSN: 2278-067X, p-ISSN: 2278-800X, www.ijerd.com
Volume 9, Issue 7 (January 2014), PP. 40-49

Solution of Fuzzy Maximal Flow Network Problem Based on Generalized Trapezoidal Fuzzy Numbers with Rank and Mode

M. K. Alam[1], M. K. Hasan[2]
[1]Lecturer in Mathematics, Department of Mathematics and Statistics, Bangladesh University of Business and Technology, Dhaka, Bangladesh.
[2]Assistant Professor of Mathematics, Department of Mathematics and Statistics, Bangladesh University of Business and Technology, Dhaka, Bangladesh.

Abstract:- Network-flow problems can be solved by several methods. Labeling techniques can be used to solve wide variety of network problems. A new algorithm to find the fuzzy maximal flow between source and sink was proposed by Kumar et el. [19]. They have represented normal triangular fuzzy numbers as network flow. It is not possible to restrict the membership function to the normal form and proposed the concept of generalized fuzzy numbers in many cases [8]. Generalized trapezoidal fuzzy numbers for solving the maximal flow network problems have been used by Kumar [21]. In this paper, we have modified the existing algorithm to find fuzzy maximal network flow between source and sink for generalized trapezoidal fuzzy number. Ranking and mode function to find the highest flow for maximum flow path of generalized trapezoidal fuzzy number has been applied. A numerical example has been solved by the proposed algorithm and the other results are discussed. Mathematica programs have been applied for various arithmetic operations.

Keywords:- Mode and Ranking function, Normal Trapezoidal Fuzzy Numbers, Generalized Trapezoidal Fuzzy Numbers, Fuzzy Maximal Flow Problem, Fuzzy Residue.

I. INTRODUCTION

In 1965 Zadeh [30] introduced the concept of fuzzy set theory. Fuzzy set can provided solution to vast range of scientific problems. When the estimation of a system coefficient is imprecise and only some vague knowledge about the actual value of the parameters is available, it may be convenient to represent some or all of them with fuzzy numbers [30]. Fuzzy numbers are fuzzy subsets of the set of real numbers satisfying some additional conditions. Arithmetic operations on fuzzy numbers have also been developed according to the extension principle based on interval arithmetic [24]. Fuzzy numbers allow us to make the mathematical model of logistic variable or fuzzy environment. Opposite and reverse fuzzy numbers have not been shown by Dubois [13] and Yager [29] on the sense of group structure. Some of the interesting arithmetic works on fuzzy numbers are discussed by Dubois [14]. Arithmetic behavior of trapezoidal fuzzy numbers is not widely discussed in the literature. The aims of this paper to stimulate the inclusion of trapezoidal fuzzy numbers in applied engineering and scientific problems by extending the concept of traditional algebra into fuzzy set theory, which is described by Bansal [3]. A method for ranking of generalized trapezoidal fuzzy numbers is studied by Chen [9]. Abbasbandy [1] has introduced a new approach for ranking of trapezoidal fuzzy numbers based on the rank, mode, left and right spreads at some-levels of trapezoidal fuzzy numbers.

The maximum flow problem is one of basic problems for combinatorial optimization in weighted directed graphs. In the real life situation very useful models in a number of practical contexts including communication networks, oil pipeline systems, power systems, costs, capacities and demands are constructed by the base of maximal flow network problem. Fulkerson [15] provided the maximal flow problem and solved by the simplex method for the linear programming. The maximal flow problem have been solved by Ford [14] using augmenting path algorithm. This algorithm has been used to solve the crisp maximal flow problems [2], [4], [26]. Fuzzy numbers represent the parameters of maximal flow problems. Kim [17] is one of the first introducer on this subject. Chanas [5], [6], [7] approached this problem using minimum costs technique. An algorithm for a network with crisp stricter was presented by Chanas in their first paper. In their second paper they proposed that the flow was a real number and the capacities have upper and lower bounds had been discussed [6]. In their third paper, they had also studied the integer flow and proposed an algorithm [8]. Interval-valued versions of the max-flow min cut theorem and Karp-Edmonds algorithm was developed by Diamond [11]. Some times it arise uncertain environment. The network flow problems using fuzzy numbers were investigated by Liu [23]. Generalized fuzzy versions of maximum flow problem were considered by Ji [16] with respect to arc capacity as fuzzy variables. A new algorithm to find fuzzy maximal flow between source and sink is proposed by Kumar et al. [19] with the help of ranking function.

In this paper the existing algorithm [19] have been modified to find fuzzy maximal flow between source and sink by representing all the parameters considered as generalized trapezoidal fuzzy numbers. To illustrate the modified algorithm, a numerical example is solved. If there is no uncertainty about the flow between source and sink then the proposed algorithm gives the same result as in crisp maximal flow problems. But when we face same rank more than one arc then we have applied mode function for selected maximal flow path. In section 2 we have discussed some basic definitions, ranking function, mode function and arithmetic operations for interval and generalized trapezoidal fuzzy numbers. In section 3 we have proposed an algorithm for solving fuzzy maximal flow problems. In section 4 we have applied the proposed algorithm over a numerical example. In section 5 and 6 we have discussed results and conclusion respectively.

II. PRELIMINARIES

In this section some basic definitions, ranking function, mode function and arithmetic operations are reviewed

1.1. Definition [13]: Let X be a universal classical set of objects and a characteristic function μ_A of a classical set $A \subseteq X$ assigns a value either 0 or 1 i.e.

$$\mu_A(x) = \begin{cases} 1 & \text{if} \quad x \in A \\ 0 & \text{if} \quad x \notin A \end{cases}.$$

This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range unit interval $[0,1]$ i.e. $\mu_{\tilde{A}}: X \to [0,1]$. The assigned values indicate the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for all $x \in X$ is called fuzzy set.

