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An Analytical Study in Dynamics of Host Parasitoid Model with Allee Effect

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Abstract:- In this paper, a discrete time host parasitoid model is investigated. The fixed points in the stability are analyzed. Two biological phenomena, the Allee effect of host population aggregation of the parasitism are considered in our mathematical model. The population dynamics are compressed when Allee effect is added, the sensitivity to the initial conditions for the host parasitoid system decreased after adding Allee effect. Finally various mathematical study were discussed.

Keywords:- Host-Parasitoid, Allee effect, stability analysis, aggregation.

I. INTRODUCTION

Some ecological models, although simple in mathematical expression have been designed to study population temporal dynamics. In particular, the pioneering work in the field was initiated by May (1974, 1976) .The discrete time host parasitoid model which are usually described by difference equation can produce much richer patterns than continuous time model. In ecology host-parasitoid models can be formulated as discrete time models. Jing et all studied the dynamics of corresponding discrete models obtained by Euler method Jing $etal$ (2004,2006).The modification of Bedding ton Free Lawton model of parasite host dynamics was investigated in Ivanchikov and Nedorezov(2011),Elsadamy(2102) discussed dynamical complexities in a discrete time food chain. Dance et al.(1997) studied discrete prey-predator model, when prey grows logistically..Now biological scientist have established many complex non-linear mathematical models to account for the dynamics behavior of the interaction (kaitala and Heno1996, kaitala et al 1999). Pable.A. *et.al.*(2013) found that alle threshold in the two dimensional system is given as the boundary of the basin of attraction of an attracting positive equilibrium.

For different parameters and initial conditions we can iterate the difference equation for thousand time steps and analyze the time population to elucidate the regularity and mechanism that hidden behind the population dynamics. All these researches relied on logistic growth function to analyze the dynamics of the host parasitoid interaction and obtained some intriguing results. In this work, we study dynamics of host parasitoid model with Allee effect for the host parasitoid aggregation which natural death of parasite in the absence of host include.

II. THE MODEL

We assume that growth of host population without parasitoid follows Moran-Ricker dynamics (Moran 1950, Ricker 1954) which is given by

$$H_{t+1} = H_t e^{r(1-\frac{H_t}{K})} \tag{1}$$

Where H_t stands for the population size at time t, λ is the intrinsic growth rate and K is the carrying capacity of the environment. Hau Liu (2009) discussed the dynamics of host parasitoid model with Allee effect for the host and parasitoid aggregation are given by

$$H_{t+1} = H_t e^{r(1-\frac{H_t}{K})(H_t-c)\big/ H_t+m} \quad , \tag{2}$$

$$P_{t+1} = H_t[1-(1+\frac{aP_t}{k})^{-k}] \tag{3}$$

Now the model is described as in the system of host parasitoid (since natural death of the host in the absence of host) and is given by

$$H_{t+1} = H_t e^{r(1-\frac{H_t}{K})(H_t-c)\Big/ H_t+m} (1+\frac{aP_t}{k})^{-k} \qquad (4)$$

$$P_{t+1} = H_t[1-(1+\frac{aP_t}{k})^{-k}]-dP_t \qquad (5)$$

Where c is the lower bound for the host and m can be defined as Allee effect constant, k is clumping factor and d is death rate of the host in the absence of the host

Here we attempt to analyze stabilizing and destabilizing effects of allee effect of host and clumping effect of parasitoid in terms of the lower bound c the searching efficiency a, intrinsic growth rate λ, and clumping degree k

III. FIXED POINT AND LOCAL STABILITY

We now study the existence of fixed points of the system (4) and(5),particularly we are interested in the interior fixed point to begin and we list all possible fixed points.

i) $E_0 = (0,0)$ is trivial fixed point

ii) $E_1 = (H^\bullet, P^\bullet)$ is the interior fixed point

where

$$H^\bullet = \frac{Mk(M^{1/K}-1)}{a(M-1)}(1+d) \qquad (6)$$

and

$$P^\bullet = \frac{k(M^{1/k}-1)}{a} \qquad (7)$$

Where $M = e^{r(1-\frac{H}{K})(H-c)\Big/ H+m}$ (8)

M is net rate of the increase in the host per generation,

Note that the equilibrium point cannot be solved in a closed form.

Theorem: Let $p(\lambda) = \lambda^3 + B\lambda^2 + C\lambda + D$ be the roots of $p(\lambda) = 0$. then the following statements are true

a) If every root of the equation has absolute value less than one, then the fixed point of the System is locally asymptotically stable and fixed point is called a sink.

b) If at least one of the roots of equation has absolute value greater than one then the fixed point of the system is unstable and fixed point is called saddle.

c) If every root of the equation has absolute value greater than one then the system is a source.

d) The fixed point of the system is called hyperbolic if no root of the equation has absolute value equal to one, if there exists a root of equation with absolute value equal to one then the fixed point is called non-hyperbolic.