1.2. Remark: Throughout this paper we shall write fuzzy set μ.

1.3. Definition [18]: Suppose μ is a fuzzy set. Then for any $\alpha \in [0,1]$, the ***level set (or α cut)*** of μ is denoted by μ^α and defined by $\mu^\alpha = \{x \in X: \mu(x) \geq \alpha\}$.

1.4. Definition: [27] The special significance is fuzzy sets that are defined on the set $\mathbb{R}$ of real number is membership functions of these sets, which have the form $\mu: \mathbb{R} \to [0,1]$ is called fuzzy number if the following axioms are satisfies:

- μ must be normal fuzzy set i.e. there exist $x \in \mathbb{R}$; $\mu(x) = 1$
- μ^α must be closed interval of real number, for every $\alpha \in]0,1]$
- the support of μ must be bounded and compact i.e. $\{x \in \mathbb{R};\ \mu(x) > 0\}$ is bounded and compact.

Fuzzy number is denoted by $F(\mathbb{R})$.

1.5. Remark: Every fuzzy number is convex fuzzy sets. Also a fuzzy set μ is convex if for all $x, y \in X$; $\mu(kx + (1-k)y) \geq \min\{\mu(x), \mu(y)\}$, for all $k \in [0, 1]$.

1.6. Arithmetic Operations

In this section, we shall define addition and subtraction between two intervals.

Let $[a, b]$ and $[c, d]$ be two closed interval, then

Addition: $[a, b] + [c, d] = [a + c, b + d]$, Additive inverse: $-[a, b] = [-b, -a]$

Subtraction: $[a, b] - [c, d] = [a, b] + (-[c, d]) = [a, b] + [-d, -c] = [a - d, b - c]$

1.7. Definition: [10] A fuzzy number $A = [[a, b, c, d]]$ is said to be a trapezoidal fuzzy numbers if its membership function is given by

$$\mu(x) = \begin{cases} 0 & ; \quad -\infty < x \leq a \\ \dfrac{x-a}{b-a} & ; \quad a \leq x < b \\ 1 & ; \quad b \leq x \leq c \\ \dfrac{x-d}{c-d} & ; \quad c < x \leq d \\ 0 & ; \quad d \leq x < \infty \end{cases}, \quad \text{where } a, b, c, d \in \mathbb{R}$$

.

1.8. Definition: [10] A fuzzy number $A = [[a, b, c, d; w]]$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu(x) = \begin{cases} 0 \quad ; \quad -\infty < x \le a \\ \dfrac{w(x-a)}{b-a} \quad ; \quad a \le x < b \\ w \quad ; \quad b \le x \le c \\ \dfrac{w(x-d)}{c-d} \quad ; \quad c < x \le d \\ 0 \quad ; \quad d \le x < \infty \end{cases} \qquad \text{where } a,b,c,d \in \mathbb{R} \text{ and } w \in]0,1]$$

Arithmetic Operations

In this subsection, arithmetic operations between two generalized trapezoidal fuzzy number, defined on universal set of real numbers $\mathbb{R}$, are reviewed by Chen [10].

Let $A = [[a_1, b_1, c_1, d_1;\ w_1]]$ and $B = [[a_2, b_2, c_2, d_2;\ w_2]]$ be two generalized trapezoidal fuzzy numbers then

- $A + B = [[a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min\{w_1, w_2\}]]$
- $A - B = [[a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2; \min\{w_1, w_2\}]]$

1.9. Ranking function

A convenient method for comparing of fuzzy number is by use of ranking function [22]. A ranking function $\mathfrak{R}: F(\mathbb{R}) \to \mathbb{R}$, where $F(\mathbb{R})$ is set of all fuzzy numbers defined on set of real numbers, which maps each fuzzy number in to a real number. Let $A = [[a_1, b_1, c_1, d_1;\ w_1]]$ and $B = [[a_2, b_2, c_2, d_2;\ w_2]]$ be two generalized trapezoidal fuzzy numbers then

$\mathfrak{R}(A) = \frac{w_1(a_1+b_1+c_1+d_1)}{4}$ and $\mathfrak{R}(B) = \frac{w_2(a_2+b_2+c_2+d_2)}{4}$.

Mathematica function for rank calculation:

$$\boldsymbol{ra}[\mathrm{z\_}] := \frac{\boldsymbol{z}[[\mathbf{5}]] * (\boldsymbol{z}[[\mathbf{1}]] + \boldsymbol{z}[[\mathbf{2}]] + \boldsymbol{z}[[\mathbf{3}]] + \boldsymbol{z}[[\mathbf{4}]])}{\mathbf{4}}$$

1.9.1. If $\mathfrak{R}(A) > R(B)$ then we say $A \succ B$

1.9.2. If $\mathfrak{R}(A) < R(B)$ then we say $A \prec B$

1.9.3. If $\mathfrak{R}(A) = \mathfrak{R}(B)$ then we say $A \approx B$

1.10. Mode function

When two generalized fuzzy number $A \approx B$ with respect to ranking function then we will apply mode function for maximum flow position. A mode function $M: F(\mathbb{R}) \to \mathbb{R}$, where $F(\mathbb{R})$ is set of all fuzzy numbers defined on set of real numbers, which maps each fuzzy number in to a real number [20]. Let $A = [[a_1, b_1, c_1, d_1;\ w_1]]$ and $B = [[a_2, b_2, c_2, d_2;\ w_2]]$ be two generalized trapezoidal fuzzy numbers then

$M(A) = \frac{w_1(b_1+c_1)}{2}$ and $M(B) = \frac{w_2(b_2+c_2)}{2}$.

Mathematica function for mode calculation, $mod[\mathrm{z\_}] := \frac{z[[5]]*(z[[2]]+z[[3]])}{2}$

III. ALGORITHM

Chen has proposed that it is not possible to control the membership function to the normal form, in some case [8]. He also proposed the concept of generalize fuzzy numbers. The normal form of trapezoidal fuzzy number ware used by various papers for solving real life problems. In this paper, we will use generalized trapezoidal fuzzy number for network flow. In the section the maximal flow network problem is modified to find fuzzy maximal flow between sources and sink for generalized trapezoidal fuzzy numbers. The proposed algorithm is direct extension of existing algorithm [26], [25]. The fuzzy maximal flow algorithm is based on finding breakthrough paths with net positive flow between the source and sink nodes. Consider arc (i,j) with initial fuzzy capacities $(\overline{\mu c_{ij}}\ ,\ \overline{\mu c_{ji}})$ and fuzzy residuals capacities (or remaining fuzzy capacities) $(\mu c_{ij}\ , \mu c_{ji})$. For a node j that receives flow from node i, we will a label $[\mu a_j\ , i]$, where μa_j is the fuzzy flow from node i to j. The step of algorithm for generalized trapezoidal fuzzy number are summarized as follows:

1.11. Step 1 For all arcs (i,j), set the residual fuzzy capacity is equal to initial fuzzy capacity i.e., $(\mu c_{ij}\ , \mu c_{ji}) = (\overline{\mu c_{ij}}\ ,\ \overline{\mu c_{ji}})$. Let $\mu a_1 = [[\infty, \infty, \infty, \infty; 1]]$ and label the source node 1 with $[[\infty, \infty, \infty, \infty; 1], -]$. Set $i = 1$, and go to step 2.

1.12. Step 2 Determine S_i, the set of unlabeled nodes j that can be reached directly from node i by arcs with positive residuals capacity (i.e., μc_{ij} is non-negative fuzzy number for each $j \in S_i$). If $S_i = \emptyset$ then go to step 4, otherwise go to step 3.

1.13. Step 3 Determine $k \in S_i$ such that

$$\max_{j \in S_i}\{\mathfrak{R}(\mu c_{ij})\} = \mathfrak{R}(\mu c_{ik})$$

Set $\mu a_k = \mu c_{ik}$ and label node k with $[\mu a_k, i]$. If $k = n$, the sink node has been labeled, and a breakthrough path is found, then go to step 5. Otherwise go to step 2.
Again, if $\max_{j \in S_i}\{\Re(\mu c_{ij})\}$ is more than one fuzzy flow then we have applied mode test for maximal flow according to maximal rank test.
Mathematica program for breakthrough path according to *rank* and *mode*

```
rafμ[AA_, BB_, CC_] := Module[{a, b, c, s}, a = ra[AA]; b = ra[BB]; c = ra[CC];
  t = {a, b, c}; Print["theri ranks are ", t]; tt = Position[t, Max[t]];
  s = Sum[t[[i]], {i, 1, 3}]; tt = Position[t, Max[t]];
  If[s > 0, If[Length[tt] == 1,
    If[Flatten[tt][[1]] == 1, Print["maximum rank is 1st position which is ", AA],
     If[Flatten[tt][[1]] == 2, Print["maximum rank is 2nd position which is ", BB],
      Print["maximum rank is 3rd position which is ", CC]]]];
   If[Length[tt] == 2, If[Flatten[tt][[1]] == 1 && Flatten[tt][[2]] == 2,
     Print["maximum rank are both 1st ", AA, "and 2n position ", BB,
      "so we need mode test"], If[Flatten[tt][[1]] == 1 && Flatten[tt][[2]] == 3,
      Print["maximum rank are both 1st ", AA, "and 3rd position ", CC,
       "so we need mode test"], If[Flatten[tt][[1]] == 2 && Flatten[tt][[2]] == 3,
       Print["maximum rank are both 2nd ", BB, "and 3rd position ", CC,
        "so we need mode test"]]]]];
   If[Length[tt] == 3, Print["all rank are same with ", t, " for fuzzy number ",
     AA, " , ", BB, " and ", CC, "so we need mode test"]],
   Print["rank of fuzzy number is ", s, " so flow is not possible"]]]
 mofμ[AA_, BB_, CC_] := Module[{a, b, c, s}, a = mo[AA]; b = mo[BB]; c = mo[CC];
   t = {a, b, c}; Print["their mods are ", t]; tt = Position[t, Max[t]];
   s = Sum[t[[i]], {i, 1, 3}]; tt = Position[t, Max[t]];
   If[s > 0, If[Length[tt] == 1,
     If[Flatten[tt][[1]] == 1, Print["maximum mode is 1st position which is ", AA],
      If[Flatten[tt][[1]] == 2, Print["maximum mode is 2nd position which is ", BB],
       Print["maximum mode is 3rd position which is ", CC]]]];
    If[Length[tt] == 2, If[Flatten[tt][[1]] == 1 && Flatten[tt][[2]] == 2,
      Print["maximum mode are both 1st ", AA, "and 2n position ", BB],
      If[Flatten[tt][[1]] == 1 && Flatten[tt][[2]] == 3,
       Print["maximum mode are both 1st ", AA, "and 3rd position ", CC],
       If[Flatten[tt][[1]] == 2 && Flatten[tt][[2]] == 3,
        Print["maximum mode are both 2nd ", BB, "and 3rd position ", CC]]]]];
    If[Length[tt] == 3, Print["all mode are same with ", t, " for fuzzy number ",
      AA, " , ", BB, " and ", CC]],
    Print["mode of fuzzy number is ", s, " so flow is not possible"]]]
```

1.14. Seep 4 If $i = 1$, no *breakthrough* is possible, then go to step 6. Otherwise, let r be the node that has been labeled immediately before current node i and remove i from the set of nodes adjacent to r. Set $i = r$ and go to step 2.