IV. DYNAMIC BEHAVIOR OF THE MODEL

In this section we investigate the local behavior of the model (4) and ,(5) around each fixed point. The local stability analysis of the model (4) and ,(5) can be studied by computing the variation matrix corresponding to each fixed point . The variation matrix of the model at state variable is given by

$$H(t+1) = F_1(H_1, P_1)$$
$$P(t+1) = F_2(H_2, P_2)$$

For which the Jacobian matrix is given by

$$J(H,P)=\begin{pmatrix} \frac{\partial F_1}{\partial H} & \frac{\partial F_1}{\partial P} \\ \frac{\partial F_2}{\partial H} & \frac{\partial F_2}{\partial P} \end{pmatrix}$$

Where

$$\frac{\partial F_1}{\partial H}=(1+\frac{aP}{k})^{-k}M[1+H^S] \qquad \text{Here} \qquad S=(\frac{r(m+c+\frac{mc}{k}-\frac{H^2}{k}-\frac{2mH}{k}}{(H+m)^2}]$$

$$\frac{\partial F_1}{\partial P}=-aMH(1+\frac{aP}{k})^{-(k+1)}$$

$$\frac{\partial F_2}{\partial H}=1-(1+\frac{aP}{k})^{-k}$$

$$\frac{\partial F_2}{\partial P}=aH(1+\frac{aP}{k})^{-(k+1)}-d$$

Proposition 1 The fixed point E_0 is locally asymptotically stable if and only if $|\lambda_1|<1$

Proof:

In order to prove the result we estimate the eigenvalues of the jacobian matrix J at E_0 is given by $J(E_0)=\begin{pmatrix} e^{-cr/m} & 0 \\ 0 & 0 \end{pmatrix}$

Hence the eigen values of the J at E_0 are $\lambda_1=e^{-cr/m}$ and $\lambda_2=0$

Thus it is stable node if $|\lambda_1|<1$

Proposition 2 The fixed point E_1 stable if satisfy the condition $|\lambda_{1,2}|<1$.

Proof:

In order to prove the result consider the matrix' $A=\begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}$

Where $G_{11}=\frac{\partial F_1}{\partial H}|_{E_1}= \; 1+\frac{M^2k(M^{1/k}-1)(1+d)}{Mk(M^{1/k}-1)(1+d)+ma(M-1)}[r+\frac{rc}{K}-\frac{2rMk(M-1)}{aK(M-1)}-\ln M]$

$$G_{12}=\frac{\partial F_1}{\partial P}|_{E_1}=\frac{Mk(1-M^{-1/K})}{1-M}(1+d)$$

$$G_{21}=\frac{\partial F_2}{\partial H}|_{E_1}=1-\frac{1}{M}$$

$$G_{22}=\frac{\partial F_2}{\partial P}|_{E_1}=\frac{k(1-M^{-1/k})}{M-1}-d$$

The characteristic equation is $|A-\lambda I|=0$

$$\Rightarrow\begin{pmatrix} G_{11}-\lambda & G_{12} \\ G_{21} & G_{22}-\lambda \end{pmatrix}=0$$

This may be rewritten in the form

$$\lambda^2 - B\lambda + C = 0 \tag{9}$$

Where B= Trace of A = $G_{11} + G_{22}$

C= |A|= $G_{11}.G_{22} - G_{12}.G_{21}$

The roots of the equation (9) are

$$\lambda_{1,2} = \frac{B \pm \sqrt{B^2 - 4C}}{2}$$

Both the Eigen values are real for λ_R and $|\lambda_{1,2}| < 1$ if

$$B^2 - 4C > 0 \text{ and } -1 < \frac{B \pm \sqrt{B^2 - 4C}}{2} < 1$$

Which yields $4C < B^2 < 4C + 4$ (10)

The Eigen values $|\lambda_{1,2}| < 1$ become complex and are inside the unit circle(λ_c) in the complex plane for

$$B^2 - 4C < 0 \;\; B^2 + (4C - B^2) < 4$$

Which yields $B^2 < 4C < 4$ (11)

If the conditions (10) and (11) are satisfied then the positive equilibrium point is stable.

V. CONCLUSION AND DISCUSSION

In this paper, we arrived a new discrete-time host–parasitoid model, in which both Allee effect for the host and aggregated parasitoid are simultaneously incorporated. Generally speaking, our model is more biological reasonable than some previous host–parasitoid models. Since the strong non-linearity, we can hardly obtain any meaningful information about the stability of the equilibrium from mathematical analysis. In Particular, we can gain some basic imagination about the role of Allee effect on host–parasitoid system. Now, we will give a short discussion based on the conclusions obtained from this study. Our first conclusion is about the relationship between dynamical complexity and Allee effect. First, we have to admit that the host–parasitoid model can exhibit many kind of complex dynamics. Therefore, Allee effect can be considered as one stabilizing effect to some extent.

The properties of self similarity and fractal basin boundaries of the basins of attraction were found in many other models (Kaitala and Heino, 1996; Kaitala et al., 1999; Tang and Chen, 2002; Xu and Boyce, 2005) except host–parasitoid model with Allee effect. A fractal basin of attraction implies the dynamics of the host–parasitoid system will change alternately among different attractors, when a small external perturbation is induced. Generally, the dynamics predicted from these mathematical models is usually very intrigued, while the dynamic behavior of real data is much simpler. However, our deterministic model, where Allee effect is added, can produce simpler dynamics. Our result, although cannot fully solve the discrepancy, can at least strengthen the utility of mathematical models in exploring populations. Dynamic complexities are the common characteristics in a variety of population models. Identifying the complex dynamics in natural population data has remains a major challenge in ecological studies. Our finding suggests that dynamic complexities might be eliminated through some balancing efforts in the nature, that is the reason why the data from the natural population looks simpler.We were also discussed various mathematical analysis in the concern.

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