1.15. Step 5 Let $N_p = \{1, k_1, k_2, \dots \dots \dots, n\}$ define the nodes of the p^{th} breakthrough path from source node 1 to sink node n. Then the maximal flow along the path is completed as $\mu_p = \min\{\mu_1, \mu_{k_1}, \mu_{k_2}, \dots \dots \dots, \mu_n\}$. Mathematica function for maximal flaw for a path is $flow = \{r[f01], r[f12], r[f24], r[f45]\}$; $f\mu a = \mathrm{Min}[flow]$. The residual capacity of each arc along the breakthrough path is decreased by μ_p in the direction of the flow and increased by μ_p in the reverse direction i.e. for nodes i and j on the path, the residual flow id change from the current $(\mu c_{ij}, \mu c_{ji})$ to

1.15.1. Case 1 We shall compute $(\mu c_{ij} - \mu_p, \mu c_{ji} + \mu_p)$ if the flow is from i to j.

1.15.2. Case 2 We shall compute $(\mu c_{ij} + \mu_p, \mu c_{ji} - \mu_p)$ if the flow is from j to i.

Mathematica function for residual capacity calculation

```
resedue[ff_, gg_, hh_] :=
 Module[{a, b, c, d, e, p, r, s}, a = Delete[ff, -1]; b = Delete[gg, -1];
  c = Delete[hh, -1]; d = Min[ff[[-1]], gg[[-1]]];
  e = Min[gg[[-1]], hh[[-1]]]; p = a - Reverse[b]; r = Join[p, {d}];
  q = b + c; s = Join[q, {e}]; {r, s}]
```

1.16. **Step 6** In the step we will determine flow and residue.

1.16.1. Given that total numbers of breakthrough paths are m. Then we get total flow of a network by determining: $F = \mu_1 + \mu_2 + \mu_3 + \cdots + \mu_m$, where m is the number of iteration.

Mathematica function for total flow calculation

```
totalflow[flowall_] :=
 Module[{a, b, c, d, e, p, r, s}, p = flowall; a = Transpose[p];
  b = Transpose[Delete[a, -1]]; c = Sum[b[[i]], {i, 1, Length[b]}];
  d = Min[a[[-1]]]; e = Join[c, {d}];
  Print["sum of total flow ", p, " is = ", e]]
```

1.16.2. Using the initial and final fuzzy residuals of arc (i, j) are $\left(\overline{\mu c}_{ij}, \overline{\mu c}_{ji}\right)$ and $\left(\mu c_{ij}, \mu c_{ji}\right)$ respectively, the fuzzy optimal flow in arc (i, j) is computed as follows: Let $(\alpha, \beta) = \left(\overline{\mu c}_{ij} - \mu c_{ij}, \overline{\mu c}_{ji} - \mu c_{ji}\right)$. If $\Re(\alpha) > 0$ then the fuzzy optimal flow from i to j is α. Otherwise, if $\Re(\beta) > 0$ then the fuzzy optimal flow from j to i is β.

Mathematica function for decision flow direction

```
flowdirection[ff_, gg_, hh_, ii_] :=
 Module[{a, b, c, d, e, f, p, q, r, s, t, u, v}, a = Delete[ff, -1];
  b = Delete[gg, -1]; c = Delete[hh, -1]; d = Delete[ii, -1];
  e = a - Reverse[c]; f = b - Reverse[d]; p = Min[ff[[-1]], hh[[-1]]];
  q = Join[e, {p}]; r = Min[gg[[-1]], ii[[-1]]]; s = Join[f, {r}];
  t = {q, s}; u = {ra[q], ra[s]}; t = Append[t, u]; t = Append[t, ff];
  If[u[[1]] > 0, Print[t, " and flow → direction"],
   If[u[[1]] < 0, Print[t, " and flow <- direction"],
    If[u[[1]] == 0, Print[t, " and no flow"]]]]]
```

IV. ILLUSTRATIVE EXAMPLE

In this section the proposed algorithm is illustrated by solving a numerical example.

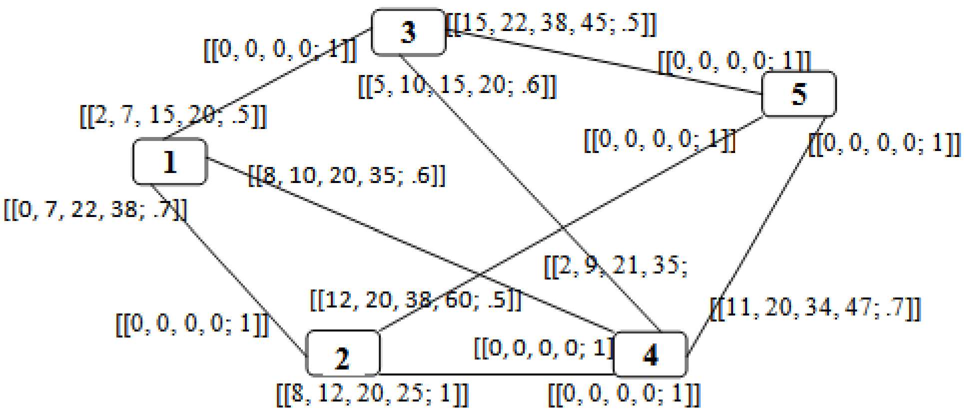

Figure: 1

Example Consider the network shown in the figure 1. We will find out the fuzzy maximal flow between source node 1 and destination node 5.

Iteration 1: Set the initial fuzzy residual $\left(\mu c_{ij}, \mu c_{ji}\right)$ equal to the initial fuzzy capacity $\left(\overline{\mu c}_{ij}, \overline{\mu c}_{ji}\right)$. Input of all flow from the given network according to Mathematica:

```
ofμ12 = fμ12 = {0, 7, 22, 38, .7}; ofμ13 = fμ13 = {2, 7, 15, 20, .5};
ofμ14 = fμ14 = {8, 10, 20, 35, .6}; ofμ21 = fμ21 = {0, 0, 0, 0, 1};
ofμ24 = fμ24 = {8, 12, 20, 25, 1.}; ofμ25 = fμ25 = {12, 20, 38, 60, .5};
ofμ31 = fμ31 = {0, 0, 0, 0, 1}; ofμ34 = fμ34 = {7, 19, 36, 55, 0.4};
ofμ35 = fμ35 = {15, 22, 38, 45, 0.5}; ofμ41 = fμ41 = {0, 0, 0, 0, 1};
ofμ42 = fμ42 = {0, 0, 0, 0, 1}; ofμ43 = fμ43 = {20, 40, 60, 80, .9};
ofμ45 = fμ45 = {30, 150, 180, 240, .3}; ofμ52 = fμ52 = {0, 0, 0, 0, 1};
ofμ53 = fμ53 = {0, 0, 0, 0, 1}; ofμ54 = fμ54 = {0, 0, 0, 0, 1};
fμ0 = {0, 0, 0, 0, 0}; tfμ = {};
```

Remarks: First four entries of each vector represents trapezoidal fuzzy number and fifth entry is value of w for generalize trapezoidal fuzzy number. We have used "**,**" in replace of ";" for calculation in Mathematica. Also we have used "**{}**" in replace of "[[ ]]" for fuzzy number to calculation in Mathematica. Also **ofμ** and **fμ** represent initial and residual flow respectively. Any bold Mathematica texts are input and other texts are output. In Mahtematica **rafμ** represents rank test function, **mofμ** represents mode test function, **resedue** represents residue function, **totalflow** represents flow addition function and **flowdirection** represents flow direction function.

Step 1: Set **fμ01={Infinity,Infinity,Infinity,Infinity,1}** in Mathematica format i.e. $\{\infty,\infty,\infty,\infty,1\}$ and label node 1 with **(fμ01,-)**. Set $i = 1$.

Step 2: $S_1 = \{2, 3, 4\} \neq \varphi$.

Step 3: We are calculating maximum ranking of the generalized trapezoidal fuzzy number using Mathematica program which defined **rafμ** in section 3.3.

```
rafμ[fμ12, fμ13, fμ14]

theri ranks are {11.725, 5.5, 10.95}

maximum rank is at 1st position which is {0, 7, 22, 38, 0.7}
```

so, set $k = 2$ and $\mu a_2 = \mu c_{12} = \text{fμ12} = [[0,7,22,38;.7]]$ and label node 2 with (fμ12, 1). Set $i = 2$ and repeat step 2.

Step 2: $S_2 = \{4, 5\}$.

Step 3: rafμ[fμ24,fμ25,fμ0]

theri ranks are {16.25,16.25,0}, maximum rank are both at 1st {8,12,20,25,1.} and 2n position {12,20,38,60,0.5} so we need mode test. **mofμ [fμ24,fμ25,fμ0]**

their mods are {16.,14.5,0}, maximum mode is 1st position which is {8,12,20,25,1.}

so set $k = 4$ and $\mu a_4 = \mu c_{24} = \text{fμ24} = [[8,12,20,25;1]]$. Now label node 4 with (fμ24,2). Set $i = 4$ and repeat step 2.

Step 2: $S_4 = \{3,5\}$.

Step 3: rafμ [fμ43,fμ45,fμ0] output: theri ranks are {45.,45.,0} maximum rank are both 1st {20,40,60,80,0.9} and 2n position {30,150,180,240,0.3} so we need mode test. **mofμ [fμ43,fμ45,fμ0]** output: their mods are {45.,49.5,0} maximum mode is 2nd position which is {30,150,180,240,0.3}.

so, set $k = 5$ and $\mu a_5 = \mu c_{45} = \text{fμ45} = [[30, 150, 180, 240; .3]]$. Now label sink node 5 with (fμ45,4). We have reached the sink node 5, and so a breakthrough path is found. Go to step 5.

Step 5: The breakthrough path is $1 \to 2 \to 4 \to 5$ and $N_1 = \{1,2,4,5\}$,

Mathematica script:

```
flowμ1 = {ra[fμ01], ra[fμ12], ra[fμ24],
    ra[fμ45]};

Print["rank of flow is ", flowμ1,
  " so minimum rank is in the position
    no ",
 Flatten[Position[flowμ1, Min[flowμ1]]]]

rank of flow is {∞, 11.725 , 16.25 , 45. }
  so minimum  rank is in the position  no
 {2}

fμa1 = fμ12
tfμ = Append[tf, fμa1]
```

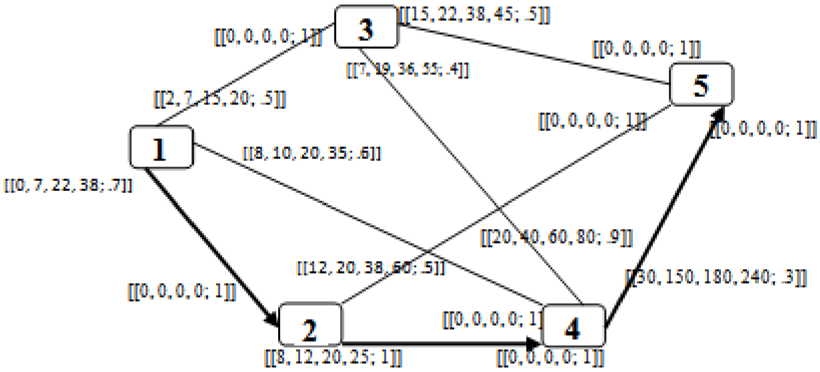

Figure: 2

We have calculated the fuzzy residual capacities along path N_1 are (using Mathematica program which has defined **residue** in section 3.5):

Input for residue	Output for residue
fμ12fμ21 = resedue[fμ12, fμa1, fμ21]	{{-38, -15, 15, 38, 0.7}, {0, 7, 22, 38, 0.7}}
fμ24fμ42 = resedue[fμ24, fμa1, fμ42]	{{-30, -10, 13, 25, 0.7}, {0, 7, 22, 38, 0.7}}
fμ45fμ54 = resedue[fμ45, fμa1, fμ54]	{{-8, 128, 173, 240, 0.3}, {0, 7, 22, 38, 0.7}}

Update network flow as follows:

```
fμ12 = fμ12fμ21[[1]]; fμ21 = fμ12fμ21[[2]]; fμ24 = fμ24fμ42[[1]];
fμ42 = fμ24fμ42[[2]]; fμ45 = fμ45fμ54[[1]];
```

Iteration 2

Repeating the procedure described in iteration 1, at the starting node 1, the obtained breakthrough path is $1 \rightarrow 4 \rightarrow 3 \rightarrow 5$ and $N_2 = \{1,4,3,5\}$. $\mu 2 = [[8, 10, 20, 35;\ .6]]$.

Only input and graph of network have been shown

```
rafμ[fμ12, fμ13, fμ14]; rafμ[fμ42, fμ43, fμ45]; rafμ[fμ35, fμ0, fμ0];
```

```
flowμ2 = {ra[fμ01], ra[fμ14], ra[fμ43], ra[fμ35]};
Print["rank of flow is ", flowμ2, " so minimum rank is
in the position no ", Flatten[Position[flowμ2, Min[flowμ2
```

```
fμa2 = fμ14
tfμ = Append[tfμ, fμa2]
fμ14fμ41 = resedue[fμ14, fμa2, fμ41]
fμ43f34 = resedue[fμ43, fμa2, fμ34]
fμ35fμ53 = resedue[fμ35, fμa2, fμ53]
fμ14 = fμ14fμ41[[1]]; fμ41 = fμ14fμ41[[2]];
fμ43 = fμ43f34[[1]]; fμ34 = fμ43f34[[2]];
fμ35 = fμ35fμ53[[1]]; fμ53 = fμ35fμ53[[2]];
```

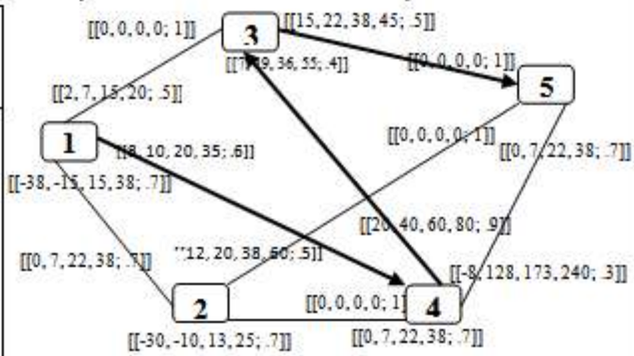

Figure: 3

Iteration 3

Repeating the procedure described in iteration 1, the obtained breakthrough path is
$1 \rightarrow 3 \rightarrow 5$ and $N_3 = \{1,3,5\}$. μ_2 =[[2, 7, 15, 20; .5]]

Only input and graph of network have shown

```
rafμ[fμ12, fμ13, fμ14]; rafμ[fμ34, fμ35, fμ0] ; rafμ[fμ42, fμ45, fμ0] ;
```

```
flowμ3 = {ra[fμ01], ra[fμ13], ra[fμ34], ra[fμ45]};
Print["rank of flow are ", flowμ3, " so minimum rank is in
 the position no ", Flatten[Position[flowμ3, Min[flowμ3]]]];
```

```
fμa3 = fμ13
tfμ = Append[tfμ, fμa3]
fμ13fμ31 = resedue[fμ13, fμa3, fμ31]
fμ34fμ43 = resedue[fμ34, fμa3, fμ43]
fμ45fμ54 = resedue[fμ45, fμa3, fμ54]
fμ13 = fμ13fμ31[[1]]; fμ31 = fμ13fμ31[[2]];
fμ34 = fμ34fμ43[[1]]; fμ43 = fμ34fμ43[[2]];
fμ45 = fμ45fμ54[[1]]; fμ54 = fμ45fμ54[[2]];
```

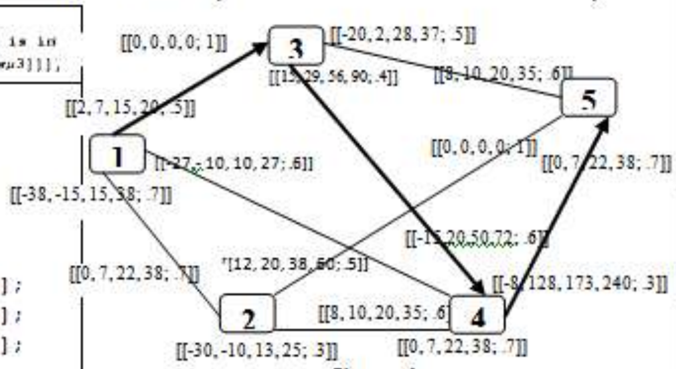

Figure: 4

Iteration 4

Repeating the procedure described in iteration 1, we get, more iteration are not possible after 3rd iteration. Because,

```
rafμ[fμ12, fμ13, fμ14]
theri ranks are {0, 0, 0}
rank of fuzzy number is
 0 so flow is not possible
mofμ[fμ12, fμ13, fμ14]
their mods are {0, 0, 0}
mode of fuzzy number is
 0 so flow is not possible
```

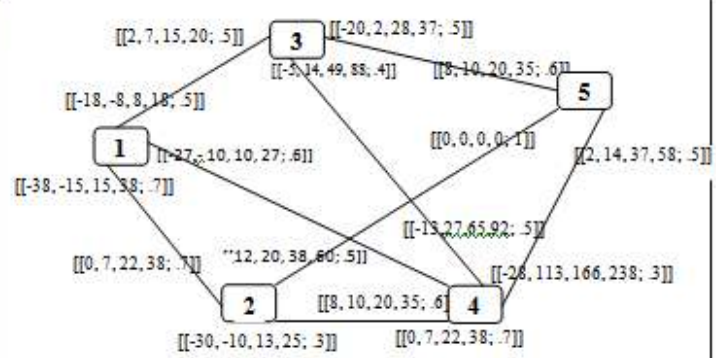

Figure: 5

Step 6: Now we calculate Fuzzy maximal flow.

```
totalflow[tfμ]
sum of total flow
 {{0, 7, 22, 38, 0.7}, {8, 10, 20, 35, 0.6}, {2, 7, 15, 20, 0.5}} is = {10, 24, 57, 93, 0.5}
```

Fuzzy maximal flow in the network is $F = f\mu a1 + f\mu a1 + f\mu a1 = [[10,24,57,93; 0.5]]$. The initial and final fuzzy residuals of arc (i,j) are $\left(\overline{\mu c}_{ij}, \overline{\mu c}_{ji}\right)$ and $\left(\mu c_{ij}, \mu c_{ji}\right)$ respectively, the fuzzy optimal flow in arc (i,j) is computed as follows: Let $(\alpha,\beta) = \left(\overline{\mu c}_{ij} - \mu c_{ij}, \overline{\mu c}_{ji} - \mu c_{ji}\right)$. If $\Re(\alpha) > 0$ then the fuzzy optimal flow from i to j is α. Otherwise, if $\Re(\beta) > 0$ then the fuzzy optimal flow from j to i is β

```
fdh = Print[" Arc", "       μ̄cij - μcij", "             ", "μ̄cji - μcji", "             ",
  "flow rank", "         ", "with flow", "      and flow direction"]

Print["(1,2),", flowdirection[ofμ12, ofμ21, fμ12, fμ21]];
Print["(1,3),", flowdirection[ofμ13, ofμ31, fμ13, fμ31]]
Print["(1,4),", flowdirection[ofμ14, ofμ41, fμ14, fμ41]];
Print["(2,4),", flowdirection[ofμ24, ofμ42, fμ24, fμ42]]
Print["(2,5),", flowdirection[ofμ25, ofμ52, fμ25, fμ52]];
Print["(3,4),", flowdirection[ofμ34, ofμ43, fμ34, fμ43]]
Print["(3,5),", flowdirection[ofμ35, ofμ53, fμ35, fμ53]];
Print["(4,2),", flowdirection[ofμ42, ofμ24, fμ42, fμ24]]
Print["(4,3),", flowdirection[ofμ43, ofμ34, fμ43, fμ34]];
Print["(4,5),", flowdirection[ofμ45, ofμ54, fμ45, fμ54]];
```

```
Arc        μ̃ẽij - μeij               μ̃ẽji - μeji              flow rank          with flow        and flow direction
(1,2),{{-38, -8, 37, 76, 0.7}, {-38, -22, -7, 0, 0.7}, {11.725 , -11.725 }, {0, 7, 22, 38, 0.7},  and flow → direction }
(1,3),{{-16, -1, 23, 38, 0.5}, {-20, -15, -7, -2, 0.5}, {5.5, -5.5}, {2, 7, 15, 20, 0.5},  and flow → direction }
(1,4),{{-19, 0, 30, 62, 0.6}, {-35, -20, -10, -8, 0.6}, {10.95 , -10.95 }, {8, 10, 20, 35, 0.6},  and flow → direction }
(2,4),{{-17, -1, 30, 55, 0.7}, {-38, -22, -7, 0, 0.7}, {11.725 , -11.725 }, {8, 12, 20, 25, 1.},  and flow → direction }
(2,5),{{-48, -18, 18, 48, 0.5}, {0, 0, 0, 0, 1}, {0, 0}, {12, 20, 38, 60, 0.5},  and no flow }
(3,4),{{-81, -30, 22, 60, 0.4}, {-72, -25, 33, 93, 0.5}, {-2.9 , 3.625 }, {7, 19, 36, 55, 0.4},  and flow <- direction }
(3,5),{{-22, -6, 36, 65, 0.5}, {-35, -20, -10, -8, 0.6}, {9.125 , -10.95 }, {15, 22, 38, 45, 0.5},  and flow → direction }
(4,2),{{-38, -22, -7, 0, 0.7}, {-17, -1, 30, 55, 0.7}, {-11.725 , 11.725 }, {0, 0, 0, 0, 1},  and flow <- direction }
(4,3),{{-72, -25, 33, 93, 0.5}, {-81, -30, 22, 60, 0.4}, {3.625 , -2.9}, {20, 40, 60, 80, 0.9},  and flow → direction }
(4,5),{{-208, -16, 67, 268, 0.3}, {-58, -37, -14, -2, 0.5}, {8.325 , -13.875 }, {30, 150, 180, 240, 0.3},  and flow → direction }
```

2. Results and Discussion

In this section, we will shortly explanation our result for the above numerical example. We have found the fuzzy maximal (optimal) flow $F = [[10, 24, 57, 93; 0.5]]$. From this result we can take the decision that the amount of flow between source and sink is 10 and 93 unit. We also can take the decision about the statement that the maximal flow will be 24 to 57 unit is 50%. The decision making for the reaming amount of flow can be obtained by

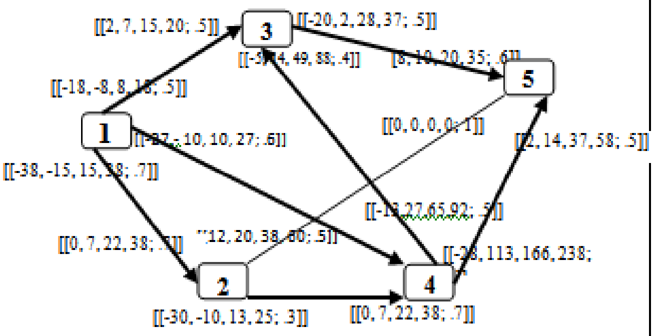

Figure: 5

$$\mu F(x) = \begin{cases} 0 \quad ; \quad -\infty < x \leq 10 \\ \dfrac{0.5(x-10)}{14} \quad ; \quad 10 \leq x < 24 \\ 0.5 \quad ; \quad 24 \leq x \leq 57 \\ \dfrac{0.5(x-93)}{36} \quad ; \quad 57 < x \leq 93 \\ 0 \quad ; \quad 93 \leq x < \infty \end{cases}$$

where, x represent the amount of flow.

V. CONCLUSION

In this paper, we have proposed algorithm for solving the fuzzy maximal (optimal) flow problems occurring in real life situation and we have shown that the flows are represented by using generalized trapezoidal fuzzy numbers. Kumar and Kaur [23] have solved fuzzy maximal flow problems using generalized trapezoidal fuzzy numbers. But they apply only ranking function for maximal flow path. To demonstrate the proposed new algorithm, we have solved a numerical example and obtain results are discussed. In the algorithm of the paper, we have used ranking function also used mode function when ranking function fails for chose the path of the flow, we have also used some Mathematica program for all mathematical calculation of these numerical example. In future, we can solve the other network problems by extending this proposed algorithm.

REFERENCES

[1]. Abbasbandy S., Hajjari T., "A new approach for ranking of trapezoidal fuzzy numbers", *Computers and Mathematics with Applications*, 57 (2009), pp. 413-419

[2]. Ahuja R. K., Magnanti T. L. and Orlin J. B., *Network Flows, Theory and Algorithm*, Prentice Hall, New Jersey (1993)

[3]. Bansal A., "Trapezoidal Fuzzy Numbers (a,b,c,d): Arithmetic Behavior", *International Journal of Physical and Mathematical Science*, (2011), pp. 39-44.

[4]. Bazarra M. S., Jarvis J. J. and Sherali H. D., *Linear Programming and Network Flows*, 2nd Edition, Wiley, New York, (1990)

[5]. Chanas S. and Kolodziejczyk W, "Maximum Flow in a Network with fuzzy arc capacities", *Fuzzy Sets and Systems*, 8: (1982), pp. 165-173

[6]. Chanas S. and Kolodziejczyk W, "Real-valued Flow in a Network with fuzzy arc capacities", *Fuzzy Sets and Systems*, 13: (1984), pp. 139-151

[7]. Chanas S. and Kolodziejczyk W, "Integer Flow in Network with Fuzzy Capacity Constrains", *Networks*, 16: (1986), pp. 17-31

[8]. Chen S. H "Operations of Fuzzy Numbers with function principal", *Tamkang Journal of Management Sciences*, 6 (1985), pp. 13-25

[9]. Chen S. J., Chen, S. M. "Fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers", *Applied Intelligence*, 26 (2007), pp. 1-11

[10]. Chen S. M. and Chen J. H., "Fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads", *Expert Systems with Applications*, vol. 36, (2009), pp. 6833-6842.

[11]. Diamond A., "A Fuzzy Max-Flow Min-Cut Theorem", *Fuzzy Sets and Systems*, 119: (2001), pp. 139-148

[12]. Dubois D., Prade H. "Fuzzy real algebra Some results", Fuzzy Sets Syatems, 2 (1979), pp. 327-349

[13]. Dubois D., Prade H, *Fuzzy Sets and Systems Theory and Applications*, New York/London/Toronto/Sydney/San Francisco; (1980)

[14]. Ford L. R. and Fulkerson D. R., "Maximal Flow Through a network", *Canadian Journal of Mathematics*, vol. 8, (1956), pp. 399-401

[15]. Fulkerson D. R. and Dantzig G. B., "Computation of Maximum Flow in Network", *Naval Research Logistics Quarterly*, vol. 2, (1955), pp. 277-283

[16]. Ji X., Yang L. and Shao Z., "Chance Constrained Maximum Flow Problem with Arc Capacities", *Lecture Notes in Computer Science*, Springer-Verlag, Berlin, Heidelberg, 4114: (2006), pp. 11-19

[17]. Kim K. and Roush F., "Fuzzy Flows on Network", *Fuzzy Sets and Systems*, 8: (1982), pp 35-38

[18]. Klir, George J. and Yuan, Bo, *Fuzzy Sets and Fuzzy Logic Theory and Applications*, Prentice, Hall of India Private Limited, New Delhi-110 001, (1997)

[19]. Kumar A., Bhatia N. and Kaur M. "A new approach for solving fuzzy maximal flow problems", *Lecture Notes in Computer Science*, Springer-Verlag, Berlin, Heidelberg, 5908 (2009), pp. 278-286

[20]. Kumar A., Singh P., Kaur A. and Kaur P., "Ranking of generalized Trapezoidal Fuzzy Numbers Based on Rank, Mode, Fivergence and Spread", *Turkish Journal of Fuzzy Systems*, Vol. 1, No. 2 (2010), pp. 141-152

[21]. Kumar A. and Kaur M. "An Algorithm for Solving Fuzzy Maximal Flow Problems Using Generalized Trapezoidal Fuzzy Numbers", *International Journal of Applied Science and Engineering*, 8, 2 (2010), pp. 109-118

[22]. Liou, T. S. and Wang, M. J. "Ranking fuzzy numbers with integral value", *Fuzzy Sets and Systems,* 50(1): (1992), pp. 247-255

[23]. Liu S. T. and Kao C., "Network Flow Problems with Fuzzy Arc Lengths", *IEEE Transactions on Systems, Man and Cybernetics*, 34: (2004), pp. 765-769

[24]. Moore R. E., *Methods and Applications of Interval Analysis*. SIAM, Philadelphia, 1979

[25]. Taha H. A., *operational Research: An Introduction,* Prentice-Hall, New Jersey, (2003)

[26]. Taha H. A., *Operation Research: An Introduction*; Eighth Edition, New Dhelhi-110 001, (2006)

[27]. Thaker P. A., Shelar D. S. and Thaker S. P., "Solving Fuzzy Linear Programming Problem", *Proceedings of the World Corld Congress on Engineering*, London, U.K., Vol II, (2009) pp. 978-988

[28]. Stephen Wolfram, *The Mathematica Book*: 5th Edition, Cambridge (2003)

[29]. Yager R., "On Solving Fuzzy Mathematical Relationships", *Information Control*, 41 (1979) 29-55

[30]. Zadeh, L., "Fuzzy sets", *Information and Control*, 8, (1965), pp. 338–353